Questions

For CRT - 21

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Max. Marks : 40
Time : 60 Minutes

Topics : **Vector Algebra**

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Q01. **(a)** If $\vec{a}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=6\hat{i}-\lambda\hat{j}+9\hat{k}$ and $\vec{a}\parallel\vec{b}$, find the value of λ.

(b) Find a unit vector in the direction perpendicular to the vectors $\hat{j}-2\hat{i}+\hat{k}$ and $2\hat{i}-\hat{j}+2\hat{k}$.

(c) Check if the points $A(-2\hat{i}+3\hat{j}+5\hat{k})$, $B(\hat{i}+2\hat{j}+3\hat{k})$ and $C(7\hat{i}-\hat{k})$ are collinear.

(d) If $\overrightarrow{PO}+\overrightarrow{OQ}=\overrightarrow{QO}+\overrightarrow{OR}$, then show that the points P, Q, R are collinear. $[1\times4=4$

Q02. **(a)** If a unit vector $\vec{p}$ makes the angle $\pi/4$ with $\hat{i}$, $\pi/3$ with $\hat{k}$ and ω, $(0<\omega<\pi/2)$ with $\hat{j}$ then, find the value of angle ω. Hence, find $\vec{p}$.

(b) *If $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{c}=3\hat{i}-\hat{j}+2\hat{k}$ then, find $[\vec{a}+\vec{b}\quad\vec{b}+\vec{c}\quad\vec{c}+\vec{a}]$.

(c) Find $|\vec{a}|$ and $|\vec{b}|$, if $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=8$ and $|\vec{a}|=8|\vec{b}|$.

(d) Show that there exists a real value of λ, so that the vectors given by $\vec{a}=2\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}-\lambda\hat{j}+\hat{k}$ and $\vec{c}=5\hat{i}+\hat{j}-3\hat{k}$ are coplanar. $[1\times4=4$

Q03. The position vectors of P, Q, R and S are $\hat{i}-3\hat{j}+\hat{k}$, $2\hat{i}+\hat{j}$, $3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ respectively. Prove that the lines PQ and RS are parallel and the ratio of their lengths is 1 : 2.

Q04. Using vectors, prove that the line segment joining the mid points of two sides of a triangle is parallel to the third side and equal to half of it.

Q05. Find the value of m such that the scalar product of vector $\hat{i}+\hat{j}+\hat{k}$ with the unit vector parallel to the sum of the vectors $m\hat{i}+2\hat{j}+3\hat{k}$ and $2\hat{i}-m\hat{j}-5\hat{k}$ is equal to 1/2.

Q06. If the vertices A, B and C of a Δ ABC are (1, 2, 3), (–1, 0, 0) and (0, 1, 2) respectively, then show that $\angle ABC+\angle CAB+\angle BCA=\pi$.

Q07. **(a)** Find the magnitude of sum of two unit vectors if it is known that their difference is also a unit vector.

(b) If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, show that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$.

Q08. If $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}$, $\vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$ then, find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ satisfying the condition $\vec{c}.\vec{d}=15$.

Q09. If $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+5\hat{j}$, $3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ are the position vectors of A, B, C and D respectively, then find the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$. Deduce that the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.

OR

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}.\vec{b}=\vec{a}.\vec{c}=0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\pi/6$, then prove that (i) $\vec{a}=\pm2(\vec{b}\times\vec{c})$ (ii) $*\left[\vec{a}+\vec{b}\quad\vec{b}+\vec{c}\quad\vec{c}+\vec{a}\right]=\pm1$.

Q10. If A, B and C are non-collinear points with position vectors $\vec{a}$, $\vec{b}$ and, $\vec{c}$ respectively.

Show that the length of perpendicular drawn from A on BC is $\dfrac{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}{|\vec{c}-\vec{b}|}$.

OR If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors of equal magnitude, prove that $\vec{a}+\vec{b}+\vec{c}$ is equally inclined with vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$. Also find the angle. $[4\times10=40$

 # Offline Test held @ THE O.P. GUPTA CLASSES in 2019-20. Sharing again, for the benefit of students in 2020-21.
 # The Questions/topics which are deleted by CBSE have been marked with *. If any error is noticed, pls inform us via WhatsApp.

Hints & Answers

Q01. **(a)** 3.

(b) $\frac{1}{\sqrt{5}}[\hat{i}+2\hat{j}]$.

(c) Show that $\overrightarrow{AB}=\lambda\overrightarrow{BC}$ which shall imply that $\overrightarrow{AB}\,||\,\overrightarrow{BC}$ but as B is a common point. So A, B, C are collinear points.

Q02. **(a)** $\omega=\frac{\pi}{3}$, $\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{2}\hat{j}+\frac{1}{2}\hat{k}$.

(b) -14.

(c) $\frac{16}{3}\sqrt{\frac{2}{7}}, \frac{2}{3}\sqrt{\frac{2}{7}}$.

Q03. See **Mathematicia** by **O.P. Gupta**.

Q04. See **Mathematicia** by **O.P. Gupta**.

Q05. $\pm\sqrt{2}$.

Q07. **(a)** $\sqrt{3}$.

Q08. $\frac{160}{3}\hat{i}-\frac{5}{3}\hat{j}-\frac{70}{3}\hat{k}$.

Q09. Required angle is π.

OR See **Mathematicia** by **O.P. Gupta**.

Q10. See **Mathematicia** by **O.P. Gupta**.

OR See **Mathematicia** by **O.P. Gupta**. Ans. $\cos^{-1}\frac{1}{\sqrt{3}}$.
