

Questions

For CRT - 18

By O.P. GUPTA

INDIRA AWARD WINNER

M.+91 9650350480

Max. Marks : 40
Time : 60 Minutes

Topics : Relations & Functions

■ The O.P. GUPTA Classes, 1st Floor, Opp. HP Petrol Pump, Thana Road, Najafgarh, New Delhi.

- Q01.** Determine whether the relation R which is defined on the set \mathbb{R} of real nos. as $R = \{(a, b) : a \leq b^2\}$ is reflexive or symmetric or transitive.
- Q02.** Let L be the set of all lines in a plane and $R : L \rightarrow L$ such that $R = \{(L_1, L_2) : L_1 \text{ is perpendicular to } L_2\}$. Examine if the relation R is reflexive, symmetric, transitive and equivalence relation or not.
- Q03.** Show that the relation R defined by $(a, b)R(c, d) \Rightarrow a + d = b + c$ on $A \times A$, where set $A = \{1, 2, 3, \dots, 10\}$ is an equivalence relation. Hence write the equivalence class $[(3, 4)]$.
- Q04.** **Let the function $f : \mathbb{R}_+ \rightarrow [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$. Show that $f(x)$ is invertible and hence, find $f^{-1}(0)$, if possible. Here \mathbb{R}_+ denotes the set of all non-negative real numbers.
- Q05.** Let $A = \mathbb{R} - \{1\}$ and $B = \mathbb{R} - \{2\}$. If $f : A \rightarrow B$ is a function defined by $f(x) = \frac{x-2}{x-1}$, show that the function f is one-one and onto.
- Q06.** (a) Let R be the equivalence relation in the set \mathbb{Z} of integers, which is given by $R = \{(a, b) : (b - a) \text{ is even}\}$. Write the equivalence class $[0]$.
- (b) *Find $f^{-1}(x)$, if $f(x) = [4 - (x - 7)^3]^{\frac{1}{4}}$.
- Q07.** A relation R is defined from $\{2, 3, 4, 5\}$ to $\{3, 6, 7, 10\}$ by $x R y \Leftrightarrow x$ is relatively prime to y , then write the domain of R .
Also state if R is reflexive relation or not? Is it transitive relation? Is R a symmetric relation?
- Q08.** Show that the relation R in the set A of points in a plane given by $R = \{(P, Q) : \text{Distance of the point } P \text{ from the origin is same as the distance of point } Q \text{ from the origin}\}$, is an equivalence relation.
- Q09.** If R be the relation defined on \mathbb{Q} (set of rational numbers) as $aRb \Leftrightarrow |a - b| \leq \frac{1}{2}$, then show that R is not an equivalence relation.
OR Show that the relation R on the set \mathbb{R} of real nos., defined as $R = \{(a, b) : a \leq b^3; a, b \in \mathbb{R}\}$ is neither reflexive nor symmetric nor transitive.
- Q10.** *If $f, g : \mathbb{R} \rightarrow \mathbb{R}$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x, \forall x \in \mathbb{R}$.
Then find $f \circ g$ and $g \circ f$. Hence, find $f \circ g(-3)$, $f \circ g(5)$ and $g \circ f(-2)$. [4×10 = 40]

** Be informed that, if a function is to be shown one-one and onto then, it is in syllabus for 2020-21. But finding the inverse of a function isn't in the syllabus. As in Q04 (above), if they ask you to "show that the function $f(x)$ is one-one and onto" - then it is perfectly in the syllabus. But if CBSE says 'show that $f(x)$ is invertible' - then it is not in syllabus.

Moreover, "finding $f \circ g$, $g \circ f$ etc. i.e., composite functions" are not in syllabus - if you see Ch 01 (NCERT). But the 'derivative of composite functions' is in syllabus (Ch 05 NCERT).

You may follow my Lectures on YouTube at the playlists -

- (1) https://www.youtube.com/playlist?list=PL9EngnKZlrSfTiShwX8ZVNfNfHqSHuE0_
- (2) <https://www.youtube.com/playlist?list=PL9EngnKZlrSfPUd4OwhwXhqwyRJ2wqh1g>

Hints & Answers

Dear students, this time we are not providing Answers/ Hints/ Solutions for a few Sums of this test. It is to ensure that you do **Practice from the books - Mathematicia by O.P. Gupta.**

Q01. See **Mathematicia** by **O.P. Gupta**. Ans. R is neither reflexive nor symmetric nor transitive relation.

Q02. See **Mathematicia** by **O.P. Gupta**.

Q03. See **Mathematicia** by **O.P. Gupta**.

Q04. $f^{-1}(x) = \frac{\sqrt{5x+54}-3}{5}$ and $f^{-1}(0) = \frac{3}{5}[\sqrt{6}-1]$.

Q05. See **Mathematicia** by **O.P. Gupta**.

Q06. (a) $[0] = \{0, \pm 2, \pm 4, \dots\}$.

(b) $f^{-1}(x) = \sqrt[3]{4-x^4} + 7$.

Q07. Here $(x, y) \in R$ implies that the factor of x and y must be 1.

Let $A = \{2, 3, 4, 5\}$ and $B = \{3, 6, 7, 10\}$. Given that $R : A \rightarrow B$.

So, $R = \{(2,3), (2,7), (3,7), (3,10), (4,3), (4,7), (5,3), (5,7)\}$.

Hence, domain of $R = \{2, 3, 4, 5\}$.

Note that $(2,2) \notin R$ though $2 \in A$. So, R is not reflexive.

Moreover, $(2,3) \in R$ and $(3,10) \in R$ but $(2,10) \notin R$. So, R is not transitive.

As $(2,3) \in R$ but $(3,2) \notin R$. So, R is not symmetric.

#Note : Two integers a and b are said to be relatively prime (or, mutually prime or coprime), if the only positive integer that divides both of them is 1.

Q08. See Example 03 of MATHEMATICIA by O.P. GUPTA.

Q09. See Example 06 of MATHEMATICIA by O.P. GUPTA.

OR See a **Similar Question**, Q27 (Exercise 13 A) of MATHEMATICIA by O.P. GUPTA.

Q10. See Example 02 of MATHEMATICIA by O.P. GUPTA.

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