Questions

For CRT - 17

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Max. Marks : 40
Time : 60 Minutes

Topics : **Differential Equations**

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Q01. **(a)** Write the order and degree (if defined) of $t = y\frac{dy}{dt} + \sqrt{a^2\left(\frac{dy}{dt}\right)^2 + b^2}$.

(b) Write the product of the degree and order of $\frac{d}{dt}\left\{1+\left(\frac{dx}{dt}\right)^2\right\} = \frac{d}{dt}\left\{\frac{d^2x}{dt^2}\right\}$. [1×2 = 2

Q02. **(a)** *Form the differential equation corresponding to the family of curves $y^2 = a(b-x)(b+x)$.

(b) *What is the differential equation corresponding to the family of curves $y = k(x-k)^2$?

(c) Find the integrating factor of $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1,\ x \neq 0$. [2×3 = 6

Q03. Solve : $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$.

Q04. *Form the differential equation of all circles which touch the x-axis at the origin. [4×2 = 8

Q05. Show that $(x-y)dy = (x+2y)dx$ is a homogeneous differential equation.
Also find the general solution of the given differential equation.

Q06. Solve : $(x+y+1)dx - (2x+2y+1)dy = 0$.

OR Solve : $\frac{dy}{dx} + y\cot x = 2x + x^2\cot x,\ x \neq 0$.

Q07. Show that $x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) + x - y\sin\left(\frac{y}{x}\right) = 0$ is homogeneous and hence, solve it.

Q08. Find the particular solution of differential equation : $y\sin x\frac{dy}{dx} = \cos x\left(\sin x - \frac{y^2}{2}\right)$, $y\left(\frac{\pi}{2}\right) = 1$.

[6×4 = 24

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https://www.youtube.com/playlist?list=PL9EngnKZlrSfPwPH1bbquhmvF03YC9vn5

Hints & Answers

Q01. **(a)** Order : 1, Degree : Not defined.

(b) $\dfrac{d}{dt}\left\{1+\left(\dfrac{dx}{dt}\right)^2\right\}=\dfrac{d}{dt}\left\{\dfrac{d^2x}{dt^2}\right\}$ $\Rightarrow 2\dfrac{dx}{dt}\times\dfrac{d}{dt}\left(\dfrac{dx}{dt}\right)=\dfrac{d^3x}{dt^3}$ $\Rightarrow 2\left(\dfrac{dx}{dt}\right)\times\left(\dfrac{d^2x}{dt^2}\right)=\dfrac{d^3x}{dt^3}$

$\therefore$ Order $=3$, Degree $=1$. So, required product is $3\times1=3$.

Q02. **(a)** $x\left\{y\dfrac{d^2y}{dx^2}+\left(\dfrac{dy}{dx}\right)^2\right\}=y\dfrac{dy}{dx}$.

(b) See **Mathematicia** by **O.P. Gupta**.

(c) $e^{2\sqrt{x}}$.

Q03. Refer to the **Mathematicia by O.P. Gupta** (General Differential Eqs.).

Q04. $(x^2-y^2)\dfrac{dy}{dx}=2xy$.

Q05. $\log\left|x^2+xy+y^2\right|-2\sqrt{3}\tan^{-1}\left(\dfrac{2y+x}{\sqrt{3}\,x}\right)=+C$.

Q06. $6y=3x+\log\left|3x+3y+2\right|+C$.

OR $y\sin x=x^2\sin x+C$.

Q07. $x\dfrac{dy}{dx}\sin\left(\dfrac{y}{x}\right)+x-y\sin\left(\dfrac{y}{x}\right)=0$ $\Rightarrow\dfrac{dy}{dx}=\dfrac{y}{x}-\operatorname{cosec}\left(\dfrac{y}{x}\right)$

Consider $f(x,y)=\dfrac{y}{x}-\operatorname{cosec}\left(\dfrac{y}{x}\right)$.

Now prove yourself that $f(x, y)$ is homogeneous.

Then to solve, put $y=vx$ $\Rightarrow\dfrac{dy}{dx}=v+x\dfrac{dv}{dx}$

$\therefore v+x\dfrac{dv}{dx}=\dfrac{vx}{x}-\operatorname{cosec}\left(\dfrac{vx}{x}\right)$ $\Rightarrow x\dfrac{dv}{dx}=-\operatorname{cosec}\ v$ $\Rightarrow-\int\sin v\ dv=\int\dfrac{dx}{x}$

$\Rightarrow\cos v=\log|x|+C$ $\therefore\cos\left(\dfrac{y}{x}\right)=\log|x|+C$.

Q08. See **Mathematicia** by **O.P. Gupta** (Linear Differential Eqs.).