Questions

For CRT - 13

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Max. Marks : 40
Time : 60 Minutes

Topics : **Indefinite Integrals & Introduction to Definite Integrals**

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Q01. If $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx = (x+a)\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{kx} + C$, then what is the value of k? Justify your answer.

Q02. Evaluate : $\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

Q03. Find : $\int \frac{x^{1/2}}{1+x^{3/4}}dx$

Q04. Find : $\int \frac{dx}{\sin^2 x - \sin 2x}$ **OR** Find : $\int \frac{1}{\sin(x-a)\cos(x-b)}dx$.

Q05. Evaluate : $\int x^2 e^{2x} dx$.

Q06. Find : $\int_0^{\pi} e^{2x}.\sin\left(\frac{\pi}{4}+x\right)dx$.

Q07. Find : $\int_0^{\pi/2} \frac{\sin 2x}{\sin^4 x + \cos^4 x}dx$. [$4\times 7 = 28$

Q08. Evaluate : $\int [\sqrt{\tan x} - \sqrt{\cot x}]dx$. **OR** Find : $\int \frac{\sin x}{\cos^3 x + \sin^3 x}dx$.

Q09. Find : $\int \sqrt{1+x^4}\frac{\{\log(1+x^4) - 4\log x\}}{x^7}dx$ **OR** Find : $\int \frac{\cos^{-1}x - \sin^{-1}x}{\cos^{-1}x + \sin^{-1}x}dx$. [$6\times 2 = 12$

Watch my Lectures on YouTube in the Playlists -

(1) https://www.youtube.com/playlist?list=PL9EngnKZlrSdHKpVZXD_MB6G53YIjeQVS

(2) https://www.youtube.com/playlist?list=PL9EngnKZlrSdva0M6M3vwwALVoOfTtYkg

 # Offline Test held @ THE O.P. GUPTA CLASSES in 2019-20. Sharing again, for the benefit of students in 2020-21.
 # The Questions/topics which are deleted by CBSE have been marked with *. If any error is noticed, pls inform us via WhatsApp.

Hints & Answers

Q01. Refer to the **Mathematicia by O.P. Gupta** (Chapter 05 Type G).

Obtain $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx = (x+a)\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{ax} + C$.

On comparing with $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx = (x+a)\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{kx} + C$, we get : $k = a$.

Q02. See **Examples 11** in Chapter 06 of **O.P. Gupta's Mathematicia**. Ans. $\frac{\pi}{2ab}$.

Q03. Let $I = \int \frac{x^{1/2}}{1+x^{3/4}}dx \quad \Rightarrow I = \int \frac{x^{3/4}}{x^{1/4}(1+x^{3/4})}dx \quad \left[\text{Put } 1+x^{3/4} = t \Rightarrow \frac{dx}{x^{1/4}} = \frac{4}{3}dt\right.$

$\therefore I = \frac{4}{3}\int \frac{t-1}{t}dt \quad \Rightarrow I = \frac{4}{3}\int\left(1-\frac{1}{t}\right)dt \quad \Rightarrow I = \frac{4}{3}\left(t - \log|t|\right) + C$

$\Rightarrow I = \frac{4}{3}\left(1+x^{3/4} - \log\left|1+x^{3/4}\right|\right) + C$ or, $I = \frac{4}{3}\left(x^{3/4} - \log\left|1+x^{3/4}\right|\right) + \lambda$, where $C + \frac{4}{3} = \lambda$.

Q04. $\frac{1}{2}\log\left|\frac{\tan x - 2}{\tan x}\right| + C$ or, $\frac{1}{2}\log|1 - 2\cot x| + C$ **OR** $\frac{1}{\cos(a-b)}\log\left|\frac{\sin(x-a)}{\cos(x-b)}\right| + C$.

Q05. $\frac{e^{2x}}{4}\left[2x^2 - 2x + 1\right] + C$.

Q06. See **Examples 07** in Chapter 06 of **O.P. Gupta's Mathematicia**. Ans. $-\frac{1}{5\sqrt{2}}(e^{2\pi}+1)$

Q07. $\frac{\pi}{2}$.

Q08. Let $I = \int[\sqrt{\tan x} - \sqrt{\cot x}]dx = \int \frac{\sin x - \cos x}{\sqrt{\sin x \cos x}}dx$

Put $-\cos x - \sin x = y \Rightarrow (\sin x - \cos x)dx = dy$.

Also, $(-\cos x - \sin x)^2 = y^2 \Rightarrow 1 + 2\sin x\cos x = y^2 \Rightarrow \sin x \cos x = \frac{y^2-1}{2}$

$\therefore I = \int \frac{\sqrt{2}\,dy}{\sqrt{y^2-1}} = \sqrt{2}\log\left|y + \sqrt{y^2-1}\right| + C = \sqrt{2}\log\left|-\sin x - \cos x + \sqrt{\sin 2x}\right| + C$.

OR Let $I = \int \frac{\sin x}{\cos^3 x + \sin^3 x}dx = \int \frac{\tan x \sec^2 x}{1+\tan^3 x}dx \quad \left[\text{Put } \tan x = y \Rightarrow \sec^2 x d = dy\right.$

$\therefore I = \int \frac{y}{1+y^3}dy = \int \frac{y}{(1+y)(1-y+y^2)}dy$

Now use Partial Fraction to get the result.

Ans. $\frac{1}{6}\log\left|\tan^2 x - \tan x + 1\right| - \frac{1}{3}\log|\tan x + 1| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x - 1}{\sqrt{3}}\right) + C$

Alternatively, $I = \int \frac{\sin x}{\cos^3 x + \sin^3 x}dx = \int \frac{\sin x}{(\cos x + \sin x)(\cos^2 x - \cos x \sin x + \sin^2 x)}dx$

$$\Rightarrow I=\frac{1}{2}\int\frac{(\cos x+\sin x)+(\sin x-\cos x)}{(\cos x+\sin x)(1-\cos x\sin x)}dx$$

$$\Rightarrow I=\frac{1}{2}\int\frac{(\cos x+\sin x)}{(\cos x+\sin x)(1-\cos x\sin x)}dx+\frac{1}{2}\int\frac{(\sin x-\cos x)}{(\cos x+\sin x)(1-\cos x\sin x)}dx$$

$$\Rightarrow I=\frac{1}{2}\int\frac{\sec^2 x}{\sec^2 x-\tan x}dx+\frac{1}{2}\int\frac{(\sin x-\cos x)}{(\cos x+\sin x)(1-\cos x\sin x)}dx$$

$$\Rightarrow I=\frac{1}{2}\int\frac{\sec^2 x}{1+\tan^2 x-\tan x}dx+\frac{1}{2}\int\frac{(\sin x-\cos x)}{(\cos x+\sin x)(1-\cos x\sin x)}dx$$

In 1st integral, put $\tan x=y\Rightarrow\sec^2 x dx=dy$.

In 2nd integral, put $\cos x+\sin x=u\Rightarrow(\sin x-\cos x)dx=-du$.

Also, $(\cos x+\sin x)^2=u^2\Rightarrow\sin x\cos x=u^2-1$.

$$\Rightarrow I=\frac{1}{2}\int\frac{dy}{y^2-y+1}-\frac{1}{2}\int\frac{du}{u(2-u^2)}\qquad\Rightarrow I=\frac{1}{2}\int\frac{dy}{\left(y-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}-\frac{1}{2}\int\frac{u\,du}{u^2(2-u^2)}$$

Put $(2-u^2)=v\Rightarrow udu=-\frac{dv}{2}$ in the 2nd integral.

$$\therefore I=\frac{1}{2}\times\frac{2}{\sqrt{3}}\tan^{-1}\frac{2y-1}{\sqrt{3}}+\frac{1}{4}\int\frac{dv}{(2-v)v}\qquad\Rightarrow I=\frac{1}{\sqrt{3}}\tan^{-1}\frac{2\tan x-1}{\sqrt{3}}+\frac{1}{8}\int\left(\frac{1}{2-v}+\frac{1}{v}\right)dv$$

$$\Rightarrow I=\frac{1}{\sqrt{3}}\tan^{-1}\frac{2\tan x-1}{\sqrt{3}}+\frac{1}{8}\left(-\log|2-v|+\log|v|\right)+C$$

$$\Rightarrow I=\frac{1}{\sqrt{3}}\tan^{-1}\frac{2\tan x-1}{\sqrt{3}}+\frac{1}{8}\log\left|\frac{2-u^2}{u^2}\right|+C\ \Rightarrow I=\frac{1}{\sqrt{3}}\tan^{-1}\frac{2\tan x-1}{\sqrt{3}}+\frac{1}{8}\log\left|\frac{2-(\cos x+\sin x)^2}{(\cos x+\sin x)^2}\right|+C$$

$$\Rightarrow I=\frac{1}{\sqrt{3}}\tan^{-1}\frac{2\tan x-1}{\sqrt{3}}+\frac{1}{8}\log\left|\frac{1-\sin 2x}{1+\sin 2x}\right|+C.$$

Q09. $\frac{[x^4+1]^{3/2}}{9x^6}-\frac{[x^4+1]^{3/2}}{6x^6}\log\left(\frac{x^4+1}{x^4}\right)+C$ **OR** See **Mathematicia by O.P. Gupta (Type G)**.
