Questions For CRT - 07

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Max. Marks : **40**
Time : **60 Minutes**
Topics : **Continuity & Differentiability**

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Q01. **(a)** Discuss the continuity of $f(x) = x^2 + 2x - 3$.

(b) Discuss the continuity of $f(x) = \cos x$.

(c) At what points, the function $f(x) = \dfrac{1}{x-3}$ is discontinuous?

(d) What are the point (s) of non-differentiability for $f(x) = |x+3|$?

Q02. **(a)** Discuss the continuity of $f(x) = |x|$ at $x = 0$.

(b) Write the interval of discontinuity for $f(x) = \log x$.

Q03. Discuss the continuity of $f(x) = \begin{cases} \dfrac{\sin x}{x} + \cos x, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$ at $x = 0$.

OR Show that the function defined by $g(x) = [x]$ is discontinuous at all integral points.

Q04. Prove that the **greatest integer function** defined by $f(x) = [x]$ is not differentiable at $x = 3$.

Q05. For what value of λ, $f(x) = \begin{cases} \dfrac{\log(1+ax) - \log(1-bx)}{x}, & \text{if } x \neq 0 \\ \lambda, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$?

Q06. Prove that $f(x) = \begin{cases} \dfrac{x}{|x| + 2x^2}, & x \neq 0 \\ -1, & x = 0 \end{cases}$ remains discontinuous at $x = 0$.

Q07. Discuss the differentiability of $f(x) = |x|$. [4×7 = 28

Q08. Determine the value of a, so that the following function $f(x)$ is continuous at $x = 0$:

$$f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2}, & \text{when } x < 0 \\ a, & \text{when } x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}} - 4}, & \text{when } x > 0 \end{cases}.$$

Q09. Discuss the continuity of $f(x) = \begin{cases} \dfrac{e^{1/x} - 1}{e^{1/x} + 1}, & \text{if } x \neq 0 \\ -1, & \text{if } x = 0 \end{cases}$ at $x = 0$.

OR Show that $f(x) = \begin{cases} x \sin\left(\dfrac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$. [6×2 = 12

Watch my Lectures on YouTube in the Playlist -

https://www.youtube.com/playlist?list=PL9EngnKZlrSf7FJ611FY1zLXOwqb7vH9T

Offline Test held @ THE O.P. GUPTA CLASSES in 2019-20. Sharing again, for the benefit of students in 2020-21.

The Questions/topics which are deleted by CBSE have been marked with *. If any error is noticed, pls inform us via WhatsApp.

LEVEL - 1

Hints & Answers

Q01. **(a)** Note that the domain of the polynomial function is always $x \in R$. Also, every function is continuous at each point of its domain. Hence, $f(x)$ is continuous for all $x \in R$.

(b) As the domain of $f(x) = \cos x$ is $x \in R$, hence, $f(x)$ is continuous for all $x \in R$.

(c) At $x = 3$.

(d) $x = -3$.

Q02. **(a)** Continuous at $x = 0$.

(b) The function $f(x)$ is discontinuous at $x \in (-\infty, 0]$.

Q03. Discontinuous.

OR The given function g (x) is defined at all integral points.

Let n be an integer. Then g (n) = [n] = n.

LHL (at x = n) : $\lim\limits_{x \to n^-} g(x) = \lim\limits_{x \to n^-} [x] = (n-1)$.

RHL (at x = n) : $\lim\limits_{x \to n^+} g(x) = \lim\limits_{x \to n^+} [x] = n$.

Since LHL (at x = n) $\neq$ RHL (at x = n).

Therefore g is not continuous at x = n i.e., g (x) is discontinuous at all integral points.

Q04. We did a similar sum in the Class. Now, check your Notebook :-)

Q05. See Ex03 Page 97 in **Mathematicia.** Ans. $\lambda = a + b$.

Q06. Show that $\lim\limits_{x \to 0^-} f(x) \neq \lim\limits_{x \to 0^+} f(x)$.

Q07. Non-differentiable at $x = 0$.

Q08. See Ex. 3 C Q07 in **Mathematicia.** Ans. $a = 8$.

Q09. Discontinuous.

OR See a Similar Question in the Ex. 3 B Q02 in **Mathematicia.**
