Questions For CRT - 06

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Max. Marks : **40**
Time : **60 Minutes**
Topics : **Inverse Trigo. Functions (Complete)**

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Q01. **(a)** **Find the value of x, if $\tan^{-1} x - \cot^{-1} x = \pi$.

(b) **Find the value of $x(1-y^2)$, if $\sin^{-1}\left(\frac{2y}{1+y^2}\right) + \cos^{-1}\left(\frac{1-y^2}{1+y^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$.

(c) **Write the value of $\tan^{-1} x + \tan^{-1} y$, if $\cot^{-1} x + \cot^{-1} y = \frac{\pi}{5}$.

(d) Minimum value of x for which $\tan^{-1}\left(\frac{x}{\pi}\right) > \frac{\pi}{4}$, $x \in N$, is valid is 5. State True or False? Why?

(e) Find the value of $\sin^{-1} \sin 12 + \cos^{-1} \cos 12$. [$2 \times 5 = 10$

Q02. Simplify : $\cos^{-1}[x^{3/2} - \sqrt{1-x^2}\sqrt{1-x}]$.

Q03. Simplify : $\cos \tan^{-1} \cot \sin^{-1} x$.

Q04. Evaluate : $\sin\left(\cos^{-1}\frac{4}{5} + \cot^{-1}\frac{3}{2}\right)$. [$4 \times 3 = 12$

Q05. Find the values of x which satisfy the equation $\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$.

Q06. **Evaluate : $\cot\left\{\sum_{n=1}^{23} \cot^{-1}\left(1 + \sum_{k=1}^{n} 2k\right)\right\}$

OR Simplify : $\tan^{-1}\left(\frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}}\right)$, $x \in \left(\pi, \frac{3\pi}{2}\right)$.

Q07. Prove that : $\cot\left(\frac{\pi}{4} - 2\cot^{-1} 3\right) = 7$.

OR Show that : $\tan^{-1} x = 2\tan^{-1}\left[\operatorname{cosec} \tan^{-1} x - \tan \cot^{-1} x\right]$. [$6 \times 3 = 18$

** Note that, if you can think of any alternative method (which is possible) - then, these maybe asked in CBSE exams. Apart from that, since the derivative of Inverse Trig. Functions are in Syllabus. So, we believe you know how to simplify the inverse trig. functions using relevant properties. That's why CBSE may still ask questions from those topics which are deleted from Ch 2 but, needs to be taught in Ch 5 (just as an example).

Watch my Lectures on YouTube in the Playlist -

https://www.youtube.com/playlist?list=PL9EngnKZlrSf2oeKp_aAFlCmOetu8KMbL

Offline Test held @ THE O.P. GUPTA CLASSES in 2019-20. Sharing again, for the benefit of students in 2020-21.
The Questions/topics which are deleted by CBSE have been marked with *. If any error is noticed, pls inform us via WhatsApp.

LEVEL - 2

Hints & Answers

Q01. **(a)** $\tan^{-1}x-\left(\frac{\pi}{2}-\tan^{-1}x\right)=\pi$ $\Rightarrow 2\tan^{-1}x=\frac{3\pi}{2}$ $\Rightarrow \tan^{-1}x=\frac{3\pi}{4}$ $\Rightarrow x=\tan\frac{3\pi}{4}=-1$

But $x=-1$ doesn't satisfy the given equation. So, the equation has no solution.

Aliter, $\tan^{-1}x-\cot^{-1}x=\pi$ $\Rightarrow \tan^{-1}x-\tan^{-1}\frac{1}{x}=\pi$ $\Rightarrow \tan^{-1}\frac{x-\frac{1}{x}}{1+x.\frac{1}{x}}=\pi$

$\Rightarrow \frac{x^2-1}{2x}=\tan\pi=0$ $\Rightarrow x^2=1$ $\Rightarrow x=\pm1$

$\because x=\pm1$ doesn't satisfy the given equation. So, the equation has no solution.

(b) $2\tan^{-1}y+2\tan^{-1}y=2\tan^{-1}x$ $\Rightarrow \tan^{-1}\frac{2y}{1-y^2}=\tan^{-1}x$ $\therefore x(1-y^2)=2y$.

(c) $\cot^{-1}x+\cot^{-1}y=\frac{\pi}{5}$ $\Rightarrow \frac{\pi}{2}-\tan^{-1}x+\frac{\pi}{2}-\tan^{-1}y=\frac{\pi}{5}$ $\therefore \tan^{-1}x+\tan^{-1}y=\frac{4\pi}{5}$.

(d) False because, $\tan^{-1}\frac{x}{\pi}>\frac{\pi}{4}$ $\Rightarrow \frac{x}{\pi}>\tan\frac{\pi}{4}$ $\Rightarrow \frac{x}{\pi}>1$ $\Rightarrow x>\pi$

$\therefore x=4$ is the minimum value of x.

(e) $\sin^{-1}\sin 12+\cos^{-1}\cos 12=\sin^{-1}(-\sin(4\pi-12))+\cos^{-1}(\cos(4\pi-12))$

$\Rightarrow \quad =-\sin^{-1}(\sin(4\pi-12))+(4\pi-12)=-(4\pi-12)+(4\pi-12)=0.$

Q02. Let $y=\cos^{-1}[x\sqrt{x}-\sqrt{1-x^2}\sqrt{1-(\sqrt{x})^2}]$

Put $x=\cos\alpha,\ \sqrt{x}=\cos\beta$ $\Rightarrow \alpha=\cos^{-1}x,\ \beta=\cos^{-1}\sqrt{x}$

$\therefore y=\cos^{-1}[\cos\alpha\cos\beta-\sqrt{1-\cos^2\alpha}\sqrt{1-\cos^2\beta}]=\cos^{-1}[\cos\alpha\cos\beta-\sin\alpha\sin\beta]$

$\Rightarrow y=\cos^{-1}\cos(\alpha+\beta)=\alpha+\beta=\cos^{-1}x+\cos^{-1}\sqrt{x}$.

Q03. Let $y=\cos\tan^{-1}\cot\sin^{-1}x=\cos\tan^{-1}\cot\cot^{-1}\frac{\sqrt{1-x^2}}{x}=\cos\tan^{-1}\frac{\sqrt{1-x^2}}{x}$

$\Rightarrow y=\cos\cos^{-1}x=x$.

Q04. Let $y=\sin\left(\cos^{-1}\frac{4}{5}+\cot^{-1}\frac{3}{2}\right)=\sin\cos^{-1}\frac{4}{5}\cos\cot^{-1}\frac{3}{2}+\cos\cos^{-1}\frac{4}{5}\sin\cot^{-1}\frac{3}{2}$

$\Rightarrow y=\sin\sin^{-1}\frac{3}{5}\cos\cos^{-1}\frac{3}{\sqrt{13}}+\cos\cos^{-1}\frac{4}{5}\sin\sin^{-1}\frac{2}{\sqrt{13}}$

$\Rightarrow y=\frac{3}{5}\times\frac{3}{\sqrt{13}}+\frac{4}{5}\times\frac{2}{\sqrt{13}}=\frac{17}{5\sqrt{13}}$.

Q05. $\sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x$ $\Rightarrow \frac{\pi}{2}-\cos^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x$

$\Rightarrow \frac{\pi}{2}+\sin^{-1}(1-x)=2\cos^{-1}x$ $\Rightarrow \cos\left(\frac{\pi}{2}+\sin^{-1}(1-x)\right)=\cos\left(2\cos^{-1}x\right)$

$\Rightarrow -\sin\sin^{-1}(1-x)=2\cos^2(\cos^{-1}x)-1$ $\Rightarrow x-1=2x^2-1$ $\Rightarrow x-2x^2=0$ $\Rightarrow x(1-2x)=0$

Either $x=0,\ (1-2x)=0$ i.e., $x=0,\ x=1/2$ are the required solutions.

Q06. Let $y=\cot\left\{\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right\}$

$\Rightarrow y=\cot\left\{\sum_{n=1}^{23}\cot^{-1}\left(1+2+4+6+...+2n\right)\right\}$

$$\Rightarrow y=\cot\left\{\sum_{n=1}^{23}\cot^{-1}\left(1+2(1+2+3+...+n)\right)\right\}$$

$$\Rightarrow y=\cot\left\{\sum_{n=1}^{23}\cot^{-1}\left(1+2\times\frac{n(n+1)}{2}\right)\right\}$$

$$\Rightarrow y=\cot\left\{\sum_{n=1}^{23}\cot^{-1}\left(1+n(n+1)\right)\right\}$$

$$\Rightarrow y=\cot\left\{\sum_{n=1}^{23}\tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)\right\}$$

$$\Rightarrow y=\cot\left\{\sum_{n=1}^{23}\left[\tan^{-1}(n+1)-\tan^{-1}n\right]\right\}$$

$$\Rightarrow y=\cot\left[\tan^{-1}24-\tan^{-1}1\right]$$

$$\Rightarrow y=\cot\left[\tan^{-1}\frac{24-1}{1+24\times 1}\right]=\cot\cot^{-1}\frac{25}{23}=\frac{25}{23}.$$

OR Let $y=\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)=\tan^{-1}\left(\frac{\sqrt{2}\left|\cos\frac{x}{2}\right|+\sqrt{2}\left|\sin\frac{x}{2}\right|}{\sqrt{2}\left|\cos\frac{x}{2}\right|-\sqrt{2}\left|\sin\frac{x}{2}\right|}\right)$

$$\Rightarrow y=\tan^{-1}\left(\frac{\left|\cos\frac{x}{2}\right|+\left|\sin\frac{x}{2}\right|}{\left|\cos\frac{x}{2}\right|-\left|\sin\frac{x}{2}\right|}\right)\qquad\left[\because \pi<x<\frac{3\pi}{2}\quad\Rightarrow\frac{\pi}{2}<\frac{x}{2}<\frac{3\pi}{4}\right.$$

$$\Rightarrow y=\tan^{-1}\left(\frac{-\cos\frac{x}{2}+\sin\frac{x}{2}}{-\cos\frac{x}{2}-\sin\frac{x}{2}}\right)=\tan^{-1}\left(\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right)=\tan^{-1}\left(\frac{1-\tan\frac{x}{2}}{1+1.\tan\frac{x}{2}}\right)$$

$$\Rightarrow y=\tan^{-1}\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)=\frac{\pi}{4}-\frac{x}{2}.\qquad\left[\because -\frac{3\pi}{4}<-\frac{x}{2}<-\frac{\pi}{2}\quad\Rightarrow-\frac{\pi}{2}<\left(\frac{\pi}{4}-\frac{x}{2}\right)<-\frac{\pi}{4}\right.$$

Q07. LHS : $y=\cot\left(\frac{\pi}{4}-2\cot^{-1}3\right)$

$$\Rightarrow y=\tan\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-2\cot^{-1}3\right)\right)$$

$$\Rightarrow y=\tan\left(\frac{\pi}{4}+2\cot^{-1}3\right)$$

$$\Rightarrow y=\frac{\tan\frac{\pi}{4}+\tan\left(2\cot^{-1}3\right)}{1-\tan\frac{\pi}{4}\,.\,\tan\left(2\cot^{-1}3\right)}$$

$$\Rightarrow y=\frac{1+\tan\left(2\tan^{-1}\frac{1}{3}\right)}{1-1\,.\,\tan\left(2\tan^{-1}\frac{1}{3}\right)}$$

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$$\Rightarrow y = \frac{1 + \tan\left(\tan^{-1}\dfrac{\frac{2}{3}}{1-\frac{1}{9}}\right)}{1 - 1\,.\,\tan\left(\tan^{-1}\dfrac{6}{8}\right)}$$

$$\Rightarrow y = \frac{1+\frac{3}{4}}{1-\frac{3}{4}} = \frac{7/4}{1/4} = 7 = \text{RHS}.$$

OR See **Mathematicia by O.P. Gupta** Ex. 2D (Q011).