Questions For CRT - 03

By O.P. Gupta

Max. Marks : **40** INDIRA AWARD WINNER
Time : **60 Minutes** M.+91 9650350480

Topics : **Algebra Of Matrices & Determinants (Up to Ex.1 M)**

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Q01. **(a)** If A = $[a_{ij}]$ is a 3×3 matrix and A_{ij} denotes the co-factors of the corresponding elements a_{ij}'s then, what is the value of $a_{21}A_{11}+a_{22}A_{12}+a_{23}A_{13}$?

(b) If $A=\begin{pmatrix}2 & -3\\ 4 & -7\end{pmatrix}$, $B=\begin{pmatrix}-1 & 5\\ 2 & -8\end{pmatrix}$, then find $\left|A\,|B|+|A|\,B\right|$.

(c) If $x\in R, 0\le x\le\frac{\pi}{2}$, and $\begin{vmatrix}2\sin x & -1\\ 1 & \sin x\end{vmatrix}=\begin{vmatrix}3 & 0\\ -4 & \sin x\end{vmatrix}$, then find the values of x.

(d) If A = $[a_{ij}]$ is a matrix of order 2×2, such that |A| = –15 and C_{ij} represents the cofactor of a_{ij}, then find $a_{21}C_{21}+a_{22}C_{22}$.

(e) If A is skew symmetric matrix of order 3, then the value of det. A is …………….

(f) Value of $\begin{vmatrix}x+y & x & x\\ 5x+4y & 4x & 2x\\ 10x+8y & 8x & 3x\end{vmatrix}$ is x^3. True/False? [1×6 = 6

Q02. **(a)** Find the maximum and minimum values of det.A, where $A=\begin{pmatrix}1 & 1 & 1\\ 1 & 1+\sin\theta & 1\\ 1 & 1 & 1+\cos\theta\end{pmatrix}$.

(b) Without actually expanding, evaluate $\begin{vmatrix}0 & x-y & x-z\\ y-x & 0 & y-z\\ z-x & z-y & 0\end{vmatrix}$. [2×2 = 4

Q03. *Solve : $\begin{vmatrix}3x-8 & 3 & 3\\ 3 & 3x-8 & 3\\ 3 & 3 & 3x-8\end{vmatrix}=0$.

Q04. *Prove that $\begin{vmatrix}-a^2 & ab & ac\\ ba & -b^2 & bc\\ ac & bc & -c^2\end{vmatrix}$ is a perfect square.

Q05. *Using properties of determinants, prove that $\begin{vmatrix}1 & 1 & 1+3x\\ 1+3y & 1 & 1\\ 1 & 1+3z & 1\end{vmatrix}=9(3xyz+xy+yz+zx)$.

Q06. *Show that $\begin{vmatrix}Ax & x^2 & 1\\ By & y^2 & 1\\ Cz & z^2 & 1\end{vmatrix}=\begin{vmatrix}A & B & C\\ x & y & z\\ zy & zx & xy\end{vmatrix}$.

Q07. If $\begin{vmatrix}x & \sin\alpha & \cos\alpha\\ -\sin\alpha & -x & 1\\ \cos\alpha & 1 & x\end{vmatrix}=8$, then find x^2.

Q08. * $\begin{vmatrix}x & x^2 & 1+px^3\\ y & y^2 & 1+py^3\\ z & z^2 & 1+pz^3\end{vmatrix}=(1+pxyz)(x-y)(y-z)(z-x)$ [4×6 = 24

Q09. *Show that $\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}<0$, if $a,b,c\in R^+$ and $a\ne b\ne c$. [6×1 = 6

 # Offline Test held @ THE O.P. GUPTA CLASSES in 2019-20. Sharing again, for the benefit of students in 2020-21.
 # The Questions/topics which are deleted by CBSE have been marked with *. If any error is noticed, pls inform us via WhatsApp.

Hints & Answers

Q01. **(a)** $a_{21}A_{11}+a_{22}A_{12}+a_{23}A_{13}=a_{21}(a_{22}a_{33}-a_{32}a_{23})+a_{22}[-(a_{21}a_{33}-a_{31}a_{23})]+a_{23}(a_{21}a_{32}-a_{31}a_{22})=0$.

(b) $\left||A|B|+|A|B|\right|=\left|\begin{pmatrix}2 & -3\\4 & -7\end{pmatrix}(-2)+(-2)\begin{pmatrix}-1 & 5\\2 & -8\end{pmatrix}\right|=\left|-2\begin{pmatrix}1 & 2\\6 & -15\end{pmatrix}\right|=(-2)^2\times(-15-12)=-108$

(c) See **Mathematicia by O.P. Gupta** Exercise 1 K Q04. Ans. $x=\dfrac{\pi}{2},\dfrac{\pi}{6}$.

(d) As $A=[a_{ij}]_{2\times2}=\begin{bmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{bmatrix}$.

Consider $|A|=\begin{vmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{vmatrix}=a_{21}C_{21}+a_{22}C_{22}$ (if we expand along R_2

$\therefore |A|=a_{21}C_{21}+a_{22}C_{22}=-15$.

(e) 0, as the determinant of a skew symmetric matrix of odd order is zero.

(f) Let $\Delta=\begin{vmatrix}x+y & x & x\\5x+4y & 4x & 2x\\10x+8y & 8x & 3x\end{vmatrix}$

By $R_2\to R_2-4R_1,\ R_3\to R_3-8R_1$

$\Rightarrow\Delta=\begin{vmatrix}x+y & x & x\\x & 0 & -2x\\2x & 0 & -5x\end{vmatrix}$

Expanding along C_2,

$\Rightarrow\Delta=-x(-5x^2+4x^2)=x^3$.

Clearly, the given statement is true.

Note that, we may solve this det. By directly expanding. Be informed that Properties of determinants are deleted by CBSE for 2020-21.

Q02. **(a)** 1/2, −1/2

(b) See a **similar Example in Mathematicia by O.P. Gupta** (Example 04 Page 39).

Q03. $x=\dfrac{2}{3},\dfrac{11}{3}$ # Can you think of solving it without using properties of det.? I know it will be complex.

Q04. Show that $\Delta=4a^2b^2c^2$, which is $(2abc)^2$, and so Δ is perfect square.

Q05. See **Example 11** Page 43 **in Mathematicia by O.P. Gupta**.

Q06. Consider $\Delta_1=\begin{vmatrix}A & B & C\\x & y & z\\zy & zx & xy\end{vmatrix}$

Multiply C_1, C_2, C_3 by x, y, z respectively.

$\Rightarrow\Delta_1=\dfrac{1}{xyz}\begin{vmatrix}Ax & By & Cz\\x^2 & y^2 & z^2\\xyz & xyz & xyz\end{vmatrix}$

Taking xyz common from R_3.

$\Rightarrow\Delta_1=\dfrac{xyz}{xyz}\begin{vmatrix}Ax & By & Cz\\x^2 & y^2 & z^2\\1 & 1 & 1\end{vmatrix}$

Interchanging rows and columns

$$\Rightarrow \Delta_1 = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix}$$

$\therefore \Delta = \Delta_1$.

Can you think of solving it without using properties of det.? If so, then this question maybe asked in CBSE Exams.

Q07. See Exercise 1 L Q05 in **Mathematicia by O.P. Gupta**.

Q08. See Exercise 1 M in **Mathematicia by O.P. Gupta**.

Q09. See Exercise 1 M in **Mathematicia by O.P. Gupta**.
