CALCULUS

CONTINUITY

Note, this Assignment contains selected portion of **MATHEMATICIA By O.P. GUPTA** (Class 12).

(Issued in the interest of Students & Educators)

EXERCISE 3 A

VERY SHORT ANSWER TYPE QUESTIONS

Q01. Discuss the continuity of the function $f(x)=\frac{1}{\log_e x}$.

Q02. Write the interval in which $f(x)$ is continuous where $f(x)=e^x \log|x|$.

Q03. Write the interval in which $f(x)=\frac{\log x}{\sqrt{1-9x^2}}$ is continuous.

Q04. At what point $f(x)=|3x-5|$ is not differentiable?

Q05. Find 'a' if $f(x)=\begin{cases}2x-1, & \text{if } x<2\\ a, & \text{if } x=2\\ x+1, & \text{if } x>2\end{cases}$ is continuous at $x=2$.

Q06. Find the value of 'a' if $f(x)=\begin{cases}ax^2, & \text{if } x\le 2\\ 3, & \text{if } x>2\end{cases}$ is continuous at $x=2$.

Q07. If $f(x)=\begin{cases}\frac{x^2-25}{x-5}, & \text{if } x\neq 5\\ \lambda, & \text{if } x=5\end{cases}$ is continuous at $x=5$ then, find the value of λ.

Q08. At what point the **signum function** $f(x)=\begin{cases}\frac{|x|}{x}, & x\neq 0\\ 0, & x=0\end{cases}$ is discontinuous?

Q09. For what value of f (0), the function $f(x)=\frac{3x-\sin^{-1}x}{3x+\sin^{-1}x}$, $x\neq 0$ is continuous at each point in its domain?

EXERCISE 3 B

SHORT ANSWER TYPE QUESTIONS

Examine the continuity of the following functions at the indicated point (Q01 - Q04) :

Q01. $f(x)=\begin{cases}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}, & \text{if } x\neq 0\\ 1, & \text{if } x=0\end{cases}$ at $x=0$

Q02. $f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right), & \text{if } x\neq 0 \\ 0, & \text{if } x=0 \end{cases}$ at $x=0$

Q03. $f(x)=\begin{cases} \frac{x-|x|}{2}, & \text{if } x\neq 0 \\ 2, & \text{if } x=0 \end{cases}$ at $x=0$

Q04. $f(x)=\begin{cases} \frac{e^x-1}{\log(1+2x)}, & \text{if } x\neq 0 \\ 7, & \text{if } x=0 \end{cases}$ at $x=0$.

Q05. Find the value of k, if $f(x)=\begin{cases} \frac{1-\cos 4x}{8x^2}, & \text{if } x\neq 0 \\ k, & \text{if } x=0 \end{cases}$ is continuous at $x=0$.

Q06. Find 'a' and 'b' if $f(x)=\begin{cases} \frac{x-4}{|x-4|}+a, & \text{if } x<4 \\ a+b, & \text{if } x=4 \\ \frac{x-4}{|x-4|}+b, & \text{if } x>4 \end{cases}$ is continuous at $x=4$.

Q07. Check the continuity of $f(x)=\begin{cases} 3x-2, & \text{if } x\leq 0 \\ x+1, & \text{if } x>0 \end{cases}$ at $x=0$. Hence sketch the graph for f (x).

Q08. Find k, such that $f(x)=\begin{cases} \frac{x(3^x-1)}{1-\cos x}, & \text{if } x\neq 0 \\ \log k, & \text{if } x=0 \end{cases}$ is continuous at $x=0$.

Q09. Determine the value of k, such that $f(x)=\begin{cases} (x-1)\tan\frac{\pi x}{2}, & \text{if } x\neq 1 \\ k, & \text{if } x=1 \end{cases}$ is continuous at $x=1$.

EXERCISE 3 C

SHORT ANSWER TYPE QUESTIONS

Q01. If $f(x)=\begin{cases} x^2+ax+b, & \text{if } 0\leq x\leq 2 \\ 3x+2, & \text{if } 2\leq x\leq 4 \\ 2ax+5b, & \text{if } 4\leq x\leq 8 \end{cases}$ is a continuous function on the interval $[0,8]$ then, find the value of 'a' and 'b'.

Q02. Find the value of 'a' and 'b' so that $f(x)=\begin{cases} x+a\sqrt{2}\sin x, & 0\le x<\frac{\pi}{4} \\ 2x\cot x+b, & \frac{\pi}{4}\le x<\frac{\pi}{2} \\ a\cos 2x-b\sin x, & \frac{\pi}{2}\le x\le\pi \end{cases}$ is continuous on $[0,\pi]$.

Q03. Is f (x) continuous at $x=0$, where $f(x)=\begin{cases} \frac{\cos ax-\cos bx}{x^2}, & \text{when } x\neq 0 \\ \frac{b^2-a^2}{2}, & \text{when } x=0 \end{cases}$?

Q04. A function f (x) is defined as follows :

$$f(x)=\frac{\sin x}{x},\ \text{if } x\neq 0$$
$$=2,\ \text{if } x=0$$

Is f (x) continuous at $x=0$? If not, what should be the value of f (x) at $x=0$ so that f (x) becomes continuous at $x=0$?

Q05. Let $f(x)=\begin{cases} \frac{1-\cos 4x}{x^2}, & \text{when } x<0 \\ a, & \text{when } x=0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & \text{when } x>0 \end{cases}$. Determine the value of a; if possible so that the function $f(x)$ is continuous at $x=0$.

Q06. Prove that the function f given by $f(x)=|x-1|,\ x\in R$ is not differentiable at $x=1$.

Q07. Show that $f(x)=|x-3|,\ x\in R$, is continuous but not differentiable at $x=3$.

Q08. Discuss the differentiability of $f(x)=|x-1|+|x-2|$.

Q09. If $f(x)=\begin{cases} x^2+3x+a, & \text{if } x\le 1 \\ bx+2, & \text{if } x>1 \end{cases}$ is differentiable at $x=1$, find the value (s) of a and b.

Q10. Prove that the function f (x) defined by $f(x)=[x],\ 0<x<3$ is not differentiable at $x=1$ and $x=2$, where [.] denotes the greatest integer function.

Q11. The value (s) of p and q, for which the function f (x) given below is continuous for all x in R:

$$f(x)=\begin{cases} \frac{\sin(p+1)x+\sin x}{x}, & x<0 \\ q, & x=0 \\ \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}, & x>0 \end{cases}.$$

Q12. If $f(x)=\dfrac{\tan\left(\frac{\pi}{4}-x\right)}{\cot 2x}$ for $x\neq\dfrac{\pi}{4}$, find the value which can be assigned to $f(x)$ at $x=\dfrac{\pi}{4}$ so that the function $f(x)$ becomes continuous at every point in $\left[0,\dfrac{\pi}{2}\right]$.

Q13. Show that the function f given by $f(x)=\begin{cases}\dfrac{e^{1/x}-1}{e^{1/x}+1}, \text{ if } x\neq 0\\ -1, \text{ if } x=0\end{cases}$ is discontinuous at $x=0$.

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