Chapter 01
BASIC CONCEPTS Of SET THEORY

Without Mathematics, there's nothing you can do.
Everything around you is Mathematics.
Everything around you is numbers!

Overview

In this chapter, we shall learn

- ✓ Definition of sets, basic terminology and, their representation
- ✓ Types of sets
- ✓ Understanding intervals and their types
- ✓ Subset, Power set, Universal set
- ✓ Operations on sets
- ✓ Venn-diagram & application
- ✓ Properties of sets
- ✓ Application based real life problems

◘ INTRODUCTION

The theory of sets is extremely vital and fundamental part of modern day Mathematics. The concept of sets is widely used in the foundation of relations, functions, logic, sequences, theory of probability etc. In day to day life, we very often use the word *collections* or *aggregates of objects of some kinds*. The theory of sets was developed by a German Mathematician Georg Cantor (1845-1918 AD). The word *'set'* was first of all used by him. Cantor used the word *'set'* for a collection of objects of any kind. According to him, *"A set is any collection into a whole of definite and distinct objects of our intuition or thought."* By the objects being 'distinct', Cantor meant that *no object should be repeated in the collection* and by the objects being 'definite', he meant that *given an object we must be able to decide whether that object belongs to the collection or not*. By the term 'collection into a whole', he meant that *the objects should have certain properties*. But Cantor's definition of set was not acceptable to some mathematicians as there may be collections of objects such that objects of the collection do not satisfy some common properties.

Dear learners, I want to tell you that this chapter shall seem very easy to you initially but, it shall get a bit tricky as you proceed through this chapter! So you have to be little alert and give it more attention than you can afford. (How do we do so? Think!)

IMPORTANT TERMS & DEFINITIONS

01. Set :

A set is *well-defined* collection of objects of any kind of our intuition or thought which are distinct and distinguishable. By the word 'distinct' we mean that ***no object should be repeated*** and by the word distinguishable, we mean that ***the object of the collection must be known*** *i.e.*, given any object ***we must be able to decide whether that object belongs to the collection or not***. The objects of a set are taken as distinct only on the ground of simplicity. Generally *objects of a set have a common property* and the objects outside this collection *do not* have this property. The objects of the collection are called the ***elements or members of the set***.

e.g. Let A = {a, e, i, o, u}. Here the elements of A are distinguishable as well as distinct. Hence A is a set.

Let B = {x : x is an intelligent person of Delhi}. Here elements are not distinguishable because if we select any person of Delhi, we can't say with the certainty whether he belongs to B or not, as there is no standard scale for evaluation of intelligence. Hence B is not a set.

Let M = {x : x represents a natural number less than 10}. Clearly, M is a set as, the elements of M are distinguishable as well as distinct. Note that, M = {1, 2, 3, …, 9}.

☞ A set is represented by listing all its elements between the braces **{ }** and by separating them from each other by **commas** (if there are more than one element).
☞ Sets are **denoted by capital letters** of English alphabet *viz.* A, B, C, S, U, X, Y, Z etc., while the elements are in general denoted by small letters *viz.* a, b, x, y etc.
☞ If 'x' is an element of a set A then we write $x \in A$ (read as x belongs to A). Also if 'y' is not an element of set A then we write $y \notin A$ (read as y does not belong to A). The **symbol $\in$ is called the membership relation**.

02. Methods of representing a set :

(a) Tabular form or Roster form :
In this method of describing a set, all the elements of a set are listed separated by commas and are enclosed within braces { }.
e.g. The set of all positive odd integers lesser than 10 can be described by {1, 3, 5, 7, 9}.

☞ ***Note*** *that in roster form of a set an element is not generally repeated. Also the order in which the elements of a set are written is immaterial. Thus the set {1, 3, 5, 7, 9} and {3, 1, 9, 5, 7} are same.*
In fact, Repetition of elements and order of elements in roster form is immaterial.

(b) Set Builder form or Rule Method :
In set builder form, all the elements of a set posses a single common property. A variable x which stands for each element of the set is written inside braces and then after giving a colon ":" or oblique line "/", the property or properties p(x), possessed by each element of the set is written within the braces itself. In this description the braces stand for ***"the set of all"*** and the colon stands for ***"such that"***.
e.g. The set A = {1, 3, 5} is written as A = {x: $x \in N$, x is an odd number and $x \le 5$ } in the set builder form. It is read as "the set of all x such that x is a positive odd number less than or equal to 5".

03. Types of set :

(a) The Empty set (or Null set or Void set) :
A set which does not contain any element is called the empty set and it is denoted by ϕ.
e.g. Let $A = \{x : x \text{ is an even prime number greater than } 2\}$. Then A is an empty set.

☞ *Note that the set {0} is not an empty set as it contains one element 0.*

(b) Singleton set :
A set having single element is called a singleton set. It is represented by writing down the element within the braces.
e.g. Let A = {The set of present prime minister of India}. Then A is a singleton set.

(c) Finite set and Infinite set :
A set which consists of a finite (definite) number of elements is called finite set, otherwise the set is called infinite set.
e.g. Let $A = \{1, 2, 3, 4\}$ is a finite set and $B = \{1, 2, 3, ...\}$ is an infinite set.

☞ *Note that an empty set is a finite set as it has no element!*

◘ ***Cardinal number of a finite set :*** *The number of elements in a finite set A is called the cardinal number of set A and is denoted by n(A).* ***e.g.*** *Let A= {a, e, i, o, u} then, n(A) = 5.*

☞ **Cardinal number** of a set is also called its **order.**

(d) Equal sets and Equivalent sets :
Two sets A and B are said to be equal if they have exactly the same elements and we write A = B, i.e., sets A and B are equal if each element of A is an element of B and each element of B is an element of A. Otherwise the sets are said to be unequal and we write $A \neq B$. Also note that the order in which the elements in the two sets have been written down is immaterial.

e.g. Let $A = \{a, b, c, d, e\}$ and $B = \{c, d, a, b, e\}$, then $A = B$.

☞ *Note that two sets A and B are equal if* $x \in A \Rightarrow x \in B$ and $x \in B \Rightarrow x \in A$.

Two finite sets A and B are said to be equivalent if they have the same number of elements *i.e.* ***same cardinal number***. Thus sets A and B are equivalent iff $n(A) = n(B)$ and, we write $A \approx B$.

e.g. Let $A = \{a, e, i, o, u\}$ and $B = \{1, 2, 3, 4, 5\}$, then $n(A) = n(B) = 5$. Therefore, $A \approx B$.

☞ ***Note*** *that all the equal sets are equivalent but equivalent sets* ***may or may not*** *be equal.*

04. Subsets, Supersets and Proper subsets :
If every element of a set A is also an element of a set B, then **A is called a subset of B** (or **A is contained in B**) and we write $\mathbf{A \subseteq B}$. If there exist at least one element of A which does not belong to B, then A is not a subset of B and we write $\mathbf{A \not\subset B}$.
Thus $A \subseteq B \Leftrightarrow \{x \in A \Rightarrow x \in B\}$.

e.g. Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$. Then $A \subseteq B$.

<u>Superset of a set</u> The statement $A \subseteq B$ can also be expressed equivalently by writing $B \supseteq A$ (read as '**B is a superset of A**'). Also a set A is said to be a superset of set B, if B is a subset of A *i.e.,* each element of B is an element of A. If A is a superset of B, we write $A \supseteq B$.

e.g. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 5\}$. Here B is a subset of A, therefore, A is a superset of B.

<u>Proper subset of a set</u> A set A is said to be a proper subset of a set B if A is a subset of B and $A \neq B$ *i.e.,* if

(a) every element of A is an element of B and
(b) B has at least one element which is not an element of set A.

Thus $A \subset B$ or $B \supset A$ (read as '**A is a proper subset of B**'). Also if A is not a proper subset of B, then we write $A \not\subset B$.

e.g. Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$. Then $A \subset B$. Also let $A = \{1, 2, 3\}$ and $B = \{2, 3, 1\}$. Then $A \not\subset B$ as $A = B$.

◆ Note the followings :
(a) If $A \subseteq B$ and $B \subseteq A \Leftrightarrow A = B$.
(b) As the empty set ϕ has no elements, so we can say that ϕ is a subset of every set.
(c) Every set A is a subset of itself, i.e., $A \subset A$.
(d) If A and B are two sets such that $A \subset B$ and $A \neq B$, then A is called a proper subset of B and B is called superset of A.
(e) If $n(A) = n$, then number of subsets of A are 2^n.

◘ Ever wondered why a set having 'n' number of elements has a total of 2^n subsets? Let's see.

Let A be a given set having n number of elements.
Then, empty set i.e., ϕ is one of the subsets of A i.e, no. of subsets of A containing no element, is ${}^nC_0$.

No. of subsets of A containing 1 element, is $= {}^nC_1$.

No. of subsets of A containing 2 elements, are $= {}^nC_2$.

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No. of subsets of A containing n elements, are $= {}^{n}C_{n}$.

$\therefore$ Total no. of subsets of $A = {}^{n}C_{0} + {}^{n}C_{1} + {}^{n}C_{2} + ... + {}^{n}C_{n}$

$\Rightarrow \qquad = 2^{n}$. (Using Binomial Theorem, Ch-08)

05. Intervals as subset of real numbers :

Consider $a, b, x \in R$ and $a < b$. Here R represents the set of real numbers.

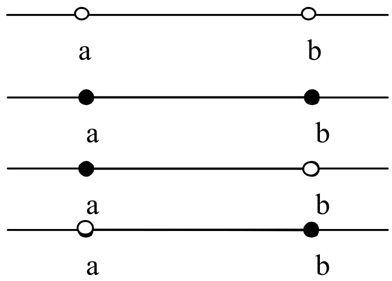
(a) Open interval : (a,b) or $]a,b[$ or $a < x < b$

(b) Closed interval : $[a,b]$ or $a \le x \le b$

(c) Semi-open and semi-closed intervals :

(i) $[a,b)$ or $[a,b[$ or $a \le x < b$

(ii) $(a,b]$ or $]a,b]$ or $a < x \le b$.



06. Power set :

The set or family of all the subsets of a given set A is called the power set of A and is denoted by P(A). Note that in P(A), every element is a set.

Thus symbolically, $P(A) = \{X : X \subseteq A\}$. Hence $X \in P(A) \Leftrightarrow X \subseteq A$.

Also, $\phi \in P(A)$ and $A \in P(A)$ for all set A.

e.g. Let $A = \{1, 2\}$ then $P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$.

☞ *Note that as the set A with m number of elements has a total of 2^{m} subsets so, P(A) has 2^{m} elements i.e., $n[P(A)] = 2^{m}$, where $m = n(A)$.*

07. Universal set :

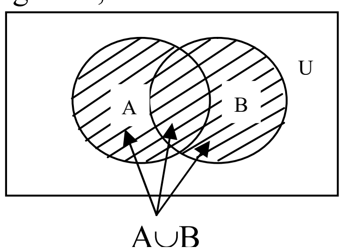
In any discussion in set theory we need a set such that all the sets under consideration in that discussion are its subsets. Such a set is called the universal set for that discussion. In other words, any set which is superset of all the sets under consideration is called the universal set and is **denoted by S or U**. A universal set can be chosen arbitrarily for any discussion of given sets, but once chosen, it is fixed for that discussion of the sets.

e.g. Let A = {1, 2, 3}, B = {3, 4, 6, 9} and C = {0, 1}. In this case we can take U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} as the universal set.

◘ Note that the universal set is not unique. For example, for the set of all integers **Z**, the universal set can be the set **Q** of rational numbers or the set of **R** real numbers.

08. Operation on sets :

(a) Union of sets : The union of two sets A and B is the set of all those elements which are either in set A or B or in both. This set is denoted by $A \cup B$. Thus $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

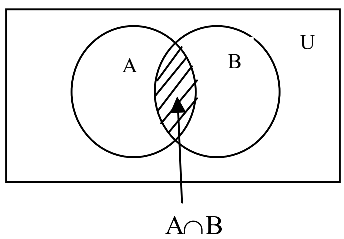


Clearly $x \in A \cup B \Leftrightarrow x \in A$ or $x \in B$.

e.g. Let A = {0, 1} and B = {2, 3} then $A \cup B$ = {0, 1, 2, 3}.

☞ *Note that if $A \subseteq B$ then $A \cup B = B$.*

(b) Intersection of sets : The intersection of set A and B is the set of all the elements which are common to both A and B. This set is denoted by $A \cap B$. Thus $A \cap B = \{x : x \in A \text{ and } x \in B\}$.



Clearly $x \in A \cap B \Leftrightarrow x \in A$ and $x \in B$.

e.g. Let A = {0, 1, 2, 3} and B = {2, 3, 4, 5} then $A \cap B$ = {2, 3}.

☞ Note the followings :

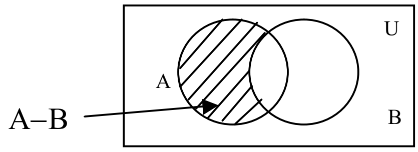
(i) If $A \subseteq B$ then $A \cap B = A$ and if $B \subseteq A$ then $A \cap B = B$.

(ii) $(A \cap B) \cup A = A$ and $(A \cap B) \cup B = B$.

(iii) $(A \cup B) \cap A = A$ and $(A \cup B) \cap B = B$.

*(iv) If $A \cap B = \phi$ then, A and B are **disjoint sets**.*

(c) Difference of sets : The difference of set A and B, in this order, is the set of all those elements of A which are not the elements of B. It is denoted by (A – B).



Thus $A - B = \{x : x \in A \text{ and } x \notin B\}$.

Clearly, $x \in A - B \Leftrightarrow x \in A \text{ and } x \notin B$.

❖ *Note that $A - B = A \cap B'$ i.e. $A - B = A \cap \overline{B}$.*

◘ Extension of **union of sets** and **intersection of sets** : Let $A_1, A_2, A_3, ..., A_n$ be n number of sets. Then, symbolically we may write union of these of sets as $\bigcup_{i=1}^{n} A_i$ i.e., $A_1 \cup A_2 \cup ... \cup A_n = \bigcup_{i=1}^{n} A_i$.

Similarly, their intersection can be written as $\bigcap_{i=1}^{n} A_i$ i.e., $A_1 \cap A_2 \cap ... \cap A_n = \bigcap_{i=1}^{n} A_i$.

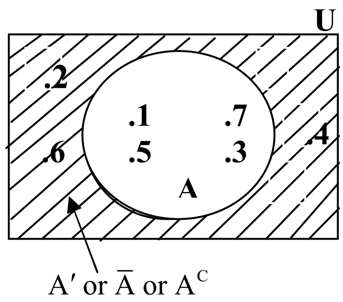
> **Venn diagram** is the pictorial representation of sets in which a set is represented by the region within a closed curve, usually circle or ellipse, inside the universal set. The universal set U is represented by a rectangular region. An element of a set *A* is represented by a point within the circle which represents *A*.

(d) Complement of a set :

Let U be the universal set and *A* is a subset of U. Then the complement of A (denoted by A' or A^C or $\overline{A}$) with respect to U is the set of all those elements of U which are not the elements of A.

Thus $A' = \{x : x \in U \text{ and } x \notin A\}$.

e.g. Let $U = \{1, 2, 3, 4, 5, 6, 7\}$ and $A = \{1, 3, 5, 7\}$ then, $A' = \{2, 4, 6\}$.



◆ Note the followings :

(i) $\phi' = U$ **(ii)** $U' = \phi$

(iii) $(A')' = A$ **(iv)** $A' = U - A$.

> ***Disjoint sets:*** *Two sets A and B are disjoint if $A \cap B = \phi$. That is, for disjoint sets A and B, there won't be any element which is present in both (i.e., there won't be any common element).*

☞ **Note :** If $A \cap B \neq \phi$ then, the sets A and B are said to be **overlapping sets** or **intersecting sets** or **not disjoint sets**.

09. Relations between sets for the Application based problems :

If sets are not disjoint, then we have following relations,

(a) $n(A \text{ or } B) = n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(b) $n(A \text{ and } B) = n(A \cap B) = n(A) + n(B) - n(A \cup B)$

(c) $n(A \text{ or } B \text{ or } C) = n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C)$
$- n(C \cap A) + n(A \cap B \cap C)$

10. Some Important Laws :

(a) Commutative laws

$A \cup B = B \cup A,\ A \cap B = B \cap A$

(b) Associative laws

$(A \cup B) \cup C = A \cup (B \cup C),\ (A \cap B) \cap C = A \cap (B \cap C)$

(c) Distributive laws

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C),\ A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(d) De Morgan's laws

$(A \cup B)' = A' \cap B'$, $(A \cap B)' = A' \cup B'$

(e) Idempotent laws

$A \cup A = A,\ A \cap A = A$

(f) Complement laws

$A \cup A' = U,\ A \cap A' = \phi$

(g) Identity laws

$A \cup \phi = A,\ A \cap U = A,\ A \cap \phi = \phi,\ A \cup U = U$

(h) Double complementation law (Involution law)

$(A')' = A$

(i) Laws of empty set and universal set

$\phi' = U,\ U' = \phi$.

For sets A and B, $(A - B) \cup (B - A)$ is called **symmetric difference** of sets A and B. It is denoted by $A \Delta B$.

11. Symbols and their meanings :

S. No.	**Symbol**	**Meaning**
01.	N	Set of natural numbers
02.	I or Z	Set of integers
03.	Q	Set of rational numbers
04.	T	Set of irrational numbers
05.	R	Set of real numbers
06.	C	Set of complex numbers
07.	$\in$	is an element of (or belongs to)
08.	$\notin$	is not an element of (or does not belong to)
09.	S or ξ or U	Universal set
10.	: or /	Such that
11.	ϕ	Empty set or Null set
12.	$\subseteq$	is subset of
13.	$\supseteq$	is superset of
14.	$\subset$	is proper subset of
15.	$\supset$	is proper superset of
16.	$\cup$	Union
17.	$\cap$	Intersection
18.	$\forall$	For all
19.	$\Rightarrow$	Implies
20.	$\Leftarrow$	is implied by

21.	$\Leftrightarrow$	Implies that and implied by
22.	ξ	Zai (Greek Letter)
23.	$\exists$	There exists
24.	ς	Zeta

WORKED OUT ILLUSTRATIVE EXAMPLES

Ex01. How many elements has P(A), if A = ϕ ?

Sol. Since $A = \phi$ so, number of elements in set A = 0.

Therefore, number of elements in P(A) = $2^0 = 1$.

Ex02. If A = {3, 5, 7, 9, 11}, B = {7, 9, 11, 13}, C = {11, 13, 15}and D = {15, 17}; find

(a) A ∩ (B ∪ D) (b) (A ∪ D) ∩ (B ∪ C).

Sol. Given A = {3, 5, 7, 9, 11}, B = {7, 9, 11, 13}, C = {11, 13, 15} and D = {15, 17}.

(a) We have $B \cup D = \{7,9,11,13,15,17\}$

So, $A \cap (B \cup D) = \{7,9,11\}$.

(b) We have $A \cup D = \{3,5,7,9,11,15,17\}$, $B \cup C = \{7,9,11,13,15\}$

So, $(A \cup D) \cap (B \cup C) = \{7,9,11,15\}$.

Ex03. Let A, B, and C be the sets such that A ∪ B = A ∪ C and A ∩ B = A ∩ C. Show that B = C.

Sol. Given $A \cup B = A \cup C$

$\Rightarrow \quad (A \cup B) \cap B = (A \cup C) \cap B$

$\Rightarrow \quad B = (A \cap B) \cup (C \cap B)$

$\Rightarrow \quad B = (A \cap B) \cup (B \cap C) \quad \ldots(i)$

Also, $A \cup B = A \cup C$

$\Rightarrow \quad (A \cup B) \cap C = (A \cup C) \cap C$

$\Rightarrow \quad (A \cap C) \cup (B \cap C) = C$

$\Rightarrow \quad (A \cap B) \cup (B \cap C) = C \quad \ldots(ii)$ [Given $A \cap B = A \cap C$

By (i) and (ii), we have: B = C.

Ex04. Show that if A ⊂ B, then C – B ⊂ C – A.

Sol. Given $A \subset B$. Let $x \in (C - B)$. Then,

$x \in (C - B) \Rightarrow x \in C$ and $x \notin B$

$\Rightarrow x \in C$ and $x \notin A$ [As $A \subset B$

$\Rightarrow x \in (C - A)$

$\therefore \quad (C - B) \subset (C - A)$.

Ex05. Assume that P (A) = P (B). Show that A = B.

Sol. Let $x \in A$. Then there is a subset X of set A such that $x \in X$.

Now $X \subset A \Rightarrow X \in P(A)$

$\Rightarrow X \in P(B)$ [$\because P(A) = P(B)$

$\Rightarrow X \subset B$

$\Rightarrow x \in B$. [$\because x \in X$ and $X \subset B \Rightarrow x \in B$

Thus, $x \in A \Rightarrow x \in B$.

$\therefore \quad A \subset B \quad \ldots(i)$

Similarly if $y \in B$. Then there is a subset Y of set B such that $y \in Y$.

Now $Y \subset B \Rightarrow Y \in P(B)$

$\Rightarrow Y \in P(A)$ [$\because P(A) = P(B)$

$\Rightarrow Y \subset A$

$\Rightarrow y \in A$. [$\because y \in Y$ and $Y \subset A \Rightarrow y \in A$

Thus, $y \in B \Rightarrow y \in A$.

$\therefore \quad B \subset A$ ...(ii)

By (i) & (ii), we have: A = B.

Ex06. Using properties of sets, show that

(i) $A \cup (A \cap B) = A$ (ii) $A \cap (A \cup B) = A$.

Sol. Assume that U denotes the universal set.

(i) $A \cup (A \cap B) = (A \cap U) \cup (A \cap B)$ [As $A \cap U = A$

$= A \cap (U \cup B)$ [By distributive law

$= A \cap U$ [As $U \cup B = U$

$\therefore \quad A \cup (A \cap B) = A$.

(ii) $A \cap (A \cup B) = (A \cup \phi) \cap (A \cup B)$ [As $A \cup \phi = A$

$= A \cup (\phi \cap B)$ [By distributive law

$= A \cup \phi$ [As $\phi \cap B = \phi$

$\therefore \quad A \cap (A \cup B) = A$.

Ex07. Let A and B be sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, show that A = B.

Sol. Given $A \cup X = B \cup X$ for some set X.

$A \cap (A \cup X) = A \cap (B \cup X)$

$\Rightarrow \quad A = (A \cap B) \cup (A \cap X)$

$\Rightarrow \quad A = (A \cap B) \cup \phi$ [Given $A \cap X = \phi$

$\Rightarrow \quad A = A \cap B$ ...(i)

Also $A \cup X = B \cup X$ for some set X.

$B \cap (A \cup X) = B \cap (B \cup X)$

$\Rightarrow \quad (B \cap A) \cup (B \cap X) = B$

$\Rightarrow \quad (B \cap A) \cup \phi = B$ [Given $B \cap X = \phi$

$\Rightarrow \quad A \cap B = B$ ...(i) [As $A \cap B = B \cap A$

By (i) and (ii), we get: A = B.

Ex08. In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee?

Sol. Let C and T denote the set of students taking coffee and tea respectively.

Then, n(T) = 150, n(C) = 225, $n(T \cap C) = 100$, $n(T \cup C) = ?$

Using $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, we get:

$n(T \cup C) = 150 + 225 - 100$

$\Rightarrow \quad n(T \cup C) = 275$.

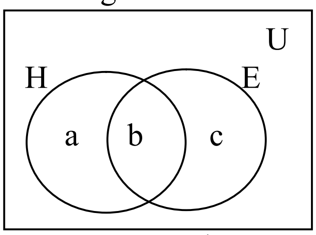
$\therefore \quad n(T \cup C)' = 600 - 275 = 325$.

So, 325 students were taking neither tea nor coffee.

Ex09. In a group of students, 100 students know Hindi, 50 know English and 25 know both. Each of the students knows either Hindi or English. How many students are there in the group?

Sol. Let H and E denote respectively the set of students knowing Hindi and English.

Consider the Venn diagram shown.



We have n(H) = a + b = 100, n(E) = b + c = 50, $n(H \cap E) = b = 25$.

Solving these eqs., we get: a = 75, b = 25 and c = 25.

So, $n(H \cup E) = a + b + c = 75 + 25 + 25 = 125$.

Hence, there are 125 students in the group of students.

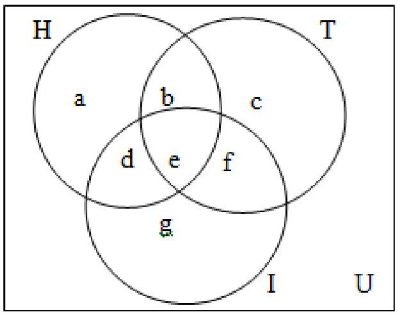
Ex10. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find the number of people who read:

(i) at least one of the newspapers.

(ii) exactly one newspaper.

Sol. Let H, T and I denote the set of people reading the

newspaper H, T and I respectively.
Consider the Venn diagram shown.



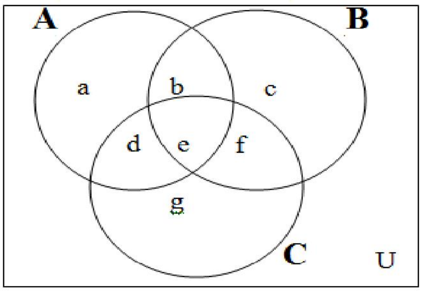
We have n(H) = a + b + d + e = 25,
n(T) = b + c + e + f = 26,
n(I) = d + e + f + g = 26,
$n(H \cap I) = d + e = 9$,
$n(H \cap T) = b + e = 11$,
$n(T \cap I) = e + f = 8$,
and $n(H \cap T \cap I) = e = 3$.
On solving all these equations simultaneously, we get:
a = 8, b = 8, c = 10, d = 6, e = 3, f = 5 and g = 12.

(i) The no. of people who read at least one of the newspaper = a + b + c + d + e + f + g
= 8 + 8 + 10 + 6 + 3 + 5 + 12 = 52.

(ii) The no. of people who read exactly one newspaper = a + c + g
= 8 + 10 + 12 = 30.

Ex11. In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only.

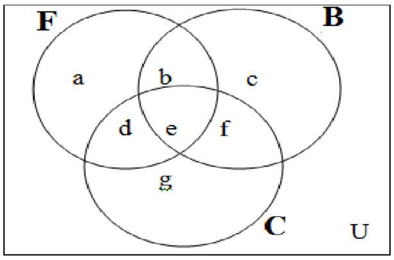
Sol. Let A, B and C respectively denote the set of people who liked the product A, B and C.
Consider the Venn diagram shown.



We have n(A) = a + b + d + e = 21,
n(B) = b + c + e + f = 26,
n(C) = d + e + f + g = 29,
$n(A \cap B) = b + e = 14$,
$n(A \cap C) = d + e = 12$,
$n(B \cap C) = e + f = 14$,
$n(A \cap B \cap C) = e = 8$.
On solving these equations, we get: g = 11.
So, the number of people who liked product C only = g = 11.

Ex12. A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of the three sports?

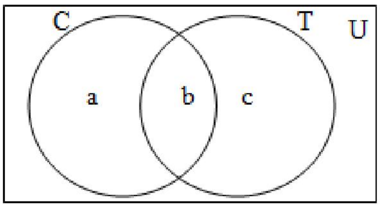
Sol. Let F, B and C denote the set of men who received the medals in football, basketball and cricket respectively.
Consider the Venn diagram shown.



We have n(F) = a + b + d + e = 38 …(i)
n(B) = b + c + e + f = 15 …(ii)
n(C) = d + e + f + g = 20 …(iii)
$n(F \cup B \cup C) = a + b + c + d + e + f + g = 58$ …(iv)
$n(F \cap B \cap C) = e = 3$ …(v)
Now by (i) + (ii) + (iii) + (v) – (iv), we get:
(a + 2b + c + 2d + 4e + 2f + g) – (a + b + c + d + e + f + g) = (38 + 15 + 20 + 3) – 58
$\Rightarrow$ b + d + 3e + f = 18
$\Rightarrow$ b + d + f = 18 – 3 (3) [By using (v)
$\Rightarrow$ b + d + f = 18 – 9
$\Rightarrow$ b + d + f = 9.
Hence the number of men who received the medals in exactly two of the three sports is 9.

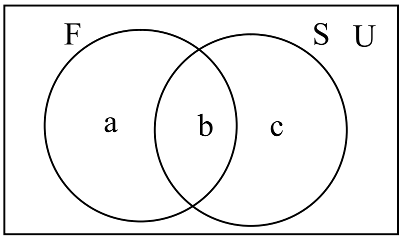
Ex13. In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

Sol. Let C and T denote the set of people liking cricket and tennis respectively.
Consider the Venn diagram.
We have n(C) = a + b = 40,



$n(C \cap T) = b = 10$,
$n(C \cup T) = a + b + c = 65$.
Solving these eqs., we get :
$a = 30, b = 10, c = 25$.
(i) No. of people liking tennis only and not cricket = c = 25.
(ii) No. of people liking tennis = b + c = 10 + 25 = 35.

Ex14. In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?
Sol. Let S and F represent the set of people speaking Spanish and French respectively.



Consider the Venn diagram.
We have, $n(F) = a + b = 50$,
$n(S) = b + c = 20$,
$n(F \cap S) = b = 10$,
$n(F \cup S) = ?$
On solving these equations we get: a = 40, c = 10.
So, $n(F \cup S) = a + b + c = 40 + 10 + 10 = 60$.
Hence the number of people speaking at least one of these two languages is 60.
Ex15. State whether the sets $\{x : x \text{ is an even integer}\}$ and $\{x : x \text{ is an odd integer}\}$ are pair of disjoint sets or not.
Sol. Let $A = \{x : x \text{ is an even integer}\}$ and $B = \{x : x \text{ is an odd integer}\}$.
So, $A = \{..., -2, 0, 2, 4, ...\}$ and $B = \{..., -3, -1, 1, 3, ...\}$.
Clearly $A \cap B = \phi$ $\therefore$ A and B are disjoint sets.

EXERCISE FOR PRACTICE

TYPE-A
Basic Introductive Questions

Q01. Which of the followings is a set? Justify your answer in each case.
(a) The collection of fat boys in your area
(b) The collection of five beautiful girls in your class
(c) The collection of Maths teachers in your school
(d) The collection of difficult topics in Mathematics
(e) The collection of smart boys of your school.

Q02. Write the following sets in the tabular form or roster form:
a) $\{x : x \text{ is a non-negative integer and } x^2 < 50\}$ b) $\{x : x \text{ is an integer satisfying } x^2 + x - 2 = 0\}$
c) $\{x : x \text{ is a letter in COMBINATIONS}\}$ d) $\{x : x \text{ is an odd integer and } 3 \le x < 13\}$
e) The set of all natural numbers 'x' such that $4x + 9 < 39$.
f) The set of all positive integers 'x' s.t. $|x - 3| < 7$.
g) {x : x is a two digit number *s.t.* the sum of its digits is nine}
h) The set of all vowels in the English alphabet, which precede r.

Q03. Write the following sets in the set-builder form:
a) $\{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$ b) $\{0, 3, 8, 15, 24, 35\}$
c) $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$ d) {0, 2, 10, 30, 68, 130}
e) {–2, 2} f) {14, 21, 28, 35, …, 84}
g) {53, 59, 61, 67, 71, 73, 79, 83, 89, 97} h) {1, 5, 10, 15, …}
i) $\left\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, ...\right\}$ j) {–2, –1, 0, 1, 2}

k) {1, 2, 3} l) $\{3, 9, 27, 81, 243\}$.

Q04. Match each of the sets on the left in the tabular (roster) form with the same set on the right given in the set builder form:

(i) {5, 10, 15, …} (a) $\{x : x$ is a letter of the word COLLEGE$\}$

(ii) {C, O, L, E, G} (b) $\{x : x$ is an odd natural number less than 10$\}$

(iii) {1, 3, 5, 7, 9} (c) $\{x : x = 5n,\ n \in N\}$

(iv) {2, 3} (d) $\{x : x$ is a prime number and a divisor of 12$\}$

Q05. Let A = {1, 2, 3, 4, 5, 6}. Insert the appropriate symbol $\in$ or $\notin$ in the blank spaces:

a) 5 … A b) 8 … A c) 4 … A d) 0 … A

Q06. List the elements of the following sets :

(a) Let $Y = \{y : y$ is a cube of natural nos. less than 6 except 4$\}$

(b) $\{x : x \in Z \text{ and } x^2 \le 9\}$

(c) {x : x is a vowel in English alphabet which preceds q}

(d) $\left\{p : p = \dfrac{1}{2n},\ n \in Z^+ \text{ and } n < 4\right\}$

Q07. (a) Write the set of all even integers whose cube is odd.

(b) Write the set of 'all real numbers which can not be written as the quotient of two integers' in the set-builder form.

Q08. If $X = \{1, 2, 3, 4, 5, 6, 7\}$ and n represents any member of X, write $n \in X$ but $2n \notin X$ as a set.

Q09. Write the set of all positive integers whose cube is odd integer.

TYPE-B

Concept Building Questions - I

Q01. Which of the followings is singleton set?

(a) $\{x : x$ is an integeral root of $x^2 - 2x + 1 = 0\}$ (b) $\{0, \{\}\}$.

Q02. Find the pair of equal sets, if any, and give reasons:

A = {0}, B = {x: x > 15 and x < 5}, C = {x: x–5 = 0}, D = {x: $x^2 = 25$}.

Q03. In the followings, state whether or not A = B :

(a) A = {x : x$\in$Z and $x^2 \le 4$}, B = {x : x$\in$R and $x^2 - 3x + 2 = 0$}

(b) A = {x : x is a letter in the word FOLLOW}, B = {y : y is a letter in the word WOLF}.

Q04. (a) Write (–5, 9] in set-builder form.

(b) Write $\{x : -3 \le x \le 7\}$ as interval.

(c) Write ϕ as an interval in set builder form.

(d) If A = {1, 3, 5}, how many elements has P(A)?

(e) Write all the possible subsets of A = {5, 6}.

Q05. Write the following intervals in set-builder form:

a) (–3, 0) b) [6, 12] c) [–5/2, 5) d) (1, 3]

Q06. Write down all the subsets of the following sets:

a) {a, b} b) {1, 2, 3} c) ϕ d) {–1, 0, 1}

Q07. Let X be the set of all the letters in the word ROOM. Write the power set of X.

Q08. Find the number of elements of the power set of the set formed by the letters of the word SWISS.

Q09. Insert appropriate symbols from "$\in, \notin, \subset, \supset, =$ or $\ne$" to fill in the blanks in the followings :

(a) 4 … {1, 2, 3, 4, 5} (b) –5 … {2, 3, 4, 5, 6}

(c) {2} … {2, 3, 4} (d) {p, q, r} … {r, p, q}

(e) {a, b, c} … {a, b, b, b, a, c} (f) {a, i, j} … {a, a, i}

(g) ϕ … $\{\phi\}$ (h) ϕ … {1, 2, 3, 4}

(i) 0 … ϕ (j) {1, 2} … {1, {2, 3}}

(k) CHENNAI … {x : x is a capital city of countries in Asia}.

Q10. Let A and B be two finite sets such that n(A) = m and n(B) = n. If the ratio of number of elements of power sets of A and B is 64 : 1 and n(A) + n(B) = 32. Find the value of m and n.

Q11. Show that $n\left[P\left\{P\left(P(\phi)\right)\right\}\right]=4$.

Q12. Which of the following sets are finite or infinite? Justify.

(a) The set of all the points on the circumference of a circle.

(b) $\{m : m \in N \text{ and } m \text{ is an even prime number}\}$.

Q13. Which of the following are empty sets? Justify.

(a) $A=\{x : x \in N \text{ and } 3<x<4\}$ (b) $B=\{y : y \in Z^+ \text{ and } y^2=y\}$.

Q14. If $A=\{p,q\}$ and $B=\{p,q,r\}$, is B a superset of A? Why?

TYPE-C
Concept Building Questions - II

Q01. If $A=\{3,5,7,9,11\}$, $B=\{7,9,11,13\}$ and, $C=\{11,13,15\}$ then, find the followings:

(a) $A\cap(B\cup C)$ (b) $(A\cap B)\cap(B\cup C)$

Q02. If $A=\{2,4,6,8,10\}$ and $B=\{4,5,6,7,8,9,10,11\}$, find: (a) A – B (b) B – A.

Q03. If R is the set of real numbers and T is the set of irrational numbers, then what is R – T?

Q04. Check which of the following pair of sets are disjoint?

a) $\{1,2,3,4\}$ and $\{x : x \text{ is a natural number and } 4\le x\le 6\}$

b) $\{a,e,i,o,u\}$ and $\{c,d,e,f\}$

c) $\{x : x \text{ is an even integer}\}$ and $\{x : x \text{ is an odd integer}\}$.

Q05. Let $U=\{1,2,3,4,5,6\}$, $A=\{2,3\}$ and $B=\{3,4,5\}$.

Find A', B', $A'\cap B'$, $A\cup B$ and hence show that $(A\cup B)'=A'\cap B'$.

Q06. Let $U=\{1,2,3,4,5,6,7,8,9\}$, $A=\{2,4,6,8\}$ and $B=\{2,3,5,7\}$, verify that:

a) $(A\cup B)'=A'\cap B'$ b) $(A\cap B)'=A'\cup B'$.

Q07. Draw appropriate venn diagram for each of the followings:

a) $(A\cup B)'$ b) $A'\cap B'$ c) $(A\cap B)'$ d) $A'\cup B'$.

Q08. Let $A_1=\{2,3,4,5\}$, $A_2=\{3,4,5,6\}$, $A_3=\{4,5,6,7\}$, find $\bigcup_{n=1}^{3}A_n$ and $\bigcap_{n=1}^{3}A_n$.

Q09. Find the smallest set A, such that $A\cup\{a,b\}=\{a,b,c,d,e\}$.

Q10. If X = set of letters of DELHI and Y = set of letters of DOLL., then find $X\cup Y$ and, $Y-X$.

Q11. On the Number line of Real numbers (Real axis), if $A=[0,3]$ and $B=[2,6]$. Then determine the followings :

(a) A' (b) $A\cup B$ (c) $A\cap B$ (d) $A-B$

Q12. Let S be the universal set of all the students of XI class of a co-educational school in Delhi. Let A be the set of all girls in this class. Find A'.

TYPE-D
Property Based Questions

Q01. Prove the followings :

(a) $n(A \text{ or } B)=n(A\cup B)=n(A)+n(B)-n(A\cap B)$.

(b) $n(A \text{ or } B \text{ or } C) = n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$
$- n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$.

Q02. Show that $A \cap \overline{B} = A - B$.
Q03. Show that $A \cup B = A \cap B$ implies $A = B$.
Q04. For any sets A and B prove that: $P(A \cap B) = P(A) \cap P(B)$.
Q05. Show that $A \cap B = A \cap C$ need not imply $B = C$.
Q06. Is it true that for any sets A and B, $P(A) \cup P(B) = P(A \cup B)$? Justify your answer.
Q07. Using properties of sets, show that:
(a) $A \cup (A \cap B) = A$ (b) $B \cup (A \cap B) = B$
(c) $A \cap (A \cup B) = A$ (d) $B \cap (A \cup B) = B$.
Q08. Show that for any sets A and B,
(a) $A = (A \cap B) \cup (A - B)$ (b) $A \cup (B - A) = (A \cup B)$.
Q09. Find sets A, B and C so that $A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets and $A \cap B \cap C$ is an empty set.
Q10. Show that the following four conditions are equivalent:
(i) $A \subset B$ (ii) $A - B = \phi$ (iii) $A \cup B = B$ (iv) $A \cap B = A$.
Q11. Let A and B be any two sets. Using properties of the sets, prove that
(a) $(A - B) \cup B = A \cup B$ (b) $(A \cup B) - A = B - A$.
Q12. Let A, B, C are any three sets. Using properties of sets and their complements, prove that
(a) $(A - B) \cup (A - C) = A - (B \cap C)$
(b) $A \cap (B - C) = (A \cap B) - (A \cap C)$
(c) $A - (B - C) = (A - B) \cup (A \cap C)$
(d) $(A - B) \cap (A - C) = A - (B \cup C)$.
Q13. Prove that $A \subset B$ and $B \subset C \Rightarrow A \subset C$.

TYPE-E
Application Based Questions

Q01. A survey shows that 84% of the Indians like grapes, whereas 45% like pineapple. What % of Indians like both grapes and pineapple?
Q02. A market research company conducted a survey of 1000 consumers and reported that 720 consumers like product A and 450 consumers like product B. What is the least number of consumers that must have liked both the products?
Q03. In a group of 800 people, 500 can speak Hindi and 320 can speak English. Find the followings:
a) How many can speak both Hindi and English?
b) How many can speak Hindi only?
Q04. If A and B are two sets such that $n(A - B) = 14 + x$, $n(B - A) = 3x$ and $n(A \cap B) = x$, draw a Venn diagram to illustrate the information. Also if $n(A) = n(B)$, then find the value of x.
Q05. There are 200 individuals with a skin disorder, 120 had been exposed to the chemical C_1, 50 to chemical C_2 and 30 to both the chemical C_1 and C_2. Find the number of individuals exposed to
a) chemical C_1 but not chemical C_2
b) chemical C_2 but not chemical C_1
c) chemical C_1 or chemical C_2.
d) none of the chemicals.
Q06. Of the members of three athletic team in a certain school, 21 are in the Basketball team, 26 in the Hockey team and 29 in the Football team. 14 play hockey and basketball, 15 play hockey and football, 12 play football and basketball and 8 play all the three. How many members are there in all?

Q07. In a university out of 100 teachers, 15 like reading newspapers only, 12 like learning computers only and 8 like watching movies only on TV in the spare time. 40 like reading news papers and watching movies, 20 like learning computer and watching movies, 10 like reading news paper and learning computer, 65 like watching movies. Draw a Venn diagram and show the various portions and hence evaluate the numbers of teachers who:
i) like reading newspapers
ii) like learning computers
iii) did not like to do any of the things mentioned above

Q08. In a survey of 400 students of a school, 100 were listed as smokers and 150 as chewers of Gum, 75 were listed as both smokers and gum chewers. Find out how many students are neither smokers nor gum chewers.

Q09. There are 20 students in a biology class and 30 students in a mathematics class. Find the number of students who are either in mathematics class or in biology class
(a) when the two classes meet at different hours and 10 students are enrolled in both the courses.
(b) when the classes meet at the same hour.

Q10. In a class of 25 students, 12 have taken mathematics, 8 have taken mathematics but not biology. Find the number of students who have taken both mathematics and biology and the number of those who have taken biology but not mathematics. It is given that each student has taken either mathematics or biology or both.

Q11. A class has 175 students. Following is the description showing the number of students studying one or more of the following subjects in this class:
Mathematics 100; Physics 70; Chemistry 46; Mathematics and Physics 30; Mathematics and Chemistry 28; Physics and Chemistry 23; Mathematics, Physics and Chemistry 18.
How many students are enrolled in Mathematics alone, Physics alone and Chemistry alone? Are there students who have not been offered any of these three subjects?

Q12. If A and B are two sets containing 3 and 6 elements respectively, what can be the maximum number of elements in $A \cup B$? Find also the minimum number of elements in $A \cup B$?

Q13. If A, B and C are three sets and U is the universal set such that n(U) = 700, n(A) = 200, n(B) = 300 and $n(A \cap B) = 100$. Find $n(A' \cap B')$.

Q14. In a survey of 25 students, it was found that 15 had taken Mathematics, 12 had taken Physics and 11 had taken Chemistry, 5 had taken Mathematics and Chemistry, 9 had taken Mathematics and Physics, 4 had taken Physics and Chemistry, and 3 had taken all the three subjects. Find the number of students that had
(a) only Chemistry
(b) only Mathematics
(c) only Physics
(d) Physics and Chemistry but not Mathematics
(e) Mathematics and Physics but not Chemistry
(f) only one of the subjects
(g) at least one of the three subjects
(h) none of the subjects

Q15. A survey provides the following information for TV viewership in an area :
60% watch program A, 50% watch program B, 47% watch program C, 28% watch program A and B, 23% watch program A and C, 18% watch program B and C, 8% watch all the three programs. Draw Venn diagram to illustrate this information and use it to find :
(a) the percentage of people who watch program A and B but not C
(b) the percentage of people who watch exactly two programs
(c) the percentage of people who do not watch any program.

Q16. In a town of 10000 families, it was found that 40% families buy newspaper A, 20% buy newspaper B and 10% buy newspaper C. Also 5% families buy newspapers A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers, find the number of families which buy the newspaper (a) A only (b) B only (c) none of A, B and C (d) exactly two newspapers (e) exactly one newspaper (f) A and C but not B (g) at least one of A, B, C. Why do you think that the students should read newspaper?

Q17. In a survey, it is found that 105 people take brand X pan-masala, 130 take brand Y pan-masala, and 145 take brand Z pan-masala. If 70 people take brand X as well as brand Y, 75 take brand Y as well as brand Z, 60 take brand X as well as brand Z and 40 take all the three brands, find how many people are surveyed who take the pan-masala of any kind? How many take brand Z pan-masala only. What measures would you suggest to spread awareness against the pan-masala in the society?

Q18. A survey of 500 television viewers produced the following information :
285 watch football, 195 watch hockey, 115 watch basketball, 45 watch football and basketball, 70 watch football and hockey, 50 watch hockey and basketball, 50 do not watch any of the three games. How many watch all the three games? How many watch exactly one of the three games? What is the importance of television in today's life?

Q19. In a beauty contest, half the number of judges voted for Miss Anamika, $2/3^{rd}$ of them voted for Miss Bhawna, 10 voted for both and 6 didn't vote for either Anamika or Bhawna. Find how many judges in all, were present there?

Q20. In a class, 18 students took English, 23 students took Hindi and 24 students took Sanskrit. Of these, 13 took both Hindi and Sanskrit, 12 took both English and Hindi and 11 took both English and Sanskrit. If 6 students were offered all the three languages, find (a) the total number of students (b) how many took Sanskrit but not Hindi (c) how many took exactly one of the three subjects?

Q21. In a survey of 450 students, it was observed that 110 play cricket, 160 play tennis and 70 play both cricket as well as tennis. How many play neither cricket nor tennis?

Q22. Two sets A and B are such that $n(A\cup B)=21$, $n(A'\cap B')=9$, $n(A\cap B)=7$, find $n(A\cap B)'$.

Q23. Let A be the set of all divisors of the number 15, B be the set of prime numbers smaller than 10 and C be the set of even numbers smaller than 9, then find the value of $(A\cup C)\cap B$.

TEST For SELF EVALUATION - 01

Time Allowed : 60 Minutes **Max. Marks : 35**

Q01. (a) If A and B are two sets such that $A \subset B$, then what is $A \cup B$?
(b) How many elements has P(A) if $A = \{\ \}$.

Q02. If P and Q are two sets s. t. $n(P) = 17,\ n(Q) = 23,\ n(P \cup Q) = 38$, then find $n(P \cap Q)$.

Q03. Write $\left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \frac{6}{37}, \frac{7}{50}\right\}$ in the set builder form. [1×3

Q04. Prove that $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$.

Q05. Prove that $A' - B' = B - A$. [4×2

Q06. Prove that $A \cap B = A \cap C$ need not imply $B = C$.

Q07. Prove that $A \subseteq B,\ B \subseteq C$ and $C \subseteq A$ implies $A = C$.

Q08. In a survey of 60 people, it was observed that 25 people read book H, 26 read book T, 26 read book I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all the three books. Find the number of people who read (i) at least one of the book (ii) exactly one of the books.

Q09. Let $W = \{1,2,3,4\}$, $X = \{3,4,5,6\}$, $Y = \{5,6,7,8\}$ and $Z = \{7,8,9,10\}$, find :
(i) $W \cup X$ (ii) $W \cup Y$ (iii) $X \cap Y$
(iv) $W \cup X \cup Y$ (v) $W \cup X \cup Z$ (vi) $X \cup Y \cup Z$. [6×4

ANSWERS

Q01. (a) B (b) 1

Q02. 2

Q03. $\left\{x : x = \frac{n}{n^2+1},\ n \in N,\ n \le 7\right\}$

Q08. (i) 52 (ii) 30.

Q09. (i) $\{1,2,3,4,5,6\}$ (ii) $\{1,2,...,8\}$ (iii) $\{5,6\}$
(iv) $\{1,2,...,8\}$ (v) $\{1,2,...,10\}$ (vi) $\{3,4,...,10\}$.

ANSWERS FOR ALL CHAPTERS

CHAPTER 01

TYPE-A

Q01. (a) Not a set (b) Not a set (c) A set (d) Not a set (e) Not a set.

Q02. (a) $\{0,1,2,3,4,5,6,7\}$ (b) $\{-2,1\}$ (c) $\{C, O, M, B, I, N, A, T, S\}$ (d) $\{3,5,7,9,11\}$

(e) $\{1,2,3,4,5,6,7\}$ (f) $\{1,2,...,9\}$ (g) $\{18, 27, 36, 45, 54, 63, 72, 81, 90\}$ (h) $\{a,e,i,o\}$

Q03. (a) $\{x : x \text{ is an integer and } -4 \le x \le 5\}$

(b) $\{x : x = n^2 - 1, \text{ where n is a natural number and } 1 \le n \le 6\}$

(c) $\left\{x : x = \dfrac{n}{n+1}, \text{ where n is a natural number and } 1 \le n \le 6\right\}$

(d) $\{x : x = n^3 + n, n \in Z - Z^-, n \le 5\}$ (e) $\{x : x \text{ is a root of } x^2 - 4 = 0\}$

(f) $\{x : x = 7m, \text{ where } m \in N, 1 < m \le 12\}$ or $\{x : x = 7n, n \in N \text{ and } 7 < x < 90\}$

(g) $\{x : x \text{ is a prime number and } 50 < x < 100\}$

(h) $\{x : x \in N, x \text{ is equal to 1 or multiple of } 5\}$

(i) $\left\{x : x = \dfrac{1}{n^2}, n \in N\right\}$ (j) $\{x : x^2 \le 4, x \in Z\}$ (k) $\{x : x^2 \le 9, x \in N\}$

(l) $\{x : x = 3^n, n \in N, n \le 5\}$ Q04. i – c, ii – a, iii – b, iv – d Q05. $\in, \notin, \in, \notin$

Q06. (a) $Y = \{1, 8, 27, 125\}$ (b) $\{-3,-2,-1, 0, 1, 2, 3\}$ (c) $\{a, e, i, o\}$ (d) $\left\{\dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{6}\right\}$

Q07. (a) $\{\ \}$ or ϕ (b) $\{x : x \in T\}$ or $\{x : x \text{ is real and irrational number}\}$.

Q08. $\{4,5,6,7\}$ Q09. $\{2m+1 : m \ge 0, m \in Z\}$

TYPE-B

Q01. (a) Singleton set. (b) Not a singleton set. Q02. No pair is equal.

Q03. (a) $A \ne B$ (b) $A = B$

Q04. (a) $\{x : x \in R, -5 < x \le 9\}$ (b) $[-3,7]$ (c) $\{x : x \in R, a < x < a\}$

(d) $2^3 = 8$ (e) $\phi, \{5\}, \{6\}, \{5,6\}$

Q05. (a) $\{x : x \in R, -3 < x < 0\}$ (b) $\{x : x \in R, 6 \le x \le 12\}$

(c) $\left\{x : x \in R, -\dfrac{5}{2} \le x < 5\right\}$ (d) $\{x : x \in R, 1 < x \le 3\}$

Q06. (a) $\phi, \{a\}, \{b\}, \{a,b\}$ (b) $\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$ (c) ϕ

(d) $\phi, \{-1\}, \{0\}, \{1\} \{-1,0\}, \{0,1\}, \{1,-1\}, \{-1,0,1\}$

Q07. $P(X) = \{\phi, \{M\}, \{O\}, \{R\}, \{M, O\}, \{O, R\}, \{R, M\}, \{M, O, R\}\}$ Q08. 8

Q09. (a) $\in$ (b) $\notin$ (c) $\subset$ (d) = (e) = (f) $\not\subset$ (g) $\subset$ (h) $\subset$ (i) $\notin$ (j) $\notin$ (k) $\notin$.

Q10. 19, 13 Q12. (a) Infinite set (b) Finite set. Q13. (a) Empty set. (b) Non empty set.

Q14. As $A \subset B$ so, B is a superset of A.

TYPE-C

Q01. (a) $\{7,9,11\}$ (b) $\{7,9,11\}$ Q02. (a) $\{2\}$ (b) $\{5, 7, 9, 11\}$

Q03. Set of rational numbers Q04. Only (c) is disjoint

Q05. $A' = \{1,4,5,6\}$, $B' = \{1,2,6\}$, $A' \cap B' = \{1,6\}$, $A \cup B = \{2,3,4,5\}$ Q08. $\{2,3,4,5,6,7\}$, $\{4,5\}$

Q09. $A = \{c, d, e\}$

Q10. $X \cup Y = \{D, E, H, I, L, O\}$, $Y - X = \{O\}$.

Q11. (a) $(-\infty, \infty) - [0,3]$ or, $(-\infty, 0) \cup (3, \infty)$ (b) [0, 6] (c) [2, 3] (d) [0, 2).

Q12. $A' =$ the set of all boys of XI class

TYPE-D

Q06. Let A = {0, 1} and B = {1, 2}. So A ∪ B = {0, 1, 2} ⇒ P(A) = {ϕ, {0}, {1}, {0, 1}},
P(B) = {ϕ, {1}, {2}, {1, 2}} ⇒ P(A ∪ B) = {ϕ, {0}, {1}, {2}, {0, 1}, {1, 2}, {0, 2}, {0, 1, 2}}
And, P(A) ∪ P(B) = {ϕ, {0}, {1}, {0, 1}, {2}, {1, 2}}. So, P(A) ∪ P(B) ≠ P(A ∪ B).

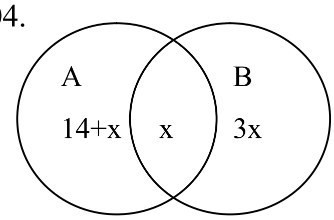
Q09. Let A = {0, 1}, B = {1, 2}, and C = {2, 0}. Accordingly, A ∩ B = {1}, B ∩ C = {2}, and A ∩ C = {0}. So A ∩ B, B ∩ C, and A ∩ C are non-empty. However it is clear that, A ∩ B ∩ C = ϕ.

TYPE-E

Q01. 29% Q02. 170

Q03. a) 20 people can speak both Hindi and English (b) 480 people can speak Hindi only.

Q04.



; x = 7.

Q05. 90, 20, 140, 60 Q06. 43 Q07. 62, 39, 1 Q08. 225 Q09. (a) 40 (b) 50

Q10. 4, 13 Q11. 60, 35, 13; 22 Q12. 9, 6 Q13. 300 Q14. 5, 4, 2, 1, 6, 11, 23, 2

Q15. 20%, 45%, 4% Q16. 33%, 14%, 40%, 8%, 48%, 4%, 58% Q17. 215, 50 Q18. 20, 325

Q19. 24 Q20. (a) 35 (b) 11 (c) 11 Q21. 250 Q22. 23 Q23. $\{2,3,5\}$

CHAPTER 02

TYPE-A

Q01. (a) $m = 3,\ n = -1$ (b) $x = 2,\ y = 1$ (c) $a = 2,\ b = -3$

Q02. x = 3, y = 2 Q03. 15, 4. Q04. 6, 6, 9

Q07. $\{(a,1),(a,2),(a,5),(b,2)(b,5),(b,1)\}$ Q08. $A = \{x, y, z\}, B = \{1, 2\}$

Q09. $A = \{-1, 0, 1\}$, remaining elements of $A \times A$ are $(-1,-1),(-1,1),(0,-1),(0,0),(1,-1),(1,0),(1,1)$

Q10. $A \times A \times A = \{(-1,-1,-1),(-1,-1,1),(-1,1,-1),(-1,1,1),(1,-1,-1),(1,-1,1),(1,1,-1),(1,1,1)\}$

Q11. $R \times R = \{(x, y) : x, y \in R\}$ represents the coordinates of all the points in two dimensional space. Also, $R \times R \times R = \{(x, y, z) : x, y, z \in R\}$ represents the coordinates of all the points in 3 dimensional space.

Chapter 01 *Solutions*

TYPE-A

Q01. (a) As it is not well defined, so it is not a set.
(b) As it is not well defined, so it is not a set.
(c) Since it's well defined so, it is a set.
(d) As it is not well defined, so it is not a set. (as level of difficulty depends upon the individual).
(e) As it is not well defined, so it is not a set.

Q02. (a) $\{0,1,2,3,4,5,6,7\}$ (b) $\{-2,1\}$
(c) $\{C, O, M, B, I, N, A, T, S\}$ (d) $\{3,5,7,9,11\}$.
(e) $\{1,2,3,4,5,6,7\}$ (f) $\{1,2,...,9\}$.
(g) The tabular form of {x : x is a two digit number *s.t.* the sum of its digits is nine} is given as {18, 27, 36, 45, 54, 63, 72, 81, 90} (h) $\{a,e,i,o\}$

Q03. (a) $\{x : x \text{ is an integer and } -4 \le x \le 5\}$
(b) $\{x : x = n^2 - 1, \text{ where n is a natural number and } 1 \le n \le 6\}$
(c) $\left\{x : x = \dfrac{n}{n+1}, \text{ where n is a natural number and } 1 \le n \le 6\right\}$
(d) $\{x : x = n^3 + n, n \in Z - Z^-, \; n \le 5\}$
(e) $\{x : x \text{ is a root of } x^2 - 4 = 0\}$
(f) $\{x : x = 7m, \text{ where } m \in N, 1 < m \le 12\}$ or $\{x : x = 7n, n \in N \text{ and } 7 < x < 90\}$
(g) $\{x : x \text{ is a prime number and } 50 < x < 100\}$
(h) $\{x : x \in N, x \text{ is equal to 1 or multiple of 5}\}$
(i) $\left\{x : x = \dfrac{1}{n^2}, n \in N\right\}$
(j) $\{x : x^2 \le 4, x \in Z\}$
(k) $\{x : x^2 \le 9, x \in N\}$
(l) $\{x : x = 3^n, n \in N, n \le 5\}$

Q04. i – c, ii – a, iii – b, iv – d

Q05. (a) $\in$ (b) $\notin$ (c) $\in$ (d) $\notin$

Q06. (a) $Y = \{1, 8, 27, 125\}$ (b) $\{-3, -2, -1, 0, 1, 2, 3\}$
(c) {a, e, i, o} (d) $\left\{\dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{6}\right\}$

Q07. (a) $\{\;\}$ or ϕ
(b) $\{x : x \in T\}$ or $\{x : x \text{ is real and irrational number}\}$

Q08. Given that $X = \{1,2,3,4,5,6,7\}$, let $A = \{n \in X \text{ but } 2n \notin X\}$
Note that, if we put $n = 1, 2$ or $3 \in X$ then $2n = 2, 4$ or $6 \in X$. Whereas if we put $n = 4, 5, 6$ or $7 \in X$ then $2n = 8, 10, 12$ or $14 \notin X$.
Clearly, $A = \{4,5,6,7\}$.

Q09. The elements of the required set are not even integers (as cube of an even integer is always an even integer). That is, the elements of the required set are all positive odd integers.
Hence the set builder form of the required set is $\{2m+1 : m \geq 0,\ m \in Z\}$.

TYPE-B

Q01. (a) As $\{x : x \text{ is an integeral root of } x^2 - 2x + 1 = 0\} = \{1\}$ has only one element so, it is a singleton set.
(b) Let $A = \{0, \{\}\}$. The cardinal number of set A is 2. So it is not singleton set.

Q02. We have A = {0}, B = {x: x > 15 and x < 5} = { } = ϕ, C = {x: x – 5 = 0} = {5} and,
D = {x: $x^2 = 25$} = {–5, 5}.
So it is clear that no pair of the sets are equal (as for equal sets A and B *say*, every element of set A must be present in B and every element of set B must be in A).

Q03. (a) Given $A = \{x : x \in Z \text{ and } x^2 \leq 4\}$, $B = \{x : x \in R \text{ and } x^2 - 3x + 2 = 0\}$
So, A = {–2, –1, 0, 1, 2}, B = {1, 2}. $\therefore A \neq B$ $[\because n(A) \neq n(B)]$
(b) Given A = {x : x is a letter in the word FOLLOW}, B = {y : y is a letter in the word WOLF}.
So, A = {F, O, L, W} and B = {W, O, L, F}
$\because n(A) = n(B)$, for all $x \in A,\ x \in B$ and for all $y \in B,\ y \in A$ $\therefore A = B$.

Q04. (a) $\{x : x \in R, -5 < x \leq 9\}$
(b) $[-3, 7]$
(c) $\{x : x \in R,\ a < x < a\}$
(d) $\because n(A) = 3 \quad \therefore n[P(A)] = 2^3 = 8$
(e) $\phi, \{5\}, \{6\}, \{5,6\}$

Q05. (a) $\{x : x \in R,\ -3 < x < 0\}$ (b) $\{x : x \in R,\ 6 \leq x \leq 12\}$
(c) $\left\{x : x \in R,\ -\frac{5}{2} \leq x < 5\right\}$ (d) $\{x : x \in R,\ 1 < x \leq 3\}$

Q06. (a) Let A = $\{a, b\}$. Since n(A) = 2 so, total number of subsets of A = 2^2 = 4.
$\therefore$ subsets of $A = \phi, \{a\}, \{b\}, \{a, b\}$
(b) Let A = {1, 2, 3}. Since n(A) = 3 so, total no, of subsets of A = 2^3 = 8.
$\therefore$ subsets of $A = \phi, \{1\}, \{2\}, \{3\}\ \{1,2\}, \{2,3\}, \{3,1\}, \{1,2,3\}$
(c) Let $A = \phi$. Since n(A) = 0 so, total number of subsets of A = 2^0 = 1.
$\therefore$ subsets of $A = \phi$
(d) Let A = {–1, 0, 1}. Since n(A) = 3 so, total no. of subsets of A = 2^3 = 8.
$\therefore$ subsets of $A = \phi, \{-1\}, \{0\}, \{1\}\ \{-1,0\}, \{0,1\}, \{1,-1\}, \{-1,0,1\}$

Q07. We have X = {M, O, R}
$\therefore$ P(X) = { ϕ, {M}, {O}, {R}, {M, O}, {O, R}, {R, M}, {M, O, R}}.

Q08. Here A = {I, S, W}.
Clearly total number of elements in set A is 3.
Therefore, n[P(A)] = 2^3 = 8.

Q09. (a) $\in$ (b) $\notin$ (c) $\subset$ (d) = (e) = (f) $\not\subset$ (g) $\subset$ (h) $\subset$ (i) $\notin$ (j) $\notin$ (k) $\notin$

Q10. Given that n(A) = m and n(B) = n $\therefore n[P(A)] = 2^m$ and $n[P(B)] = 2^n$
Now, $2^m : 2^n = 64 : 1$ and, $n(A) + n(B) = m + n = 32$
That implies, $2^{m-n} = 2^6 \Rightarrow m - n = 6$...(i) and, $m + n = 32$...(ii)
Adding (i) & (ii), $2m = 38 \Rightarrow m = 19$.
Replacing $m = 19$ in (ii), we get $n = 13$

Therefore, $m = 19$, $n = 13$.

Q11. Let $A = \phi$ $\quad \therefore n(A) = 0$. So, $P(A) = \{\phi\} = B$ *say.*

Then $P(B) = \{\phi, \{\phi\}\} = C$ *say.* $\therefore n(C) = 2 \Rightarrow n[P(C)] = n\left[P\{P(P(A))\}\right] = 2^2 = 4$.

Q12. (a) It is an infinite set because circle is a collection of infinite points whose distances from the centre is constant.

(b) It is a finite set as $\{m : m \in N$ and m is an even prime number$\} = \{2\}$.

Q13. (a) Set A is empty set as there is no natural number lying between 3 and 4.

(b) Set B is non empty set as $B = \{1\}$.

Q14. Since $A \subset B$ so, B is a superset of A.

TYPE-C

Q01. Given $A = \{3,5,7,9,11\}$, $B = \{7,9,11,13\}$ and, $C = \{11,13,15\}$

(a) We have $B \cup C = \{7,9,11,13,15\}$

$\therefore A \cap (B \cup C) = \{3,5,7,9,11\} \cap \{7,9,11,13,15\} = \{7,9,11\}$

(b) We have $A \cap B = \{7,9,11\}$

$\therefore (A \cap B) \cap (B \cup C) = \{7,9,11\} \cap \{7,9,11,13,15\} = \{7,9,11\}$.

Q02. We have $A = \{2,4,6,8,10\}$ and $B = \{4,5,6,7,8,9,10,11\}$

(a) $A - B = \{2\}$ (b) $B - A = \{5,7,9,11\}$.

Q03. $R - T = Q$, i.e., Set of rational numbers.

Q04. (a) We've $\{1,2,3,4\}$ and $\{x : x$ is a natural number and $4 \le x \le 6\}$

That is, $\{1,2,3,4\}$ and $\{4,5,6\}$. Since the element 4 is common to both the sets so, these sets are not disjoint.

(b) We have $\{a,e,i,o,u\}$ and $\{c,d,e,f\}$. Since the element e is common to both the sets so, these sets are not disjoint.

(c) We've $\{x : x$ is an even integer$\}$ and $\{x : x$ is an odd integer$\}$.

That is $\{0, \pm 2, \pm 4, \pm 6, ...\}$ and $\{\pm 1, \pm 3, \pm 5, ...\}$.

It is clear that $\{0, \pm 2, \pm 4, \pm 6, ...\} \cap \{\pm 1, \pm 3, \pm 5, ...\} = \phi$.

Q05. We have $U = \{1,2,3,4,5,6\}$, $A = \{2,3\}$ and $B = \{3,4,5\}$

$\therefore A' = U - A = \{1,4,5,6\}$, $B' = \{1,2,6\}$, $A' \cap B' = \{1,6\}$, $A \cup B = \{2,3,4,5\}$, $(A \cup B)' = \{1,6\}$

So it is clear that $A' \cap B' = (A \cup B)'$.

Q06. We have $U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{2,4,6,8\}$ and $B = \{2,3,5,7\}$.

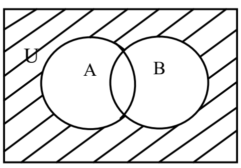
(a) $\because A' = \{1,3,5,7,9\}$, $B' = \{1,4,6,8,9\}$, $A \cup B = \{2,3,4,5,6,7,8\}$ and $A' \cap B' = \{1,9\}$

Also $(A \cup B)' = \{1,9\}$. So it is clear that $(A \cup B)' = A' \cap B'$.

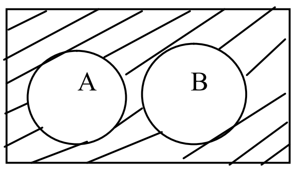
(b) We have $A \cap B = \{2\}$, $(A \cap B)' = \{1,3,4,5,6,7,8,9\}$, $A' \cup B' = \{1,3,4,5,6,7,8,9\}$

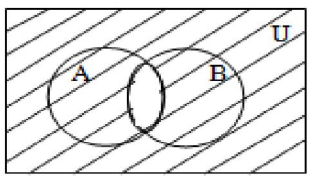
So it is clear that $(A \cap B)' = A' \cup B'$.

Q07. (a) $(A \cup B)'$ & (b) $A' \cap B'$ (c) $(A \cap B)'$ & (d) $A' \cup B'$



or





Q08. Here $A_1 = \{2,3,4,5\}$, $A_2 = \{3,4,5,6\}$, $A_3 = \{4,5,6,7\}$.

Now $\bigcup_{n=1}^{3} A_n = A_1 \cup A_2 \cup A_3 = \{2,3,4,5,6,7\}$.

Also, $\bigcap_{n=1}^{3} A_n = A_1 \cap A_2 \cap A_3 = \{4,5\}$.

Q09. Since $A \cup \{a,b\} = \{a,b,c,d,e\}$ so, set A must have elements c, d and e.
Also A is to be the smallest set so, $A = \{c,d,e\}$.

Q10. Here $X = \{D,E,H,I,L\}$ and $Y = \{D,O,L\}$.
So, $X \cup Y = \{D,E,H,I,L,O\}$, $Y - X = \{O\}$.

Q11. Here $A = [0,3]$ and $B = [2,6]$

(a) $A' = R - A$ i.e., $(-\infty,\infty) - [0,3]$ or, $(-\infty,0) \cup (3,\infty)$

(b) $A \cup B = [0,3] \cup [2,6] = [0, 6]$

(c) $A \cap B = [0,3] \cap [2,6] = [2, 3]$

(d) $A - B = [0,3] - [2,6] = [0, 2)$.

Q12. $A' =$ the set of all boys of XI class

TYPE-D

Q01. Consider the Venn diagram. It is clear that

(a) $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$

$= n(A - B) + n(A \cap B) + n(B - A) + n(A \cap B) - n(A \cap B)$

[Adding & subtracting $n(A \cap B)$

$= [n(A - B) + n(A \cap B)] + [n(B - A) + n(A \cap B)] - n(A \cap B)$

$= n(A) + n(B) - n(A \cap B)$. H.P.

(b) $n(A \cup B \cup C) = n[(A \cup B) \cup C] = n(A \cup B) + n(C) - n[(A \cup B) \cap C]$

$= \{n(A) + n(B) - n(A \cap B)\} + n(C) - n[(A \cap C) \cup (B \cap C)]$

$= \{n(A) + n(B) + n(C) - n(A \cap B)\} - \{n(A \cap C) + n(B \cap C) - n(A \cap C \cap B \cap C)\}$

$= \{n(A) + n(B) + n(C) - n(A \cap B)\} - \{n(A \cap C) + n(B \cap C) - n(A \cap C \cap B \cap C)\}$

$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)\}$. H. P.

Q02. Let x be any arbitrary element of $A \cap \overline{B}$. Then $x \in (A \cap \overline{B})$
That implies, $x \in A$ and $x \in \overline{B}$ i.e., $x \in A$ and $x \notin B$ $\Rightarrow x \in (A - B)$
$\therefore A \cap \overline{B} \subseteq (A - B) \ldots$(i)
Similarly, let y be any arbitrary element of $A - B$. Then $y \in (A - B)$
That implies, $y \in A$ and $y \notin B$ $\Rightarrow y \in A$ and $y \in \overline{B}$ $\Rightarrow y \in (A \cap \overline{B})$
$\therefore (A - B) \subseteq (A \cap \overline{B}) \ldots$(ii)
By (i) and (ii), we get : $A \cap \overline{B} = A - B$.

Q03. Let a ∈ A. Then a ∈ A ∪ B. Since A ∪ B = A ∩ B , a ∈ A ∩ B. So a ∈ B.
Therefore, A ⊂ B. Similarly, if b ∈ B, then b ∈ A ∪ B.
Since A ∪ B = A ∩ B, b ∈ A ∩ B. So, b ∈ A.
Therefore, B ⊂ A. Thus, A = B.

Q04. Let X ∈ P(A ∩ B). Then X ⊂ A ∩ B. So, X ⊂ A and X ⊂ B.
Therefore, X ∈ P(A) and X ∈ P(B) which implies X ∈ P(A) ∩ P(B).
This gives P(A ∩ B) ⊂ P(A) ∩ P(B).
Let Y ∈ P(A) ∩ P(B). Then Y ∈ P(A) and Y ∈ P(B). So, Y ⊂ A and Y ⊂ B.
Therefore, Y ⊂ A ∩ B, which implies Y ∈ P(A ∩ B).
This gives P(A) ∩ P(B) ⊂ P(A ∩ B).

Hence P(A ∩ B) = P(A) ∩ P(B).

Q05. Let A = {0, 1}, B = {0, 2, 3}, and C = {0, 4, 5}.
Accordingly, A ∩ B = {0} and A ∩ C = {0}
Here, A ∩ B = A ∩ C = {0}. However, B ≠ C [∵ 2 ∈ B but 2 ∉ C.

Q06. Let A = {0, 1} and B = {1, 2}. So A ∪ B = {0, 1, 2}
$\Rightarrow$ P(A) = {ϕ, {0}, {1}, {0, 1}},
P(B) = {ϕ, {1}, {2}, {1, 2}}
and P(A ∪ B) = {ϕ, {0}, {1}, {2}, {0, 1}, {1, 2}, {0, 2}, {0, 1, 2}}
Also, P(A) ∪ P(B) = {ϕ, {0}, {1}, {0, 1}, {2}, {1, 2}}.
So it is clear that, P(A) ∪ P(B) ≠ P(A ∪ B).

Q07. Assume that U denotes the universal set.

(a) $A\cup(A\cap B)=(A\cap U)\cup(A\cap B)$ [As $A\cap U=A$
$=A\cap(U\cup B)$ [By distributive law
$=A\cap U$ [As $U\cup B=U$
$\therefore$ $A\cup(A\cap B)=A$.

(b) $B\cup(A\cap B)=(B\cap U)\cup(A\cap B)$ [As $B\cap U=B$
$=B\cap(U\cup A)$ [By distributive law, $A\cap B=B\cap A$
$=B\cap U$ [As $U\cup B=U$
$\therefore$ $B\cup(A\cap B)=B$.

(c) $A\cap(A\cup B)=(A\cup\phi)\cap(A\cup B)$ [As $A\cup\phi=A$
$=A\cup(\phi\cap B)$ [By distributive law
$=A\cup\phi$ [As $\phi\cap B=\phi$
$\therefore$ $A\cap(A\cup B)=A$.

(d) $B\cap(A\cup B)=(B\cup\phi)\cap(A\cup B)$ [∵ $B\cup\phi=B$
$=B\cup(\phi\cap A)$ [By distributive law, $A\cup B=B\cup A$
$=B\cup\phi$ [As $\phi\cap B=\phi$
$\therefore$ $B\cap(A\cup B)=B$.

Q08. **(a) Method 1 :**
To show: A = (A ∩ B) ∪ (A – B)
Let x ∈ A. We have to show that x ∈ (A ∩ B) ∪ (A – B)
Case I x ∈ A ∩ B ⇒ Then, x ∈ (A ∩ B) ⊂ (A ∪ B) ∪ (A – B)
Case II x ∉ A ∩ B ⇒ x ∉ A or x ∉ B. ∴ x ∉ B [As x ∉ A.
So x ∉ A – B ⊂ (A ∪ B) ∪ (A – B).
∴ A ⊂ (A ∩ B) ∪ (A – B) …(i)
It is clear that A ∩ B ⊂ A and (A – B) ⊂ A
∴ (A ∩ B) ∪ (A – B) ⊂ A …(ii)
From (i) and (ii), we obtain A = (A ∩ B) ∪ (A – B)

Method 2 :
RHS : (A ∩ B) ∪ (A – B) = (A ∩ B) ∪ (A ∩ B′) = A ∩ (B∪ B′) = A ∩ U = A = LHS.

(b) Method 1 :
To show: A ∪ (B – A) ⊂ A ∪ B
Let x ∈ A ∪ (B – A) ⇒ x ∈ A or x ∈ (B – A) ⇒ x ∈ A or (x ∈ B and x ∉ A) ⇒ (x ∈ A or x ∈ B) and (x ∈ A or x ∉ A) ⇒ x ∈ (A ∪ B).
So A ∪ (B – A) ⊂ (A ∪ B) …(i)
Next, we show that (A ∪ B) ⊂ A ∪ (B – A).
Let y ∈ A ∪ B ⇒ y ∈ A or y ∈ B ⇒ (y ∈ A or y ∈ B) and (y ∈ A or y ∉ A) ⇒ y ∈ A or (y ∈ B and y ∉ A) ⇒ y ∈ A ∪ (B – A).
So A ∪ B ⊂ A ∪ (B – A) …(ii)
Hence, from (i) and (ii), we obtain A ∪ (B – A) = A ∪ B.

Method 2 :

RHS : $A \cup (B - A) = A \cup (B \cap A') = (A \cup B) \cap (A \cup A') = (A \cup B) \cap U = (A \cup B) = LHS$.

Q09. Let $A = \{0, 1\}$, $B = \{1, 2\}$, and $C = \{2, 0\}$.
Accordingly, $A \cap B = \{1\}$, $B \cap C = \{2\}$, and $A \cap C = \{0\}$.
So, $A \cap B$, $B \cap C$, and $A \cap C$ are non-empty.
However it is clear that, $A \cap B \cap C = \phi$.

Q10. First, we have to show that (i) $\Leftrightarrow$ (ii).
Let $A \subset B$. To show: $A - B = \phi$. If possible, suppose $A - B \neq \phi$.
This means that there exists $x \in A$, $x \neq B$, which is not possible as $A \subset B$.
$\therefore A - B = \phi$. So $A \subset B \Rightarrow A - B = \phi$.
Let $A - B = \phi$.
To show: $A \subset B$. Let $x \in A$. Clearly, $x \in B$ because if $x \notin B$, then $A - B \neq \phi$.
So $A - B = \phi \Rightarrow A \subset B$. Hence (i) $\Leftrightarrow$ (ii).
Let $A \subset B$.
To show: $A \cup B = B$. Clearly, $B \subset A \cup B$. Let $x \in (A \cup B) \Rightarrow x \in A$ or $x \in B$
Case I: $x \in A \Rightarrow x \in B$ $[\because A \subset B]$ $\therefore (A \cup B) \subset B$.
Case II: $x \in B \Rightarrow$ Then, $A \cup B = B$.
Conversely, let $A \cup B = B$. Let $x \in A \Rightarrow x \in A \cup B$ $[\because A \subset A \cup B$
$\Rightarrow x \in B$ $[\because A \cup B = B]$ $\therefore A \subset B$.
Hence, (i) $\Leftrightarrow$ (iii).
Now, we have to show that (i) $\Leftrightarrow$ (iv).
Let $A \subset B$. Clearly $A \cap B \subset A$. Let $x \in A$. We have to show that $x \in A \cap B$.
As $A \subset B$, $x \in B \Rightarrow x \in A \cap B$. $\therefore A \subset A \cap B$. Hence, $A = A \cap B$.
Conversely, suppose $A \cap B = A$.
Let $x \in A \Rightarrow x \in A \cap B \Rightarrow x \in A$ and $x \in B \Rightarrow x \in B$ $\therefore A \subset B$.
Hence, (i) $\Leftrightarrow$ (iv).

Q11. (a) Consider LHS : $(A - B) \cup B = (A \cap B') \cup B = (A \cup B) \cap (B' \cup B)$ [By Distributive law
$\Rightarrow = (A \cup B) \cap (U) = A \cap B = RHS$.
(b) Consider LHS : $(A \cup B) - A = (A \cup B) \cap A' = (A \cap A') \cup (B \cap A')$
$\Rightarrow = \phi \cup (B \cap A') = B \cap A' = B - A = RHS$.

Q12. (a) LHS : $(A - B) \cup (A - C) = (A \cap B') \cup (A \cap C')$
$\Rightarrow = A \cap (B' \cup C') = A \cap (B \cap C)' = A - (B \cap C) = RHS$
(b) RHS : $(A \cap B) - (A \cap C) = (A \cap B) \cap (A \cap C)'$
$\Rightarrow = (A \cap B) \cap (A' \cup C') = [(A \cap B) \cap A'] \cup [(A \cap B) \cap C']$
$\Rightarrow = \phi \cup [A \cap (B \cap C')] = A \cap (B \cap C') = A \cap (B - C) = LHS$.
(c) $LHS = A - (B - C) = A - (B \cap C') = A \cap (B \cap C')' = A \cap (B' \cup C)$
$\Rightarrow = (A \cap B') \cup (A \cap C) = (A - B) \cup (A \cap C) = RHS$.
(d) RHS : $A - (B \cup C) = A \cap (B \cup C)' = A \cap (B' \cap C')$
$\Rightarrow = (A \cap B') \cap (A \cap C') = (A - B) \cap (A - C) = LHS$.

Q13. Let $a \in A$. Then $a \in B$ (as $A \subset B$
This implies, $a \in C$ (as $B \subset C$
Therefore, $a \in A$ as well as $a \in C$ so, $A \subset C$.

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TYPE-E

Q01. Let A and B denote the set of Indians who like grapes and pineapple respectively.
Given $n(A) = 84\%$, $n(B) = 45\%$, $n(A \cup B) = 100\%$, $n(A \cap B) = ?$
$\therefore n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$\Rightarrow n(A \cap B) = 84\% + 45\% - 100\% = 29\%$

So, 29% of the Indians like both grapes and pineapple.

Q02. Here n (A) = 720, n (B) = 450, $n(A \cup B) = 1000$, $n(A \cap B) = ?$, where A and B denote the set of consumers who like product A and B respectively.

$\therefore n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$\Rightarrow n(A \cap B) = 720 + 450 - 1000$

That is, $n(A \cap B) = 170$

Therefore at least 170 consumers must have liked both the products.

Q03. Let H and E denote the set of people who can speak Hindi and English respectively.

Given $n(H) = 500$, $n(E) = 320$, $n(H \cup E) = 800$, $n(H \cap E) = ?$

$\therefore n(H \cap E) = n(H) + n(E) - n(H \cup E)$

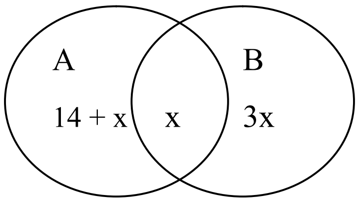
$\Rightarrow n(H \cap E) = 500 + 320 - 800 = 20$

(a) So, 20 people can speak both Hindi and English.

(b) $n(H - E) = n(H) - n(H \cap E) = 500 - 20 = 480$.

Therefore, 480 people can speak Hindi only.

Q04.



Venn-Diagram

$\because n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\because n(A) + n(B - A) = n(A - B) + n(A \cap B) + n(A) - x$

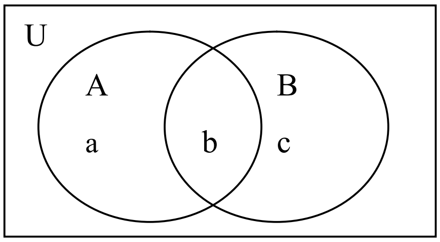
$\Rightarrow 3x = 14 + x + x - x$

$\Rightarrow 3x - x = 14$

$\therefore x = 7$

Q05. Let C_1 and C_2 denote the set of people who had been exposed to chemical C_1 and C_2 respectively.

Consider the adjacent venn diagram.



Given $n(U) = 200$, $n(C_1) = a + b = 120$,

$n(C_2) = b + c = 50, n(C_1 \cap C_2) = b = 30$

Solving these eqs., we get : $a = 90$, $b = 30$, $c = 20$

(a) The number of people who were exposed to chemical C_1 but not chemical C_2

$= n(C_1 - C_2) = a = 90$

(b) Number of people who were exposed to chemical C_2 but not chemical C_1

$= n(C_2 - C_1) = c = 20$

(c) Number of people who were exposed to chemical C_1 or chemical C_2

$= n(C_1 \cup C_2) = a + b + c = 90 + 30 + 20 = 140$

(d) Number of people who were exposed to none of the chemicals

$= n(U) - n(C_1 \cup C_2) = 200 - 140 = 60$.

Q06. Let B, H and F denote the set of Basketball team, Hockey team and Football team respectively.

Given $n(B) = 21$, $n(H) = 26$, $n(F) = 29$, $n(H \cap B) = 14$,

$n(H \cap F) = 15$, $n(F \cap B) = 12$, $n(H \cap B \cap F) = 8$.

Also, $n(H \cup B \cup F) = ?$

$\because n(H \cup B \cup F) = n(H) + n(B) + n(F) - n(H \cap B) - n(B \cap F) - n(H \cap F) + n(H \cap B \cap F)$

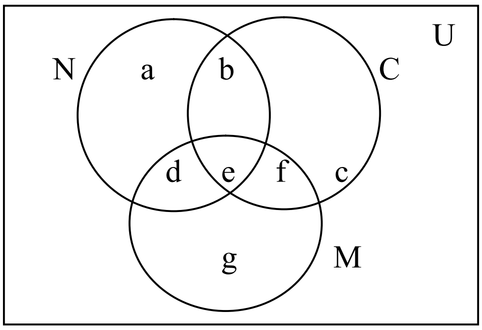
$\Rightarrow n(H \cup B \cup F) = 26 + 21 + 29 - 14 - 12 - 15 + 8$

$\therefore n(H \cup B \cup F) = 43$

Hence 43 members are there in all.

Q07. Let N, C, M denote the set of teachers who like reading newspapers, learning computers and watching movies on TV respectively.

Consider the Venn diagram shown.



$\therefore n(U)=100,\ n(N \text{ only})=a=15,$

$n(C \text{ only})=c=12,$

$n(M \text{ only})=g=8,$

$n(N\cap M)=d+e=40,$

$n(C\cap M)=f+e=20,$

$n(C\cap N)=b+e=10,$

$n(M)=d+e+f+g=65$

Solving these equations simultaneously, we get : b = 7, d = 37, e = 3, f = 17.

(i) Numbers of teachers who like reading newspapers = a + b + d +e = 62

(ii) Numbers of teachers who like learning computers = b + c + e + f = 39

(iii) Numbers of teachers who did not like to do any of the things mentioned above

$$=n(U)-n(N\cup C\cup M)=100-(a+b+c+d+e+f+g)=1.$$

Q08. Let A and B denote the set of students who were listed as smokers and chewers of Gum respectively.

$\because n(A)=100,\ n(B)=150,\ n(A\cap B)=75$

$\therefore n(A\cup B)=n(A)+n(B)-n(A\cap B)=100+150-75=175$

Therefore, $n(A'\cap B')=n[(A\cup B)']=n(U)-n(A\cup B)=400-175=225$.

Hence 225 students are neither smokers nor gum chewers.

Q09. Let B and M denote the set of students in a Biology class and in Mathematics class respectively. Also given that n (B) = 20, n (M) = 30 and, n ($B\cap M$) = 10,

So n($B\cup M$) = 20 + 30 – 10 = 40.

(a) Therefore 40 students are either in Mathematics or in Biology class when the classes meet at different hours.

(b) Since $B\cap M=\phi\ \therefore n(B\cap M)=0$

So, $n(B\cup M)=20+30-0=50$.

Therefore 50 students are either in Mathematics or in Biology class when the classes meet at the same hour.

Q10. Let M and B denote the set of people who have taken Mathematics and Biology respectively.

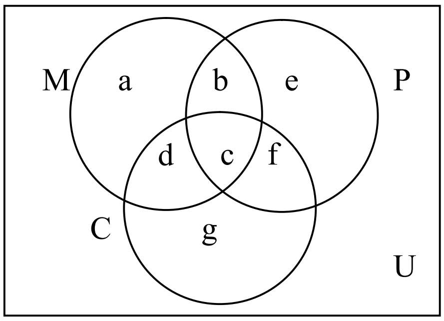
$\therefore n(M)=12,\ n(M-B)=8,\ n(M\cup B)=25,\ n(M\cap B)=?,\ n(B-M)=?$

So, $n(M\cap B)=n(M)-n(M-B)=12-8=4$

and, $n(B-M)=n(M\cup B)-n(M)=25-12=13$.

Note that you can make Venn diagram for the purpose of understanding.

Q11. Let M, P and C denote the sets of the students enrolled in Mathematics, Physics and Chemistry respectively. Consider the Venn diagram.



$\therefore n(U)=175,\ n(M)=a+b+c+d=100,$

$n(P)=b+c+e+f=70,$

$n(C)=d+c+g+f=46,$

$n(M\cap P)=b+c=30,$

$n(M\cap C)=d+c=28,$

$n(P\cap C)=f+c=23,$

$n(P\cap C\cap M)=c=18$

Solving these equations simultaneously, we get :

$a = 60,\ b = 12,\ c = 18,\ d = 10,\ e = 35,\ f = 5,\ g = 13$

So, no. of students enrolled in Mathematics alone $= a = 60$,

No. of students enrolled in Physics alone $= e = 35$,

No. of students enrolled in Chemistry alone $= g = 13$,

And, no. of students who have not been offered any of these subjects

$$= n(U) - n(M \cup P \cup C) = 175 - (a + b + c + d + e + f + g) = 22.$$

Q12. Given $n(A) = 3,\ n(B) = 6$.

Since $n(A \cup B)$ will be maximum when $n(A \cap B) = 0$.

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B) = 3 + 6 - 0 = 9$.

Also $n(A \cup B)$ will be minimum when $n(A \cap B)$ is maximum i.e., when $n(A \cap B) = 3$

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B) = 3 + 6 - 3 = 6$

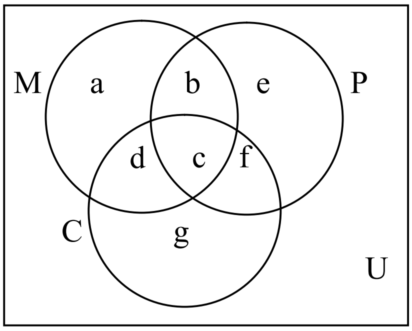
Q13. Given n(U) = 700, n(A) = 200, n(B) = 300 and $n(A \cap B)$ = 100.

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B) = 200 + 300 - 100 = 400$

So, $n(A' \cap B') = n[(A \cup B)'] = n(U) - n(A \cup B) = 700 - 400 = 300$.

Q14. Let M, P and C denote the sets of the students enrolled in Mathematics, Physics and Chemistry respectively.

Consider the Venn diagram.



$\therefore n(U) = 25,\ n(M) = a + b + c + d = 15$,

$n(P) = b + c + e + f = 12$,

$n(C) = d + c + g + f = 11$,

$n(M \cap P) = b + c = 9$,

$n(M \cap C) = d + c = 5$,

$n(P \cap C) = f + c = 4$,

$n(P \cap C \cap M) = c = 3$

Solving these equations simultaneously, we get :

$a = 4,\ b = 6,\ c = 3,\ d = 2,\ e = 2,\ f = 1,\ g = 5$

So, no. of students that had

(a) only Chemistry $= g = 5$,

(b) only Mathematics $= a = 4$,

(c) only Physics $= e = 2$,

(d) Physics and Chemistry but not Mathematics $= f = 1$,

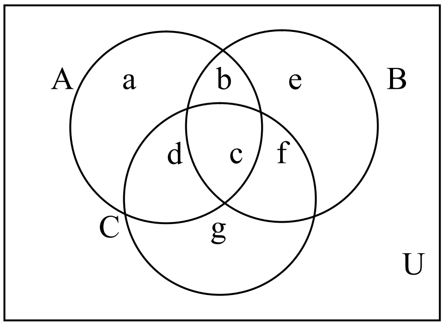
(e) Mathematics and Physics but not Chemistry $= b = 6$,

(f) only one of the subjects $= a + e + g = 4 + 2 + 5 = 11$,

(g) at least one of the three subjects $= a + b + c + d + e + f + g = 23$,

(h) none of the subjects $n(U) - n(M \cup P \cup C) = 25 - (a + b + c + d + e + f + g) = 2$.

Q15. Let A, B and C denote the sets of the people watching program A, program B and program C respectively. Consider the Venn diagram.



$\therefore n(U) = 100\%,\ n(A) = a + b + c + d = 60\%$,

$n(B) = b + c + e + f = 50\%$,

$n(C) = d + c + g + f = 47\%$,

$n(A \cap B) = b + c = 28\%$,

$n(A \cap C) = d + c = 23\%$,

$n(B \cap C) = f + c = 18\%$,

$n(B \cap C \cap A) = c = 8\%$

Solving these equations simultaneously, we get :

$a = 17\%,\ b = 20\%,\ c = 8\%,\ d = 15\%,\ e = 12\%,\ f = 10\%,\ g = 14\%$.

(a) The percentage of people who watch program A and B but not C $= b = 20\%$

(b) The percentage of people who watch exactly two programs $= b + d + f = 45\%$

(c) The percentage of people who do not watch any program

$$= n(U) - n(M \cup P \cup C) = 100\% - (a + b + c + d + e + f + g) = 4\%.$$

Q16. Do yourself. Ans. : (a) 33%, (b) 14%, (c) 40% (d) 8% (e) 48% (f) 4% (g) 58%.

Q17. Do yourself. Ans. : 215, 50.

Q18. Do yourself. Ans. : 20, 325.

Q19. Let total no. of judges be x.

Also let set A : no. of judges who voted for Anamika, B : no. of judges who voted for Bhawna.

Clearly, $n(U) = x,\ n(A) = \frac{x}{2},\ n(B) = \frac{2x}{3},\ n(A \cap B) = 10,\ n(U) - n(A \cup B) = 6$

$\because n(U) - [n(A) + n(B) - n(A \cap B)] = 6 \qquad \Rightarrow x - \left[\frac{x}{2} + \frac{2x}{3} - 10\right] = 6 \qquad \therefore x = 24.$

So there were 24 judges were present.

Q20. Do yourself. Ans. (a) 35 (b) 11 (c) 11.

Q21. $n[(C \cup T)'] = n(U) - n(C \cup T) = 450 - [n(C) + n(T) - n(C \cap T)] = 450 - 200 = 250.$

Q22. $\because n(A \cap B)' = n(U) - n(A \cap B) = n(A \cup B) + n(A \cup B)' - n(A \cap B)$

$\Rightarrow n(A \cap B)' = n(A \cup B) + n(A' \cap B') - n(A \cap B) = 21 + 9 - 7 = 23.$

Q23. Here $A = \{1, 3, 5, 15\},\ B = \{2, 3, 5, 7\},\ C = \{2, 4, 6, 8\}$

$\therefore (A \cup C) \cap B = (\{1, 3, 5, 15\} \cup \{2, 4, 6, 8\}) \cap \{2, 3, 5, 7\} = \{1, 2, 3, 4, 5, 6, 8, 15\} \cap \{2, 3, 5, 7\} = \{2, 3, 5\}.$