Your Name : ______________________________

School Code | | | | | |

Date Of Birth | d | d | m | m | y | y | y | y |

CBSE Roll No. | | | | | | | |

Code No. 19/2/5

Candidates must write the Code on the title page of the answer-book.

Series : ATS/05

ACHIEVERS TEST SERIES XII – 05

A Compilation By : O.P. Gupta (WhatsApp @ +91 9650350480)

For more stuffs on Maths, please visit : www.theOPGupta.com

Time Allowed : 180 Minutes **Max. Marks : 100**

SECTION A

Q01. Find the integrating factor of $\cos x\,dy + y\,dx = 2\sin x\,dx$.

Q02. Write the value of $\sin^{-1}\cos(\pi/9)$?

Q03. Find the angle between x-axis and $\hat{i}+\hat{j}+\hat{k}$.

Q04. Evaluate : $\begin{vmatrix} x & 2x & 3x \\ a & b & c \\ y & 2y & 3y \end{vmatrix}$.

OR If A is a matrix of order 2 and $|A^{-1}| = 1/2$ then, find $|2A|$.

SECTION B

Q05. Write the differential equation of the family of parabolas corresponding to $y^2 = 4ax,\ a > 0$.

OR Given that $dy/dx = ye^x$ and x = 0, y = e. Find the value of y when x = 1.

Q06. Find the value of $(y-x)$ in the matrix equation : $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$.

OR Define symmetric matrix and skew symmetric matrix.

Q07. Find the angle θ, between the line $\frac{x-2}{3} = \frac{y-3}{5} = \frac{z-4}{4}$ and the plane $2x-2y+z-5=0$.

Q08. The vectors $\vec{a} = 3\hat{i}+x\hat{j}$ and $\vec{b} = 2\hat{i}+\hat{j}+y\hat{k}$ are mutually perpendicular. If $|\vec{a}| = |\vec{b}|$, then find the value of y.

OR Write the vector projection of the vector $2\hat{i}+3\hat{j}-\hat{k}$ along $\hat{i}+\hat{j}$.

Q09. Write the domain of $\sec^{-1}(2x+1)$.

Q10. Find $\int \lambda dx$, if $\lambda = 2x$ s.t. $\sin\cot^{-1}(1+x) = \cos\tan^{-1}x$.

Q11. Let $f = \{(1,2),(3,5),(4,1)\}$ and $g = \{(2,3),(5,1),(1,3)\}$, the write the value of fog.

Q12. If $P(A) = 3/8,\ P(B) = 1/2,\ P(A\cap B) = 1/4$, find $P(\bar{A}\,|\,\bar{B})$.

SECTION C

Q13. For the function $f(x) = x^3 - bx^2 + a\,x,\ x\in[1,3]$, Rolle's Theorem holds with $c = 2+3^{-1/2}$. Find the values of a and b.

Q14. Differentiate $\cos^{-1}(x) + \cos^{-1}\left(\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right)$ w.r.t. x.

Q15. An amount of ₹600 crores is spent by the government in three schemes. Scheme A is for saving girl child from the cruel parents who don't want girl child and get the abortion before her birth. Scheme B is for saving of newlywed girls from death due to dowry. Scheme C is planning for good health for senior citizen. Now twice the amount spent on Scheme C together with amount spent on Scheme A is ₹700 crores. And three times the amount spent on Scheme A together with amount spent on Scheme B and Scheme C is ₹1200 crores. Find the amount spent on each schemes using matrices.

Q16. Evaluate $\int \frac{x\,dx}{1+x\tan x}$.

Q17. Evaluate : $\int \cos 2\theta \log\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right) d\theta$.

Q18. Solve the differential equation $dy-(3y\cot x+\sin 2x)dx=0$ given $y=2$ when $x=\pi/2$.

OR Solve : $\left(x\sin^2(y/x)-y\right)dx+xdy=0$ given $y=\pi/4$ when $x=1$.

Q19. Scalar product of $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c}=\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ and hence find the unit vector along $\vec{b}+\vec{c}$.

OR Find a unit vector perpendicular to the plane of triangle ABC, where the coordinates of its vertices are A(3, –1, 2), B(1, –1, –3) and C(4, –3, 1).

Q20. Find the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R. Also find the maximum volume.

Q21. A manufacturing company makes two type of teaching aids A and B of Mathematics of class XII. Each type of A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 hours, respectively. The profit on type A and B is ₹80 and ₹120 per piece, respectively. Formulate an LPP for the company to maximize its profit. Hence, solve it.

Q22. A class has 15 students whose ages are 14,17,15,14,21,17,19,20,16,18,20,17,16,19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded.
What is the probability distribution of the random variable X? Find the mean of X.

OR An urn has 2 white and 2 black balls. A ball is drawn at random. If it is white, it is not replaced into the urn. Otherwise, it is replaced with another ball of the same colour. The process is repeated. Find the probability that the third ball drawn is black.

Q23. There are 3 coins. One is two-headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tails 40% of the times. One of the three coins is chosen at random and tossed, and it shows heads. What is the probability that it was the two-headed coin?

SECTION D

Q24. Evaluate : $\int_0^1 \sin^{-1}(x\sqrt{1-x}-\sqrt{x-x^3})\,dx$.

Q25. On the set {0, 1, 2, 3, 4, 5, 6}, a binary operation * is defined as : $a*b=\begin{cases} a+b, \text{ if } a+b<7 \\ a+b-7, \text{ if } a+b\ge 7\end{cases}$.

Write the operation table of the operation * and prove that zero is the identity for this operation and each element $a\neq 0$ of the set is invertible with '7 – a' being the inverse of 'a'.

Q26. If $a+b+c\neq 0$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b\end{vmatrix}=0$, then using properties of determinants, prove that $a=b=c$.

OR If $A=\begin{bmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1\end{bmatrix}$, find A^{-1} using elementary row transformations.

Q27. If $\lambda=\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ then, prove that $\sin 2\lambda = x^2$.

OR Show that $2\tan^{-1}\left(\sqrt{\dfrac{a-b}{a+b}}\tan\dfrac{\theta}{2}\right)=\cos^{-1}\left(\dfrac{a\cos\theta+b}{a+b\cos\theta}\right)$.

Q28. Prove that the image of (3,–2, 1) in the plane 3x – y + 4z = 2 lies on plane x + y + z + 4 = 0.

OR From the point P(a, b, c), perpendiculars PL and PM are drawn to YZ and ZX planes respectively. Find the equation of the plane OLM. Hence find its distance from (a, –b, –c)

Q29. Find the area of the region $\{(x,y): x^2+y^2\le 4,\ x+y\ge 2\}$, using method of integration. ◘◘◘

SECTION A

Q01. We've $\cos x\frac{dy}{dx}+y=2\sin x$ $\Rightarrow \frac{dy}{dx}+\sec x.y=2\tan x$

Here $P(x)=\sec x,\ Q(x)=2\tan x$

$\therefore$ Integrating Factor $=e^{\int \sec x dx}=e^{\log(\sec x+\tan x)}=\sec x+\tan x$.

Q02. $\sin^{-1}\cos\frac{\pi}{9}=\frac{\pi}{2}-\cos^{-1}\cos\frac{\pi}{9}=\frac{7\pi}{18}$.

Q03. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ **Q04.** 0 **OR** $|A^{-1}|=\frac{1}{|A|}=\frac{1}{2}\Rightarrow|A|=2\ \therefore|2A|=2^2|A|=8$.

SECTION B

Q05. We have $y^2=4ax,\ a>0$ $\Rightarrow \frac{y^2}{x}=4a$ $\Rightarrow \frac{x\times 2yy'-y^2\times 1}{x^2}=0$

Therefore, the required diff. eq. is $2x\frac{dy}{dx}-y=0$.

OR e^e

Q06. We have $2\begin{bmatrix}x & 5\\ 7 & y-3\end{bmatrix}+\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}=\begin{bmatrix}7 & 6\\ 15 & 14\end{bmatrix}$ $\Rightarrow \begin{bmatrix}2x+3 & 6\\ 15 & 2y-4\end{bmatrix}=\begin{bmatrix}7 & 6\\ 15 & 14\end{bmatrix}$

By equality of matrices, we get : $2x+3=7,\ 2y-4=14$ $\Rightarrow x=2, y=9$.

Hence $(y-x)=9-2=7$.

Q07. The d.r.'s of the given line $\frac{x-2}{3}=\frac{y-3}{5}=\frac{z-4}{4}$ is 3, 5, 4 $\therefore \vec{b}=3\hat{i}+5\hat{j}+4\hat{k}$.

Also the d.r.'s of the normal to the plane $2x-2y+z-5=0$ is 2, –2, 1 $\therefore \vec{m}=2\hat{i}-2\hat{j}+\hat{k}$.

Angle between the line and the plane is, $\sin\theta=\frac{\vec{b}.\vec{m}}{|\vec{b}||\vec{m}|}$ $\Rightarrow \sin\theta=\frac{(3\hat{i}+5\hat{j}+4\hat{k}).(2\hat{i}-2\hat{j}+\hat{k})}{\sqrt{9+25+16}\sqrt{4+4+1}}$

$\Rightarrow \sin\theta=\frac{6-10+4}{5\sqrt{2}\times 3}=0$ $\therefore \theta=0^{o}$.

Q08. $\because \vec{a}=3\hat{i}+x\hat{j}$ & $\vec{b}=2\hat{i}+\hat{j}+y\hat{k}$ are mutually $\perp^{er}$ so, $\vec{a}.\vec{b}=0$ i.e., $(3\hat{i}+x\hat{j}).(2\hat{i}+\hat{j}+y\hat{k})=0$

$\therefore 6+x=0\Rightarrow x=-6$

Also $|\vec{a}|=|\vec{b}|$ so, $\sqrt{3^2+x^2}=\sqrt{2^2+1^2+y^2}$ $\Rightarrow 4+x^2=y^2$

$\Rightarrow 4+(-6)^2=y^2$ $\Rightarrow 40=y^2$ $\therefore y=\pm 2\sqrt{10}$.

OR Vector projection of $2\hat{i}+3\hat{j}-\hat{k}$ along the vector $\hat{i}+\hat{j}$ is $\left(\frac{(2\hat{i}+3\hat{j}-\hat{k}).(\hat{i}+\hat{j})}{|\hat{i}+\hat{j}|}\right)\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

$\Rightarrow =\left(\frac{2+3-0}{\sqrt{1^2+1^2}}\right)\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)=\frac{5(\hat{i}+\hat{j})}{2}$.

We have used, vector projection of $\vec{a}$ on $\vec{b}=(\vec{a}.\hat{b})\hat{b}$.

Q09. $x\in(-\infty,-1]\cup[0,\infty)$.

Q10. We have $\sin\sin^{-1}\frac{1}{\sqrt{2+x^2+2x}}=\cos\cos^{-1}\frac{1}{\sqrt{1+x^2}}\Rightarrow x=-\frac{1}{2}$ i.e., $2x=-1=\lambda$.

$\therefore \int\lambda dx=\int -1dx=-x+C$.

Q11. {(2, 5), (5, 2), (1, 5)}.

Q12. As $P(\bar{A}|\bar{B})=\dfrac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}=\dfrac{1-P(A\cup B)}{1-P(B)}=\dfrac{1-\{P(A)+P(B)-P(A\cap B)\}}{1-P(B)}=\dfrac{3}{4}$.

SECTION C

Q13. See Mathematicia by O.P. Gupta. Ans. $a=11,\ b=6$.

Q14. $\dfrac{d}{dx}\left(\cos^{-1}(x)+\cos^{-1}\left(\dfrac{x}{2}+\dfrac{\sqrt{3-3x^2}}{2}\right)\right)=\dfrac{d}{dx}\left(\dfrac{\pi}{3}\right)=0$.

Q15. ₹300 crores, ₹100 crores and ₹200 crores respectively for Scheme A, B and C.

Q16. Let $I=\int\dfrac{x\ dx}{1+x\tan x}$ $\Rightarrow I=\int\dfrac{x\cos x\ dx}{\cos x+x\sin x}$ $\left[\begin{array}{l}\text{Put } \cos x+x\sin x=t\\ \Rightarrow(-\sin x+x\cos x+\sin x)dx=x\cos x dx=dt\end{array}\right.$

$\Rightarrow I=\int\dfrac{dt}{t}=\log|t|+C$ $\therefore I=\log|\cos x+x\sin x|+C$, where C is integral constant.

Q17. $\dfrac{\sin 2\theta}{2}\log\tan\left(\dfrac{\pi}{4}+\theta\right)-\dfrac{1}{2}\log\sec 2\theta+k$

Q18. We have $\dfrac{dy}{dx}-3y\cot x=\sin 2x$

On comparing with $\dfrac{dy}{dx}+P(x)y=Q(x)$ we get : $P(x)=-3\cot x,\ Q(x)=\sin 2x$

Now I. F. $=e^{\int P(x)dx}=e^{\int -3\cot x dx}=e^{-3\log\sin x}=e^{\log(\sin x)^{-3}}=(\sin x)^{-3}=\dfrac{1}{\sin^3 x}$

The solution is given by $y\,(\text{I. F.})=\int Q(x)\,(\text{I. F.})dx+C$

$\Rightarrow y\left(\dfrac{1}{\sin^3 x}\right)=\int\sin 2x\left(\dfrac{1}{\sin^3 x}\right)dx+C$ $\Rightarrow\dfrac{y}{\sin^3 x}=2\int\sin x\cos x\left(\dfrac{1}{\sin^3 x}\right)dx+C$

$\Rightarrow\dfrac{y}{\sin^3 x}=2\int\operatorname{cosec} x\cot x\,dx+C$ $\Rightarrow\dfrac{y}{\sin^3 x}=-2\operatorname{cosec} x+C$

Given $y=2$ when $x=\dfrac{\pi}{2}$ so, $\dfrac{2}{\sin^3(\pi/2)}=-2\operatorname{cosec}\dfrac{\pi}{2}+C$ $\Rightarrow C=4$

Hence the required solution is given as follow :

$\dfrac{y}{\sin^3 x}=4-2\operatorname{cosec} x$ or, $y=4\sin^3 x-2\sin^2 x$.

OR Given $\left(x\sin^2\left(\dfrac{y}{x}\right)-y\right)dx+xdy=0$ $\Rightarrow xdy=-\left(x\sin^2\left(\dfrac{y}{x}\right)-y\right)dx$

$\Rightarrow\dfrac{dy}{dx}=\dfrac{y}{x}-\sin^2\left(\dfrac{y}{x}\right)$...(i) Put $y=vx\Rightarrow\dfrac{dy}{dx}=v+x\dfrac{dv}{dx}$

Replacing value of $\dfrac{dy}{dx}$ in (i), we get : $v+x\dfrac{dv}{dx}=v-\sin^2 v$ $\Rightarrow x\dfrac{dv}{dx}=-\sin^2 v$

$\Rightarrow\int-\operatorname{cosec}^2 v\,dv=\int\dfrac{dx}{x}$ $\Rightarrow\cot v=\log|x|+C$ $\Rightarrow\cot\left(\dfrac{y}{x}\right)=\log|x|+C$

Given that $y=\dfrac{\pi}{4}$ when $x=1$ so, $\cot\left(\dfrac{\pi/4}{1}\right)=\log|1|+C$ $\Rightarrow C=1$

Hence the required solution is $\cot\left(\dfrac{y}{x}\right)=\log|x|+1$ or $y=x\cot^{-1}(\log|x|+1)$.

Q19. Value of $\lambda=1$, Unit vector $=\dfrac{3\hat{i}+6\hat{j}-2\hat{k}}{7}$.

OR Given vertices of triangle ABC are A(3, –1, 2), B(1, –1, –3) and C(4, –3, 1).

Let $\vec{p}=\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\hat{i}-\hat{j}-3\hat{k}-3\hat{i}+\hat{j}-2\hat{k}=-2\hat{i}-5\hat{k}$,

and $\vec{q}=\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=4\hat{i}-3\hat{j}+\hat{k}-3\hat{i}+\hat{j}-2\hat{k}=\hat{i}-2\hat{j}-\hat{k}$

So, required unit vector $\hat{r}=\pm\dfrac{\vec{p}\times\vec{q}}{|\vec{p}\times\vec{q}|}$, where $\vec{r}=\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & 0 & -5\\ 1 & -2 & -1\end{vmatrix}=-10\hat{i}-7\hat{j}+4\hat{k}$

$\Rightarrow \hat{r}=\pm\dfrac{-10\hat{i}-7\hat{j}+4\hat{k}}{\sqrt{100+49+16}}=\pm\left(\dfrac{4\hat{k}-10\hat{i}-7\hat{j}}{\sqrt{165}}\right)$.

Q20. $\dfrac{2R}{\sqrt{3}}$, $\left(\dfrac{4\pi R^3}{3\sqrt{3}}\right)$cubic units.

Q21. Let no. of pieces of teaching aids of type A and B produced per week be x and y respectively.

To maximize : $Z=₹(80x+120y)$.

Subject to constraints : $9x+12y\le 180,\ x+3y\le 30,\ x\ge 0,\ y\ge 0$

Note that this LPP will have maximum profit of ₹1680 at (12, 6).

Q22. The probability distribution of the random variable X is :

X	14	15	16	17	18	19	20	21
P(X)	2/15	1/15	2/15	3/15	1/15	2/15	3/15	1/15

The mean of random variable $X=\sum X\,P(X)=\dfrac{263}{15}$.

OR Require probability $=P(WWB)+P(WBB)+P(BWB)+P(BBB)$

$\Rightarrow \quad =\dfrac{2}{4}\times\dfrac{1}{3}\times\dfrac{2}{2}+\dfrac{2}{4}\times\dfrac{2}{3}\times\dfrac{2}{3}+\dfrac{2}{4}\times\dfrac{2}{4}\times\dfrac{2}{3}+\dfrac{2}{4}\times\dfrac{2}{4}\times\dfrac{2}{4}=\dfrac{49}{72}$.

Q23. Let E_1 : choosing first (two headed) coin, E_2 : choosing second (biased) coin, E_3 : choosing third coin. Also, let A : the coin showing heads.

$\therefore\ P(E_1)=P(E_2)=P(E_3)=\dfrac{1}{3}$, $P(A|E_1)=1$, $P(A|E_2)=\dfrac{75}{100}$, $P(A|E_3)=\dfrac{60}{100}$.

By Bayes' Theorem, $P(E_1|A)=\dfrac{P(A|E_1)P(E_1)}{P(A|E_1)P(E_1)+P(A|E_2)P(E_2)+P(A|E_3)P(E_3)}$

$$=\dfrac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{1}{3}\times\frac{3}{4}+\frac{1}{3}\times\frac{3}{5}}=\dfrac{20}{47}.$$

SECTION D

Q24. See Mathematicia by O.P. Gupta. Ans. $\dfrac{\pi}{4}-1$.

Q25. We have $a*b=\begin{cases}a+b, \text{ if } a+b<7\\ a+b-7, \text{ if } a+b\ge 7\end{cases}$ defined on the set A = {0, 1, 2, 3, 4, 5, 6}.

Operation table of the binary operation * is given below :

*	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

Let x be the identity for an element a.

So $a * x = a$. If $a + x < 7$ then, $a + x = a$ or if $a + x \geq 7$ then, $a + x - 7 = a$

That is, $a + x < 7$ then, $x = 0 \in A$ or if $a + x \geq 7$ then, $x = 7 \notin A$

Therefore, $x = 0$ is the identity element for this operation.

Also let y be the inverse of each non-zero element a. Then $a * y = 0$.

If $a + y < 7$ then, $a + y = 0$ or if $a + y \geq 7$ then, $a + y - 7 = 0$

i.e., $a + y < 7$ then, $y = -a \notin A$ for all $a \in A$ or if $a + y \geq 7$ then, $y = 7 - a \in A$ for all $a \in A - \{0\}$

$\therefore y = 7 - a$ is the inverse of each non-zero element a.

Q26. See Mathematicia by O.P. Gupta.

OR Given $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{bmatrix}$ Using elementary row operations, we have $A = IA$

i.e., $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ By $R_2 \to R_2 - 2R_1$

$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & 7 & 3 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ By $R_1 \to R_1 + \frac{1}{2} R_3$

$\Rightarrow \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 7 & 3 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1/2 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ By $R_3 \to R_3 - \frac{2}{7} R_2$

$\Rightarrow \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 7 & 3 \\ 0 & 0 & 1/7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1/2 \\ -2 & 1 & 0 \\ 4/7 & -2/7 & 1 \end{bmatrix} A$ By $R_2 \to \frac{1}{7} R_2$

$\Rightarrow \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 3/7 \\ 0 & 0 & 1/7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1/2 \\ -2/7 & 1/7 & 0 \\ 4/7 & -2/7 & 1 \end{bmatrix} A$ By $R_3 \to 7R_3$

$\Rightarrow \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 3/7 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1/2 \\ -2/7 & 1/7 & 0 \\ 4 & -2 & 7 \end{bmatrix} A$ By $R_2 \to R_2 - \frac{3}{7} R_3$

$\Rightarrow \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1/2 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{bmatrix} A$ By $R_1 \to R_1 - \frac{1}{2} R_3$

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{bmatrix} A$ $\because I = A^{-1}A$ $\therefore A^{-1} = \begin{bmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{bmatrix}$.

Q27. See Mathematicia by O.P. Gupta **OR** See Mathematicia by O.P. Gupta

Q28. Hint : Obtain the image (0,–1,–3) of given point in 1st plane and then show that it satisfies the equation of 2nd plane.

OR Since perpendiculars PL and PM are drawn to the planes YZ and ZX respectively from P (a, b, c). So the coordinates of L and M on required plane OLM are L (0, b, c) and M (a, 0, c). Also we know that O (0, 0, 0).

Equation of required plane OLM is, $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} x-0 & y-0 & z-0 \\ 0-0 & b-0 & c-0 \\ a-0 & 0-0 & c-0 \end{vmatrix} = 0 \qquad \Rightarrow \begin{vmatrix} x & y & z \\ 0 & b & c \\ a & 0 & c \end{vmatrix} = 0$

Expanding along R_1, we get : $bcx + acy - abz = 0$ or, $\frac{x}{a} + \frac{y}{b} - \frac{z}{c} = 0$.

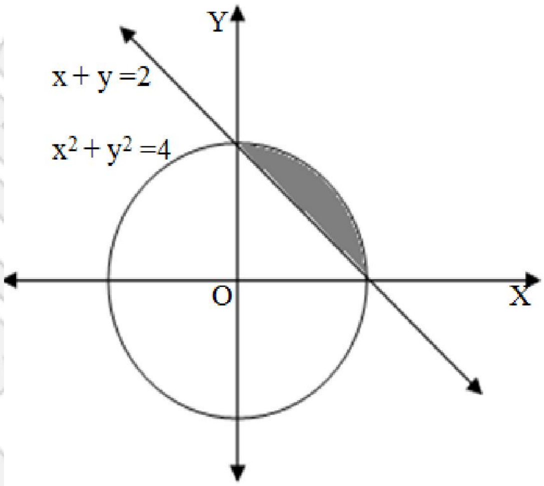
Also, the required distance from (a, –b, –c) is $= \dfrac{\left|\frac{a}{a} - \frac{b}{b} + \frac{c}{c} - 0\right|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = \dfrac{abc}{\sqrt{a^2b^2 + b^2c^2 + c^2a^2}}$ units.

Q29. We have $\{(x, y) : x^2 + y^2 \le 4,\ x + y \ge 2\}$

Let $x^2 + y^2 = 4$ ...(i) and $x + y = 2$ ...(ii)

On solving (i) & (ii) we get the point of intersection as (0, 2) and (2, 0).

Now, the area of the shaded region bounded by curves (i) & (ii) $= \int_0^2 \left(\sqrt{4-x^2} - (2-x)\right) dx$



$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2} + \frac{(2-x)^2}{2}\right]_0^2$

$= \left[0 + 2\sin^{-1}\frac{2}{2} + 0\right] - [0 + 0 + 2]$

$\Rightarrow \quad = (\pi - 2)$ sq.units.

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