Your Name : ____________________________

School Code | | | | | |

Date Of Birth | d | d | m | m | y | y | y | y |

CBSE Roll No. | | | | | | | |

Code No. 19/2/4

Candidates must write the Code on the title page of the answer-book.

Series : ATS/04

ACHIEVERS TEST SERIES XII – 04

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Time Allowed : 180 Minutes **Max. Marks : 100**

SECTION A

Q01. Write the maximum and minimum value of $f(x) = \cos(\cos x)$.

OR Find an angle θ, such that rate of change of this angle is twice as fast as its sine.

Q02. How many binary operations can be defined on the set $\{\sqrt{2}-1, \sqrt{2}+1\}$?

Q03. For a matrix A of order n, it is given that $|A| = 4$, if $|adj.A| = 16$, then find the value of n.

Q04. Write the order and degree of $y_2 + (y_1)^{1/3} + 3x = 0$.

SECTION B

Q05. If $\cos^{-1} x + \cos^{-1} y = 2\pi$, then find the value of $x^{100} + y^{200} + \frac{1}{x^{100} y^{200}}$.

OR Find the domain of the definition of the function $f(x) = \sin^{-1}\left(|x-1| - 2\right)$.

Q06. Find $\int x^{2019\,X}(1+\log x)dx$. **OR** Find $\int (2^x + 2^{-x})^2 dx$.

Q07. A bag contains 3 red and 7 black balls. Two balls are selected at random one-by-one without replacement. If the second selected ball happens to be red, what is the probability that the first selected ball is also red? **OR** If A and B are mutually exclusive events, find P(A|B).

Q08. Let $\vec{a}, \vec{b}, \vec{c}$ be the unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{x}$, $\vec{a}.\vec{x} = 1$, $\vec{b}.\vec{x} = 3/2$, $|\vec{x}| = 2$ then, find the angle between $\vec{c}$ and $\vec{x}$.

Q09. Using normal unit vector form of the equation of plane, find the length of perpendicular drawn from origin to the plane $3x + 4y - 5z = 50$.

Q10. The volume of a cube is rising at a constant rate. Prove that the increase in surface area varies inversely as the length of the edge of the cube.

Q11. If $AB = AC \Rightarrow B = C$ then, write the condition on matrix A. Given that A, B and C are non-zero square matrices of same order.

Q12. Evaluate $\int_3^4 \frac{3dx}{\sqrt{3x-8}}$.

SECTION C

Q13. An amount of ₹5000 is put into three investments at the rate of 6%, 7% and 8% per annum respectively. The total annual income is ₹358. If the combined income from the first two investments is ₹70 more than the income from the third, find the amount of each investment by using matrix method.

OR Find the values of x and y if $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} x & 1 \\ y & -1 \end{bmatrix}$ such that $A^2 + B^2 = (A+B)^2$.

Q14. Find all the points on the curve $y^2 = 2a[x + a\sin(x/a)]$ at which tangent is parallel to the x-axis.

OR If the function $f(x) = 2x^3 - 9m x^2 + 12m^2 x + 1$, where $m > 0$ attains its maximum and minimum values at p and q respectively such that $p^2 = q$, then find the value of m.

Q15. Discuss the continuity of $f(x)=\begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, \text{ if } x\neq 0 \\ 0, \text{ if } x=0 \end{cases}$ at $x=0$.

Is it differentiable at the same point?

Q16. Solve : $\tan y\dfrac{dy}{dx}=\cos(x+y)+\cos(x-y)$.

Q17. Find the vector (s) of magnitude 6 units along the bisector of the angle between the vectors $4\hat{i}-4\hat{j}+2\hat{k}$ and $2\hat{i}+4\hat{j}-4\hat{k}$. **Q18.** Solve : $\left(\sqrt{x+y}+\sqrt{x-y}\right)dx+\left(\sqrt{x-y}-\sqrt{x+y}\right)dy=0$.

Q19. For what value of k, the line $\dfrac{x-2}{3}=\dfrac{y+1}{2}=\dfrac{z-1}{-1}$ intersects $x^2+y^2=k^2,\ z=0;\ k>0$?

OR If $\dfrac{x-2}{3}=\dfrac{y-1}{-5}=\dfrac{z+2}{2}$ lies in the plane $x+3y-\alpha z+\beta=0$, then find the value of $\alpha\times\beta$.

Q20. Evaluate $\displaystyle\int\frac{e^{2x}-2e^x}{e^{2x}+1}dx$. **Q21.** Prove that : $\displaystyle\int_0^{\pi/3}\cos^4 3\theta\sin^2 6\theta d\theta=\frac{5\pi}{96}$.

Q22. A bag I contains 3 white and 2 black marbles, while another bag II contains 2 white and 4 black marbles. A bag and a marble out of it is picked at random.
What is the probability that the marble is white?

Q23. The random variable X can take only the values 0, 1, 2. Given that P(X = 0) = P(X = 1) = p and $E(X^2)=E(X)$, then find the value of p.

SECTION D

Q24. A company makes two types of belts A and B. Profits on the belt of type A and B are ₹4 and ₹3 respectively. Each belt of type A requires twice as much time as a belt of type B and if all belts made were of type B, the company could make 1000 belts per day. The supply of leather is sufficient for only 800 belts per day (for both belts of type A and B combined). At the most 400 buckles for belt of type A and 700 for those of belt of type B are available per day. How many belts of each type should the company make per day so as to maximize the profit?

Q25. Consider the binary operations $\otimes: R\times R\to R$ and $\oplus: R\times R\to R$ defined as $a\otimes b=|a+b|$ and $a\oplus b=a$ for all $a,b\in R$. Show that $\otimes$ distributes over $\oplus$. Does $\oplus$ distribute over $\otimes$?

OR Show that the relation R in the set A of points in a plane given by
R = {(P, Q) : Distance of the point P from the origin is same as the distance of point Q from the origin}, is an equivalence relation.

Q26. If $x\neq y\neq z$ and $\begin{vmatrix} x & x^3 & x^4-1 \\ y & y^3 & y^4-1 \\ z & z^3 & z^4-1 \end{vmatrix}=0$ then prove that $xyz(xy+yz+zx)=x+y+z$.

OR Let $x=\begin{vmatrix} 1 & 1 & 1 \\ {}^nC_1 & {}^{n+2}C_1 & {}^{n+4}C_1 \\ {}^nC_2 & {}^{n+2}C_2 & {}^{n+4}C_2 \end{vmatrix}$. Find the value of $\sqrt[3]{x}$.

Q27. Find $\dfrac{dy}{dx}$, if $y=\sin^{-1}\left[\dfrac{6x-4\sqrt{1-4x^2}}{5}\right]$.

Q28. Find the smallest of the two areas in which the circle $x^2+y^2=4$ is divided by $y^2=6x-3$.

OR Find the area enclosed between $|y|-|x|=1$ and $x^2+y^2=1$.

Q29. Find the value (s) of k, such that $\dfrac{x-2}{1}=\dfrac{y-3}{1}=\dfrac{4-z}{k}$ and $\dfrac{x-1}{k}=\dfrac{y-4}{2}=\dfrac{z-5}{1}$ will intersect.

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Q01. Max. value is 1 and min. value is cos1.

OR Here $\frac{d\theta}{dt}=2\times\frac{d}{dt}(\sin\theta) \Rightarrow \cos\theta=\frac{1}{2} \quad \therefore \theta=\frac{\pi}{3}$.

Q02. No. of binary operations are $2^{2^2}=2^4=16$.

Q03. As $|adj.A|=|A|^{n-1}=16 \quad \Rightarrow 4^{n-1}=4^2 \quad \Rightarrow n-1=2 \quad \therefore n=3$.

Q04. Order : 2 and Degree : 3

Q05. As $\cos^{-1}y=2\pi-\cos^{-1}x \quad \Rightarrow y=\cos(2\pi-\cos^{-1}x)=\cos\cos^{-1}x=x$.

So, $\cos^{-1}x+\cos^{-1}y=2\pi$ implies, $2\cos^{-1}x=2\pi$ i.e., $x=\cos\pi=-1 \quad \therefore x=y=-1$.

Now $x^{100}+y^{200}+\frac{1}{x^{100}y^{200}}=1+1+\frac{1}{1\times 1}=3$. **OR** $x\in[-2,0]\cup[2,4]$.

Q06. Put $x^{2019X}=t \Rightarrow 2019x\log x=\log t \Rightarrow \left(2019x\times\frac{1}{x}+2019\log x\right)dx=\frac{1}{t}dt$

$\Rightarrow x^{2019X}(1+\log x)dx=\frac{1}{2019}dt$

So, $\int x^{2019X}(1+\log x)dx=\frac{1}{2019}\int dt=\frac{t}{2019}+c=\frac{x^{2019X}}{2019}+c$.

OR $\frac{1}{\log 4}(2^{2x}-2^{-2x})+2x+c$.

Q07. Let A : getting a red ball in first draw, B : getting a red ball in second draw.

As P(B|A) = Probability of getting a red ball in 2^{nd} draw when a red ball is drawn already in the first draw

That is, $P(B|A)=\frac{{}^2C_1}{{}^9C_1}=\frac{2}{9}$.

Hence required probability $=P(A\cap B)=P(A)P(B|A)=\frac{3}{10}\times\frac{2}{9}=\frac{1}{15}$.

OR Clearly $A\cap B=\phi \quad \therefore P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{0}{P(B)}=0$.

Q08. $\cos^{-1}\left(\frac{3}{4}\right)$

Q09. Rewriting the given plane in Normal unit vector form, $\vec{r}.(3\hat{i}+4\hat{j}-5\hat{k})=50$

$\Rightarrow \vec{r}.\left(\frac{3\hat{i}+4\hat{j}-5\hat{k}}{\sqrt{50}}\right)=\frac{50}{\sqrt{50}}$ i.e., $\vec{r}.\left(\frac{3\hat{i}}{\sqrt{50}}+\frac{4\hat{j}}{\sqrt{50}}-\frac{5\hat{k}}{\sqrt{50}}\right)=\sqrt{50}$

So, the required length $=\sqrt{50}$.

Note $\vec{r}.\hat{m}=d$, where $\hat{m}$ is normal unit vector to the plane and $|d|$ is length of $\perp^{er}$ drawn from origin to the plane

Q11. A must be invertible matrix i.e., $|A|\neq 0$.

Q12. $\int_3^4\frac{3dx}{\sqrt{3x-8}}=\left[3\times\frac{2\sqrt{3x-8}}{3}\right]_3^4=2\sqrt{4}-2\sqrt{1}=2$

Q13. **Hint :** $x+y+z=5000,\ 6x+7y+8z=35800,\ 6x+7y-8z=7000;\ x=1000,\ y=2200,\ z=1800$

OR Use $A^2+B^2=(A+B)^2 \Rightarrow A^2+B^2=(A+B)(A+B) \Rightarrow A^2+B^2=A^2+AB+BA+B^2$

That is, $O = AB + BA \quad \therefore AB = -BA$. Replace values of matrices A and B to get : $x = 1, y = 4$.

Q14. OR Since the function attains its maximum and minimum at p and q respectively so, we have $f'(p) = 0$ and $f'(q) = 0$. Use these and $p^2 = q$ to get : $m = 2$.

Q15. Function is continuous at $x = 0$ but, not differentiable.

Q16. See **Hint** in Centurion Assignment Part 2 (Q01). Ans. $\sec y = 2\sin x + C$

Q17. $\frac{6}{\sqrt{10}}(3\hat{i} - \hat{k})$ and, $\frac{6}{\sqrt{26}}(\hat{i} - 4\hat{j} + 3\hat{k})$ **Q18.** $\sqrt{x^2 - y^2} + x = C$

Q19. $\sqrt{26}$ **OR** Here $\alpha = -6, \beta = 7 \quad \therefore \alpha\beta = -42$.

Q20. $\frac{1}{2}\log\left|e^{2x} + 1\right| - 2\tan^{-1} e^x + c$

Q21. See **Solution** in Centurion Assignment Part 1 (Q55)

Q22. See **Solution** in Centurion Assignment Part 1 (Q100). Ans. $\frac{7}{15}$.

Q23. Value of p is 1/2.

Q24. Let x and y number of belts of type A and B are made per day respectively.
To maximize : $Z = ₹(4x + 3y)$

Subject to constraints : $\frac{x}{500} + \frac{y}{1000} \le 1,\ x + y \le 800,\ x \le 400,\ y \le 700;\ x, y \ge 0$.

Q25. See the **Solution of** both of these Alternatives in Centurion Assignment Part 1.

Q26. See **Solution** in Centurion Assignment Part 1 (Q20)
OR Here $x = 8 \quad \therefore \sqrt[3]{x} = 2$.

Q27. $\frac{2}{\sqrt{1 - 4x^2}}$ **Q28.** $\left(\frac{4\pi - \sqrt{3}}{3}\right)$ units^2 **OR** 0. **Q29.** –3, 0.