Your Name : ______________________________

School Code | | | | | |

Date Of Birth | d | d | m | m | y | y | y | y |

Code No. 19/2/3

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Candidates must write the Code on the title page of the answer-book.

Series : ATS/03

ACHIEVERS TEST SERIES XII – 03

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Time Allowed : 180 Minutes **Max. Marks : 100**

SECTION A

Q01. If $f(x)=x^3-3|x|+\frac{2}{|x|}$, find the $f'(x)$ when $x<0$. **OR** Find $\frac{d}{dx}\{|x|\}$, $x\neq 0$.

Q02. Let $f(x)=|x|$ and $g(x)=[x]$. Write the value of $fog(-7/3)$.

Q03. If A is a a square matrix of order 3 such that |A| = 3, then write the value of |adj.(adj.A)|.

Q04. Total revenue from the sale of x items of a product is R (X) = $4x^2+25x+60$. Find the marginal revenue when value of x is 10.

SECTION B

Q05. If $\alpha+\alpha^{-1}=2$, find the principal value of $\tan^{-1}(-\alpha)$. **OR** Find $\tan^{-1}2+\tan^{-1}3$.

Q06. Find $\int(pq+qr+rp)dx$, s.t. $\cot^{-1}p+\cot^{-1}q+\cot^{-1}r=\pi$. **OR** Find $\int\sin^5 x\,dx$.

Q07. Prove that log sin x is strictly increasing on $(0,\ \pi/2)$.

OR For what values of a, the function $f(x)=\sin x-ax+b$ is decreasing on $x\in R$?

Q08. If $\vec{a}$ is any vector in the space, then write the value of $(\vec{a}.\hat{i})\hat{i}+(\vec{a}.\hat{j})\hat{j}+(\vec{a}.\hat{k})\hat{k}$.

Q09. Reduce the equation $\vec{r}.(4\hat{i}-6\hat{j}+12\hat{k})=5$ to the normal form of plane and hence, find the length of perpendicular drawn from the origin to the plane.

Q10. Find the approximate value of $\tan^{-1}0.999$, using derivatives. (Use $\pi=3.14$, if needed).

Q11. Find AA^T, where $A=\begin{bmatrix}2\\3\\4\end{bmatrix}$. **Q12.** Evaluate : $\int_{-1}^{1}\sqrt{|x|-x}\,dx$.

SECTION C

Q13. If AB = BA for any two square matrices, then prove by using principle of mathematical induction that, $(AB)^n=A^nB^n$. It is given that $AB^n=B^nA$. Here $n\in N$.

OR Find the matrix A such that $\begin{bmatrix}2&-1\\1&0\\-3&4\end{bmatrix}A=\begin{bmatrix}-1&-8&-10\\1&-2&-5\\9&22&15\end{bmatrix}$.

Q14. An apple orchard has 30 trees per acre and average yield is 400 apples per tree. For each additional tree planted per acre, yield per tree reduces by 10 apples. How many trees per acre will give the largest crop of apples? How many apples does he get per acre?

OR Find the equation of tangent & normal to the curve $y=(1+x)^y+\sin^{-1}(\sin^2 x)$ at $x=0$.

Q15. Find the value of 'a' and 'b' so that $f(x)=\begin{cases}x^2, & x\leq c\\ ax+b, & x>c\end{cases}$ is differentiable at $x=c$.

Q16. Urn I has 6 red, 4 black balls; urn II has 2 red, 6 black balls and urn III has 1 red, 8 black balls. An urn is chosen at random and a ball is drawn from that urn. The ball drawn is red. Find the probability that the ball is drawn either from urn II or urn III.

Q17. Let $\vec{a}=2\hat{i}-\hat{j}+\hat{k},\ \vec{b}=\hat{i}+3\hat{j}-\hat{k},\ \vec{c}=-2\hat{i}+\hat{j}-3\hat{k}$ and $\vec{d}=3\hat{i}+2\hat{j}+5\hat{k}$. Find the values of α, β and γ such that $\vec{d}=\alpha\vec{a}+\beta\vec{b}+\gamma\vec{c}$.

Q18. Solve : $xy\log(x/y)dx+\{y^2-x^2\log(x/y)\}dy=0$.

Q19. Find the length of the perpendicular drawn from the point P(2, –1, 5) to the following line : $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}$.

OR Find the equation of the line of intersection of $\vec{r}.(\hat{i}-\hat{j}+2\hat{k})=5$ and $\vec{r}.(3\hat{i}+\hat{j}+\hat{k})=6$.

Q20. Evaluate $\int \frac{e^{\tan^{-1}x}dx}{(1+x^2)^2}$.

Q21. Let X denote the number of colleges where you will apply after your results and P(X = x) denotes your probability of getting admission in x number of colleges. It is given that

$$P(X=x)=\begin{cases} kx, \text{ if } x=0 \text{ or } 1 \\ 2kx. \text{ if } x=2 \\ k(5-x), \text{ if } x=3 \text{ or } 4 \end{cases} \quad \text{where } k \text{ is } +\text{ve constant}.$$

(i) Find the value of k.
(ii) What is the probability that you will get admission exactly in two colleges?
(iii) Find the mean and variance of the probability distribution.

Q22. Evaluate $\int_{\pi/3}^{\pi/2} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{5/2}}dx$.

Q23. Find $\int \frac{x^2-1}{(x^2+1)\sqrt{x^4+1}}dx$.

SECTION D

Q24. A cylinder manufacturer makes small and large cylinders from a large piece of card-board. The large cylinder requires 4 square metre and small cylinder requires 3 square metre of card-board. The manufacturer is required to make at least 3 large cylinders and at least twice as many small cylinders as large cylinders. Assume that 60 square metre of card-board is in the stock and profit on the small and large cylinders are ₹25 and ₹35 respectively, formulate an LPP for this situation to maximize the profit. Also, write the maximum profit.

Q25. Let X be a non-empty set and * : $P(X)\times P(X)\to P(X)$ be defined as $A*B=(A-B)\cup(B-A)$ for all $A,B\in P(X)$. Show that ϕ is the identity for the operation * and all the elements A of P(X) are invertible with A^{-1}.

OR Find the least value of : $(\sec^{-1}x)^2+(\text{cosec}^{-1}x)^2$.

Q26. If $\begin{vmatrix} {}^xC_1 & {}^xC_2 & {}^xC_3 \\ {}^yC_1 & {}^yC_2 & {}^yC_3 \\ {}^zC_1 & {}^zC_2 & {}^zC_3 \end{vmatrix}$ is divided by $\frac{xyz}{4}$, then obtain the quotient.

OR If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$ and vectors $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$ with position vectors $\overrightarrow{OA}=\hat{i}+a\hat{j}+a^2\hat{k}$, $\overrightarrow{OB}=\hat{i}+b\hat{j}+b^2\hat{k}$ and $\overrightarrow{OC}=\hat{i}+c\hat{j}+c^2\hat{k}$ are non-coplanar, then find the value of abc.

Q27. Find $\int_0^2 [(1-x^2)y''-xy']dx$, if $y=(\sin^{-1}x)^2+(\cos^{-1}x)^2$.

OR If $5f(x)+3f(1/x)=x+2$ and $y=xf(x)$, find y'.

Q28. Sketch the rough graph of $y=4\sqrt{x-1},\ 1\le x\le 3$ and evaluate the area between the curve x axis and the line $x=3$.

Q29. Find the distance of P(–1, –5, 10) from the point of intersection of the line $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and the plane $\vec{r}.(\hat{i}-\hat{j}+\hat{k})=5$.

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SECTION A

Q01. $\because x<0$ so, $f(x)=x^3-3(-x)+\dfrac{2}{(-x)}$ $\therefore f'(x)=3x^2+3+\dfrac{2}{x^2}$.

OR $\dfrac{d}{dx}\{|x|\}=\begin{cases}\dfrac{d}{dx}\{x\}=1, \text{ if } x>0\\ \dfrac{d}{dx}\{-x\}=-1, \text{ if } x<0\end{cases}$.

Q02. $fog(-7/3)=f\left(g(-7/3)\right)=f\left(\left[-7/3\right]\right)=f(-3)=\left|-3\right|=3$.

Q03. Note $|adj.(adj.A)|=|A|^{(n-1)^2}$, where n is the order of A. So, $|adj.(adj.A)|=81$. **Q04.** 105

SECTION B

Q05. $-\pi/4$

OR Here $\tan^{-1}2+\tan^{-1}3=\dfrac{\pi}{2}-\cot^{-1}2+\dfrac{\pi}{2}-\cot^{-1}3=\pi-\left[\tan^{-1}\dfrac{1}{2}+\tan^{-1}\dfrac{1}{3}\right]$

$\Rightarrow \quad =\pi-\left[\tan^{-1}\dfrac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}\right]=\pi-\left[\tan^{-1}\dfrac{\frac{5}{6}}{\frac{5}{6}}\right]=\pi-\tan^{-1}1=\pi-\dfrac{\pi}{4}=\dfrac{3\pi}{4}$.

Q06. $x+C$

OR Let $I=\int \sin^5 x dx$ $\Rightarrow I=\int \sin^4 x \sin x dx$ $\Rightarrow I=\int (1-\cos^2 x)^2 \sin x dx$

Put $\cos x=y \Rightarrow \sin x dx=-dy$ $\therefore I=-\int (1-y^2)^2 dy$ $\Rightarrow I=-\int\left\{1-2y^2+y^4\right\}dy$

$\Rightarrow I=-\left\{y-\dfrac{2y^3}{3}+\dfrac{y^5}{5}\right\}+C$ $\therefore I=\dfrac{2\cos^3 x}{3}-\dfrac{\cos^5 x}{5}-\cos x+C$.

Q07. Here $f(x)=\log\sin x$ $\Rightarrow f'(x)=\dfrac{1}{\sin x}\times\cos x=\cot x$

As $f'(x)>0$ in $(0,\pi/2)$ so, $f(x)$ is strictly increasing in $x\in(0,\pi/2)$.

OR Here $f(x)=\sin x-ax+b$ $\Rightarrow f'(x)=\cos x-a$.

Since $f(x)$ is a decreasing function for all $x\in R$ so, $f'(x)\le 0$

Therefore, $\cos x-a\le 0$ $\Rightarrow a\ge\cos x$ ...(i)

Now for all $x\in R$, $\cos x\in[-1,1]$ i.e., $-1\le\cos x\le 1$ ...(ii)

By (i) and (ii), we can conclude that $a\in[1,\infty)$.

Q08. $\vec{a}$

Q09. Normal form : $\vec{r}.\left(\dfrac{4}{14}\hat{i}-\dfrac{6}{14}\hat{j}+\dfrac{12}{14}\hat{k}\right)=\dfrac{5}{14}$, length of $\perp^{er}=\dfrac{5}{14}$ units (comparing to $\vec{r}.\hat{n}=d$.

Q10. Let $f(x)=\tan^{-1}x$ $\Rightarrow f'(x)=\dfrac{1}{1+x^2}$

As $f(x+h)=f(x)+h f'(x)$, where $h\to 0$

Put $x=1$, $h=-0.001$ $\therefore f(1-0.001)=f(0.999)=f(1)+(-0.001)f'(1)$

$\Rightarrow \tan^{-1}(0.999)=\tan^{-1}(1)+(-0.001)\times\dfrac{1}{1+1^2}$ $\Rightarrow \tan^{-1}(0.999)=\dfrac{\pi}{4}+(-0.001)\times\dfrac{1}{2}=\dfrac{3.14}{4}-0.0005$

$\therefore \tan^{-1}(0.999)=0.785-0.0005=0.7845$.

Q11. $\begin{bmatrix} 4 & 6 & 8 \\ 6 & 9 & 12 \\ 8 & 12 & 16 \end{bmatrix}$ **Q12.** $\int_{-1}^{0}\sqrt{(-x)-x}\,dx+\int_{0}^{1}\sqrt{(x)-x}\,dx=\int_{-1}^{0}(-2x)^{1/2}dx+\int_{0}^{1}0\,dx=\frac{2\sqrt{2}}{3}$.

SECTION C

Q13. See Mathematicia by O.P. Gupta **OR** $\begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$.

Q14. Let x be the additional trees planted per acre. So, total yield is, $C=(30+x)(400-10x)$.
Ans. Number of additional trees per acre = 5 and, total no. of trees per acre = 35.
Number of apples per acre $=(30+5)(400-50)=12250$.

OR $y=(1+x)^y+\sin^{-1}(\sin^2 x)$ at $x=0 \Rightarrow y=(1+0)^y+\sin^{-1}(\sin^2 0)=1$
$\therefore$ the point of contact is P(0, 1).

Also $\frac{dy}{dx}=\dfrac{y(1+x)^{y-1}+\dfrac{2\sin x\cos x}{\sqrt{1-\sin^4 x}}}{1-(1+x)y\log(1+x)}$ $\therefore \left.\frac{dy}{dx}\right]_{at\ P}=\dfrac{1(1+0)^{1-1}+\dfrac{2\sin 0\cos 0}{\sqrt{1-\sin^4 0}}}{1-(1+0)\times 1\times\log(1+0)}=1$ $\Rightarrow m_N=-1$

Equation of tangent : $x-y+1=0$ and normal : $x+y-1=0$.

Q15. Note that a differentiable function is always continuous at $x=c$.
So, $\lim_{x\to c^-} f(x)=\lim_{x\to c^+} f(x)$ $\therefore ac+b=c^2$ ...(i)
Also, $Lf'(c)=Rf'(c)$ $\therefore a=2c$...(ii)
By (i) and (ii), we get : $b=-c^2$.

Q16. $P(E_2\,|\,A)+P(E_3\,|\,A)=1-P(E_1\,|\,A)=\frac{65}{173}$.

Q17. $\alpha=-3/2,\ \beta=1,\ \gamma=-5/2$

Q18. $\frac{x^2}{y^2}\left[\log\left(\frac{x}{y}\right)-\frac{1}{2}\right]+2\log|y|=\lambda$.

Q19. Foot of $\perp^{er}$: (1, 2, 3) and lenght $=\sqrt{14}$ units

OR Let $\pi_1: x-y+2z=5$...(i) and $\pi_2: 3x+y+z=6$...(ii)
Adding (i) and (ii), we get : $4x+3z=11$, clearly $x=2,\ z=1\Rightarrow y=-1$
$\therefore$ point on line of intersection of (i) and (ii) is $(2,-1,1)$
Hence the required line : $\frac{x-2}{-3}=\frac{y+1}{5}=\frac{z-1}{4}$.

Q20. Put $\tan^{-1}x=t\Rightarrow\frac{dx}{1+x^2}=dt$. $\therefore I=\int\frac{e^{\tan^{-1}x}dx}{(1+x^2)^2}=\int e^t\cos^2 t\,dt=\int e^t\left(\frac{1+\cos 2t}{2}\right)dt$.
Ans. $I=\frac{e^{\tan^{-1}x}}{5}\left(\frac{3+2x^2+2x}{1+x^2}\right)+C$.

Q21. (i) $k=1/8$ (ii) 1/2 (iii) $\mu=19/8,\ \sigma^2=47/64$.

Q22. Let $I=\int_{\pi/3}^{\pi/2}\frac{\sqrt{1+\cos x}}{(1-\cos x)^{5/2}}dx=\int_{\pi/3}^{\pi/2}\frac{\sqrt{2\cos^2\frac{x}{2}}}{\left(2\sin^2\frac{x}{2}\right)^{5/2}}dx=\frac{1}{4}\int_{\pi/3}^{\pi/2}\frac{\cos\frac{x}{2}}{\sin^5\frac{x}{2}}dx$ $\left[\begin{array}{l}\text{Put } \sin\frac{x}{2}=t\Rightarrow\cos\frac{x}{2}dx=2dt \\ \text{When } x=\pi/3\Rightarrow t=1/2, \\ \text{When } x=\pi/2\Rightarrow t=\frac{1}{\sqrt{2}}\end{array}\right.$

$$\Rightarrow I=\frac{1}{4}\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}}\frac{2dt}{t^5}=\frac{1}{2}\left[\frac{t^{-4}}{-4}\right]_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}}=-\frac{1}{8}\left[\frac{1}{\left(\frac{1}{\sqrt{2}}\right)^4}-\frac{1}{\left(\frac{1}{2}\right)^4}\right]=\frac{3}{2}.$$

Q23. Let $I=\int\frac{x^2-1}{(x^2+1)\sqrt{x^4+1}}dx$ $\qquad \Rightarrow I=\int\frac{\left(\frac{x^2-1}{x^2}\right)x^2}{\left(\frac{x^2+1}{x}\right)x\times x\sqrt{x^2+\frac{1}{x^2}}}dx$

$$\Rightarrow I=\int\frac{\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)\sqrt{\left(x+\frac{1}{x}\right)^2-2}}dx$$

$\left[\text{Put } x+\frac{1}{x}=t \Rightarrow \left(1-\frac{1}{x^2}\right)dx=dt\right.$

$$\Rightarrow I=\int\frac{dt}{t\sqrt{t^2-2}}=\frac{1}{2}\int\frac{2tdt}{t^2\sqrt{t^2-2}}$$

$\left[\text{Put } t^2-2=y^2 \Rightarrow 2tdt=2ydy\right.$

$\Rightarrow I=\frac{1}{2}\int\frac{2ydy}{(y^2+2)y}=\int\frac{dy}{(y^2+2)}=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{y}{\sqrt{2}}\right)+C$. Now simplify further, yourself.

SECTION D

Q24. Let the number of large cylinders and small cylinders be x and y respectively.
To maximize : Z = ₹(35x +25y).
Subject to constraints : $x\ge 0, y\ge 0, 4x+3y\le 60, x\ge 3, y\ge 2x$.

Q25. Let E be an identity element then, $A*E=E*A=A$ for all $E\in P(X)$ i.e., $(A-E)\cup(E-A)=A$
Taking $A=\phi$, we have $(\phi-E)\cup(E-\phi)=\phi\Rightarrow\phi\cup E=\phi$, which implies that $E=\phi$.
$\therefore A*\phi=\phi*A=A$ for all $\phi\in P(X)$ i.e., $(A-\phi)\cup(\phi-A)=A$.
Hence ϕ is the identity element.
Let $A\in P(X)$ be invertible then $B\in P(X)$ s.t. $A*B=B*A=\phi$
That is, $(A-B)\cup(B-A)=\phi\Rightarrow A-B=\phi$ as well as $B-A=\phi$
$\Rightarrow A\subset B$ and $B\subset A \quad \Rightarrow A=B$.
Thus for all $A\in P(X)$, $A*A=\phi$. Hence A is invertible and $A^{-1}=A$.

OR $\frac{\pi^2}{8}$.

Q26. Let $\Delta=\begin{vmatrix} {}^xC_1 & {}^xC_2 & {}^xC_3 \\ {}^yC_1 & {}^yC_2 & {}^yC_3 \\ {}^zC_1 & {}^zC_2 & {}^zC_3 \end{vmatrix}=\begin{vmatrix} x & \frac{x(x-1)}{2!} & \frac{x(x-1)(x-2)}{3!} \\ y & \frac{y(y-1)}{2!} & \frac{y(y-1)(y-2)}{3!} \\ z & \frac{z(z-1)}{2!} & \frac{z(z-1)(z-2)}{3!} \end{vmatrix}$.

Now take $\frac{1}{2}$ and $\frac{1}{6}$ common from C_2 and C_3 respectively.
Then take x, y and z common from R_1, R_2 and R_3 respectively.
Then apply $R_2\to R_2-R_1$ and $R_3\to R_3-R_1$.
Then take $(y-x)$ and $(z-x)$ common from R_2 and R_3 respectively.
Then expanding along C_1, we get : $\Delta=\frac{xyz}{12}(x-y)(y-z)(z-x)$.

Therefore, the required quotient is : $\frac{1}{3}(x-y)(y-z)(z-x)$.

OR As $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0 \quad \Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}+\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}=0$

$\Rightarrow -\begin{vmatrix} 1 & a^2 & a \\ 1 & b^2 & b \\ 1 & c^2 & c \end{vmatrix}+abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}=0$ [By $C_1 \leftrightarrow C_3$ in Det.I; Also take a, b, c common from R_1, R_2 & R_3 resp. in Det.II]

Apply $C_2 \leftrightarrow C_3$, we get : $(-1)^2\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}+abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}=0 \quad \Rightarrow \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}(1+abc)=0 \ldots(i)$

As $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are non coplanar so, $\left[\overrightarrow{OA}\ \overrightarrow{OB}\ \overrightarrow{OC}\right]=\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}\neq 0 \ldots(ii)$

By (i) and (ii), we can conclude that $(1+abc)=0 \quad \therefore abc=-1$.

Q27. Note that $\int_0^2 [(1-x^2)y''-xy']dx=\int_0^2 4dx=[4x]_0^2=8$

OR Given $5f(x)+3f\left(\frac{1}{x}\right)=x+2 \ldots(i)$

Replace x by $\frac{1}{x}$, $5f\left(\frac{1}{x}\right)+3f(x)=\frac{1}{x}+2 \ldots(ii)$

By $(i)\times 5-(ii)\times 3$, $f(x)=\frac{5x}{16}-\frac{3}{16x}+\frac{1}{4} \qquad \therefore y=\frac{5x^2}{16}-\frac{3}{16}+\frac{x}{4} \qquad \Rightarrow \frac{dy}{dx}=\frac{5x}{8}+\frac{1}{4}$.

Q28. $\frac{16\sqrt{2}}{3}$ sq.units

Q29. Point of intersection : $Q(2,-1,2)$ $\therefore PQ=\sqrt{89}$ units.

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