Your Name : ______________________________

School Code | | | | | |

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Code No. 19/1/2

Candidates must write the Code on the title page of the answer-book.

Series : ATS/02

ACHIEVERS TEST SERIES XII – 02

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Time Allowed : 180 Minutes **Max. Marks : 100**

SECTION A

Q01. Given $f(x)=\dfrac{1}{x-1}$, find the **points of discontinuity** of the composite function $f\left(f(x)\right)$.

Q02. If A is a 2×3 matrix and B is matrix such that $A'B$ and BA' are both defined, then what is the **order** of matrix B?

Q03. Evaluate $\displaystyle\int\frac{dx}{\sqrt{x}+1}$.

Q04. If $\hat{a}$ and $\hat{b}$ are two unit vectors and $\left|\hat{a}-\hat{b}\right|=1$, then find the **acute angle** between them.

OR If $\hat{a}$ and $\hat{b}$ are two unit vectors and $\left|\hat{a}-\hat{b}\right|=\left|\hat{a}+\hat{b}\right|$, then find the angle between them.

SECTION B

Q05. If R = {(x, y) : x + 2y = 8} is a relation on N, write the **domain and range** of R.

OR Let $A=\{1,2,3,...,100\}$. Let a relation R be defined on A, given by $R=\{(x,y): xy \text{ is a perfect square}\}$. Obtain the **equivalence class** [3].

Q06. Find $\displaystyle\int\frac{dx}{x^3[x^5+1]^{3/5}}$. **OR** Find $\displaystyle\int\frac{dx}{x[\sqrt{x}+1]}$.

Q07. Using **differentials**, find the approximate value of $\sqrt{.082}$.

Q08. Let ABCD is a parallelogram. If L and M are mid-points of BC and CD respectively, express $\overrightarrow{AL}$ and $\overrightarrow{AM}$ in terms of $\overrightarrow{AB}$ and $\overrightarrow{AD}$. Also show that $\overrightarrow{AL}+\overrightarrow{AM}=1.5\overrightarrow{AC}$.

Q09. Find the points on the line $\vec{r}=3\hat{k}-2\hat{i}-\hat{j}+\lambda(3\hat{i}+2\hat{j}+2\hat{k})$ at a distance of 5 units from (1, 3, 3).

Q10. If the probability that a person is **not** a swimmer is 0.3 then, find the probability that out of 5 persons, 4 are swimmers.

Q11. Find the points on $y=x^3$ at which the slope of tangent is **equal** to the y coordinate of the point.

Q12. If $A=\begin{pmatrix}1 & 2\\ 3 & 5\end{pmatrix}$, then find the value of $\left|A^{2009}-5A^{2008}\right|$.

OR Using matrix equation $\begin{pmatrix}x+y+z\\ x+z\\ y+z\end{pmatrix}=\begin{pmatrix}9\\ 5\\ 7\end{pmatrix}$, find the value of x, y and z.

SECTION C

Q13. Evaluate $\displaystyle\int\frac{\sqrt{1-\sin x}}{1+\cos x}e^{-x/2}dx,\ 0<x<\frac{\pi}{2}$.

Q14. Find $\displaystyle\int\frac{x^3}{x^6+1}dx$.

Q15. Find the intervals in which the function $f(x) = \sin 3x - \cos 3x,\ 0<x<\pi$, is **strictly increasing or strictly decreasing**.

Q16. Find dy/dx, if $y=(x)^{x^x}\sin^2(x^{\log\cos x})$.

Q17. Find the value of 'p' so that $f(x)=\begin{cases}\dfrac{\sqrt{3}\cos x+\sin x}{x+\pi/3}, & x\neq -\pi/3\\ p, & x=-\pi/3\end{cases}$ is continuous at $x=-\pi/3$.

OR If $x=\sec\theta-\cos\theta$ and $y=\sec^n\theta-\cos^n\theta$, show that $(x^2+4)y_1^2=n^2(y^2+4)$.

Q18. If $(y^3-2x^2y)dx+(2y^2x-x^3)dy=0$, prove that $xy\sqrt{y^2-x^2}$ is **constant**.

Q19. If the vectors $a\hat{i}+a\hat{j}+c\hat{k},\ \hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are **coplanar**, then show that $c^2=ab$.

OR Find the **altitude of a parallelepiped** determined by the vectors $\vec{a},\ \vec{b},\ \vec{c}$, if the base is taken as parallelogram determined by $\vec{a},\ \vec{b}$ and if $\vec{a}=\hat{i}+\hat{j}+\hat{k},\ \vec{b}=2\hat{i}+4\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}+3\hat{k}$.

Q20. A variable plane which remains at a constant distance 3p from the origin, cuts the coordinate axes at A, B and C respectively. Find the **locus of the centroid** of triangle ABC.

Q21. Let three digit numbers A28, 3B9 and 62C, where A, B and C are integers between 0 and 9, be divisible by fixed integer n. Show that $\begin{vmatrix}A & 3 & 6\\ 8 & 9 & C\\ 2 & B & 2\end{vmatrix}$ is divisible by n.

OR If A and B are square matrices of same order such that $AB=BA$, then prove by induction that $AB^n=B^nA$.

Q22. For a loaded die, the probabilities of outcomes are as under :
P(1) = P(2) = 0.2, P(3) = P(5) = P(6) = 0.1 and P(4) = 0.3.
The die is thrown two times. Let A and B be the events A : same number each time, B : a total score of 10 or more. Determine whether or not A and B are independent.

Q23. There are two bags, Bag I and Bag II. If it is given that Bag I contains 4 red and 5 black balls while Bag II contains 3 red and 7 black balls. One ball is drawn from Bag I and two balls are drawn (without replacement) from Bag II. Find the probability that out of the three balls drawn, two are black and one is red.

SECTION D

Q24. Evaluate the value of integral $\displaystyle\int_0^{\pi/2}\frac{\cos^9 x\,dx}{\cos^3 x+\sin^3 x}$.

Q25. Let $A=\{a+\sqrt{7}b : a, b\in Z\}$. Show that usual **addition and multiplication of numbers** is a binary operation on set A. **OR** Simplify : $\sin^2\left(2\tan^{-1}\sqrt{\dfrac{1+x}{1-x}}\right),\ -1\le x\le 1$.

Q26. If $a\neq p,\ b\neq q,\ c\neq r$ and $\begin{vmatrix}p & b & c\\ a & q & c\\ a & b & r\end{vmatrix}=0$, then find the value of $\dfrac{p}{p-a}+\dfrac{q}{q-b}+\dfrac{r}{r-c}$.

OR If $S_m=\alpha^m+\beta^m+\gamma^m$, then prove that $\begin{vmatrix}S_0 & S_1 & S_2\\ S_1 & S_2 & S_3\\ S_2 & S_3 & S_4\end{vmatrix}=\left[(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)\right]^2$.

Q27. Solve graphically :
Maximize : $P=30x+60y$ Subject to : $2x+y\le 70,\ x+y\le 40,\ x+3y\le 90,\ x\ge 0,\ y\ge 0$.

Q28. Using integration, find the area of the region $\left\{(x,y) : |x+2|\le y\le\sqrt{20-x^2}\right\}$.

OR Find $\displaystyle\int_2^5 (2x^2+3x+e^{2x+1})dx$, by using the **first principle of integrals**.

Q29. Prove that the line through A(0, –1, –1) and B(4, 5, 1) intersects the line through C(3, 9, 4) and D(–4, 4, 4). Also find their point of intersection. ■■■

SOLUTIONS & ANSWERS KEY (ATS-02)
MATHEMATICS CLASS XII

SECTION A

Q01. $f(f(x)) = \frac{1}{f(x)-1} = \frac{1}{\frac{1}{x-1}-1} = \frac{x-1}{2-x}$. So clearly, points of discontinuity are $x = 1, 2$.

Q02. Let order of B be $m \times n$. As $A'B$ and BA' are both defined so, $m = 2, n = 3$.
So order of B is 2×3.

Q03. Let $I = \int \frac{dx}{\sqrt{x}+1} = \int \frac{2tdt}{t+1}$, where $x = t^2 \Rightarrow dx = 2tdt$.

$\therefore I = 2\int\left(1-\frac{1}{t+1}\right)dt = 2\left(t - \log|t+1|\right) + c = 2\left(\sqrt{x} - \log\left|\sqrt{x}+1\right|\right) + c$.

Q04. As $|\hat{a}-\hat{b}| = 1$ $\quad \Rightarrow |\hat{a}-\hat{b}|^2 = (\hat{a}-\hat{b}).(\hat{a}-\hat{b}) = 1^2$ $\quad \Rightarrow |\hat{a}|^2 + |\hat{b}|^2 - 2\hat{a}.\hat{b} = 1$

$\Rightarrow 1^2 + 1^2 - 2|\hat{a}||\hat{b}|\cos\theta = 1$, where θ is the required angle.

$\Rightarrow 1+1-2\times1\times1\times\cos\theta = 1$ $\quad \Rightarrow \cos\theta = \frac{1}{2}$ $\quad \therefore \theta = \frac{\pi}{3}$.

OR $\quad |\hat{a}-\hat{b}| = |\hat{a}+\hat{b}|$ $\quad \Rightarrow |\hat{a}-\hat{b}|^2 = |\hat{a}+\hat{b}|^2$ $\quad \Rightarrow (\hat{a}-\hat{b}).(\hat{a}-\hat{b}) = (\hat{a}+\hat{b}).(\hat{a}+\hat{b})$

$\Rightarrow a^2 - 2\hat{a}.\hat{b} + b^2 = a^2 + 2\hat{a}.\hat{b} + b^2$ $\quad \Rightarrow 4\hat{a}.\hat{b} = 0$

$\Rightarrow 4ab\cos\theta = 0$, where θ is the angle between $\hat{a}$ and $\hat{b}$.

$\therefore 4\times1\times1\cos\theta = 0$ $\quad \Rightarrow \cos\theta = 0$ $\quad \therefore \theta = \pi/2$.

Hence, $\hat{a} \perp \hat{b}$ i.e., the angle between these vectors is $\pi/2$.

SECTION B

Q05. Domain $= \{2, 4, 6\}$ and Range $= \{1, 2, 3\}$.
OR For equivalence class [3], let $(3, x) \in R$ where $x \in A$.
That is, $3x$ is perfect square when $x = 3, 12, 27, 48, 75$.
Hence $[3] = \{3, 12, 27, 48, 75\}$.

Q06. Let $I = \int \frac{dx}{x^3[x^5+1]^{3/5}} = \int \frac{dx}{x^6\left[1+\frac{1}{x^5}\right]^{3/5}} = -\frac{1}{5}\int \frac{dt}{t^{3/5}}$, where $1+\frac{1}{x^5} = t \Rightarrow \frac{dx}{x^6} = \frac{dt}{-5}$.

$\therefore I = -\frac{1}{5}\times\frac{t^{-3/5+1}}{-3/5+1} = -\frac{1}{2}\left(1+\frac{1}{x^5}\right)^{2/5} + c$.

OR $\quad$ Put $x = t^2 \Rightarrow dx = 2tdt$. So, $I = \int \frac{dx}{x[\sqrt{x}+1]} = \int \frac{2tdt}{t^2(t+1)} = \int \frac{2dt}{t(t+1)} = 2\int\left(\frac{1}{t} - \frac{1}{t+1}\right)dt$

$\Rightarrow I = 2\{\log|t| - \log|t+1|\} + c = 2\log\left|\frac{\sqrt{x}}{\sqrt{x}+1}\right| + c$.

Q07. 0.286 (approx.)

Q08. $\overrightarrow{AL} = \overrightarrow{AB} + \frac{1}{2}\overrightarrow{AD}$, $\overrightarrow{AM} = \overrightarrow{AD} + \frac{1}{2}\overrightarrow{AB}$

Q09. $(-2, -1, 3)$, $(4, 3, 7)$

Q10. Let p : probability of a person being a swimmer. So, $p = 1 - q = 1 - 0.3 = 0.7$, $q = 0.3$.
Total no. of persons, n = 5.

So, $P(X=4)={}^5C_4(0.7)^4(0.3)^{5-4}=5\times\left(\frac{7}{10}\right)^4\times\frac{3}{10}=15\times\frac{2401}{10^5}$.

Q11. (0, 0), (3, 27)

Q12. As $|A|=\begin{vmatrix}1 & 2\\ 3 & 5\end{vmatrix}=5-6=-1$.

$\therefore\left|A^{2009}-5A^{2008}\right|=\left|A^{2008}(A-5I)\right|=\left|A^{2008}\right|\begin{vmatrix}-4 & 2\\ 3 & 0\end{vmatrix}=|A|^{2008}(0-6)=(-1)^{2008}(-6)=-6$.

OR $\begin{pmatrix}x+y+z\\ x+z\\ y+z\end{pmatrix}=\begin{pmatrix}9\\ 5\\ 7\end{pmatrix}$ $\Rightarrow x+y+z=9...(i),\ x+z=5...(ii),\ y+z=7...(iii)$

By (i) and (ii), $5+y=9$ $\therefore y=4$

By (i) and (iii), $x+7=9$ $\therefore x=2$

Also by (i), $2+4+z=9$ $\therefore z=3$.

SECTION C

Q13. Put $-\frac{x}{2}=t \Rightarrow dx=-2dt$.

Let $I=\int\frac{\sqrt{1-\sin x}}{1+\cos x}e^{-x/2}dx=-2\int\frac{\sqrt{1-\sin(-2t)}}{1+\cos(-2t)}e^t dt=-2\int\frac{\sqrt{1+\sin 2t}}{1+\cos 2t}e^t dt$

$\Rightarrow I=-2\int\frac{\sqrt{(\cos t+\sin t)^2}}{1+\cos 2t}e^t dt=-2\int\frac{|\cos t+\sin t|}{1+\cos 2t}e^t dt=-2\int\frac{\cos t+\sin t}{2\cos^2 t}e^t dt$ $\left[\because x\in(0,\frac{\pi}{2})\ \ \therefore t\in(-\frac{\pi}{4},0)\right.$

$\Rightarrow I=-\int(\sec t+\sec t\tan t)e^t dt=-e^t\sec t+c=-e^{-x/2}\sec(-x/2)+c$.

Q14. Let $I=\int\frac{x^3}{x^6+1}dx=\frac{1}{2}\int\frac{x^2.2x}{(x^2)^3+1}dx$ $\left[\text{Put } x^2=t\Rightarrow 2xdx=dt\right.$

$\therefore I=\frac{1}{2}\int\frac{t}{t^3+1}dt$.

Now use partial fraction, consider $\frac{t}{t^3+1}=\frac{t}{(t+1)(t^2-t+1)}=\frac{A}{t+1}+\frac{B(2t-1)}{t^2-t+1}+\frac{C}{t^2-t+1}$.

Q15. Increasing in : $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{7\pi}{12},\frac{11\pi}{12}\right)$, Decreasing in : $\left(\frac{\pi}{4},\frac{7\pi}{12}\right)\cup\left(\frac{11\pi}{12},\pi\right)$.

Q16. Here $y=(x)^{x^x}\sin^2(x^{\log\cos x})$ $\Rightarrow\frac{dy}{dx}=(x)^{x^x}\frac{d}{dx}\left\{\sin^2(x^{\log\cos x})\right\}+\sin^2(x^{\log\cos x})\frac{d}{dx}\left\{(x)^{x^x}\right\}$

$\Rightarrow\frac{dy}{dx}=(x)^{x^x}\left\{2\sin(x^{\log\cos x})\cos(x^{\log\cos x})\times\frac{d}{dx}(x^{\log\cos x})\right\}+\sin^2(x^{\log\cos x})\frac{d}{dx}\left\{(x)^{x^x}\right\}...(i)$

Let $u=x^{\log\cos x}$ $\Rightarrow\log u=\log x^{\log\cos x}$ $\Rightarrow\log u=\log\cos x\ \log x$

$\therefore\frac{1}{u}\times\frac{du}{dx}=\log\cos x\times\frac{1}{x}+\log x\times\frac{-\sin x}{\cos x}$ $\Rightarrow\frac{du}{dx}=u\left\{\frac{\log\cos x}{x}-\log x\times\tan x\right\}$

Also let $v=(x)^{x^x}$ $\Rightarrow\log v=x^x\log x$ $\Rightarrow\frac{1}{v}\times\frac{dv}{dx}=x^x\times\frac{1}{x}+\log x\times\frac{d}{dx}(x^x)$

$\Rightarrow\frac{dv}{dx}=(x)^{x^x}\left\{\frac{x^x}{x}+x^x(1+\log x)\times\log x\right\}$

By (i), $\frac{dy}{dx}=(x)^{x^x}\left\{\sin 2(x^{\log\cos x})\times(x^{\log\cos x})\left(\frac{\log\cos x}{x}-\log x\times\tan x\right)\right\}$

$$+\sin^2(x^{\log\cos x})\times(x)^{x^x}\left\{\frac{x^x}{x}+x^x(1+\log x)\times\log x\right\}$$

Q17. $f(-\pi/3)=p$...(i)

And, $\lim_{x\to-\pi/3}\frac{\sqrt{3}\cos x+\sin x}{x+\frac{\pi}{3}}=\lim_{x\to-\pi/3}\frac{2\left(\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x\right)}{x+\frac{\pi}{3}}=\lim_{x+\frac{\pi}{3}\to0}\frac{2\sin\left(x+\frac{\pi}{3}\right)}{x+\frac{\pi}{3}}=2\times1=2$...(ii).

As $f(x)$ is continuous at $x=-\pi/3$ so, by (i) and (ii), we get : $p=2$.

OR See a similar sum in Ch-03 of Mathematicia in Derivatives of Parametric Function.

Q18. $(y^3-2x^2y)dx+(2y^2x-x^3)dy=0\quad\Rightarrow\frac{dy}{dx}=\frac{y^3-2x^2y}{x^3-2y^2x}=\frac{y}{x}\times\frac{y^2-2x^2}{x^2-2y^2}$

Now put y = vx and proceed.

Q19. As the vectors $a\hat{i}+a\hat{j}+c\hat{k},\ \hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are coplanar, then $\begin{vmatrix}a&a&c\\1&0&1\\c&c&b\end{vmatrix}=0$. Now proceed.

OR Use $h=\frac{|\vec{c}.(\vec{a}\times\vec{b})|}{|\vec{a}\times\vec{b}|}$. Ans. $\frac{4}{\sqrt{38}}$ units.

Q20. Let A(a, 0, 0), B(0, b, 0) and C(0, 0, c). So, equation of plane : $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$...(i)

$\therefore$ distance of (i) from origin, $3p=\frac{\left|\frac{0}{a}+\frac{0}{b}+\frac{0}{c}-1\right|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\quad\Rightarrow\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{9p^2}$...(ii)

Let centroid of ΔABC be $(\alpha,\beta,\gamma)=\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)\quad\Rightarrow a=3\alpha,\ b=3\beta,\ c=3\gamma$

Replacing values of $\alpha,\ \beta,\ \gamma$ in (ii), we get : $\frac{1}{(3\alpha)^2}+\frac{1}{(3\beta)^2}+\frac{1}{(3\gamma)^2}=\frac{1}{9p^2}\quad\Rightarrow\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=\frac{1}{p^2}$

$\therefore$ required locus is : $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$.

Q21. As A28, 3B9 and 62C are divisible by n so,

$A28=100A+20+8=\alpha n,\ 300+10B+9=\beta n,\ 600+20+C=\gamma n$, where $\alpha,\ \beta,\ \gamma\in Z$

Let $\Delta=\begin{vmatrix}A&3&6\\8&9&C\\2&B&2\end{vmatrix}$ By $R_2\to R_2+100R_1+10R_3$

$$\Delta=\begin{vmatrix}A&3&6\\100A+20+8&300+10B+9&600+20+C\\2&B&2\end{vmatrix}$$

$\Rightarrow\Delta=\begin{vmatrix}A&3&6\\\alpha n&\beta n&\gamma n\\2&B&2\end{vmatrix}=n\begin{vmatrix}A&3&6\\\alpha&\beta&\gamma\\2&B&2\end{vmatrix}$, clearly Δ is divisible by n.

OR See Mathematicia by O.P. Gupta.

Q22. Yes, as $P(A\cap B)=P(A)P(B)=0.02$. (See NCERT Exemplar Solutions by O.P. Gupta)

Q23. See Mathematicia by O.P. Gupta. Ans. 7/15.

SECTION D

Q24. Let $I=\int_0^{\pi/2}\frac{\cos^9 x dx}{\cos^3 x+\sin^3 x}$...(i) . Also, $I=\int_0^{\pi/2}\frac{\sin^9 x dx}{\cos^3 x+\sin^3 x}$ ...(ii)

Adding (i) and (ii), we get : $2I=\int_0^{\pi/2}\frac{(\cos^9 x+\sin^9 x)dx}{\cos^3 x+\sin^3 x}$

$$\Rightarrow 2I=\int_0^{\pi/2}\frac{(\cos^3 x+\sin^3 x)(\cos^6 x+\sin^6 x-\cos^3 x\sin^3 x)dx}{\cos^3 x+\sin^3 x}$$

$$\Rightarrow 2I=\int_0^{\pi/2}(\cos^6 x+\sin^6 x-\cos^3 x\sin^3 x)dx$$

$$\Rightarrow 2I=\int_0^{\pi/2}\left\{(\cos^2 x+\sin^2 x)^3-3\cos^2 x\sin^2 x(\cos^2 x+\sin^2 x)-\cos^3 x\sin^3 x\right\}dx$$

$$\Rightarrow 2I=\int_0^{\pi/2}\left\{1-3\cos^2 x\sin^2 x-\cos^3 x\sin^3 x\right\}dx$$

$$\Rightarrow 2I=\int_0^{\pi/2}1dx-\frac{3}{4}\int_0^{\pi/2}\sin^2 2xdx-\frac{1}{8}\int_0^{\pi/2}\sin^3 2xdx$$

$$\Rightarrow 2I=\int_0^{\pi/2}1dx-\frac{3}{4}\int_0^{\pi/2}\frac{1-\cos 4x}{2}dx-\frac{1}{8}\int_0^{\pi/2}\left[\frac{3\sin 2x-\sin 6x}{4}\right]dx \qquad \therefore I=\frac{5\pi}{32}-\frac{1}{24}.$$

Q25. Let $a+\sqrt{7}b,\ c+\sqrt{7}d\in A$.

As $(a+\sqrt{7}b)+(c+\sqrt{7}d)=(a+c)+\sqrt{7}(b+d)$.

Since $(a+c)$ and $(b+d)$ both belongs to Z for all $a,b,c,d\in Z$ hence, addition is uniquely determined number in A. Therefore, addition is binary operation on A.

Also, $(a+\sqrt{7}b).(c+\sqrt{7}d)=(ac+7bd)+\sqrt{7}(ad+bc)$.

Since $(ac+7bd)$ and $(ad+bc)$ both belongs to Z for all $a,b,c,d\in Z$ hence, product is uniquely determined number in A. Therefore, multiplication is binary operation on A.

OR $1-x^2$.

Q26. We have $\begin{vmatrix}p & b & c\\ a & q & c\\ a & b & r\end{vmatrix}=0$ By $R_1\to R_1-R_2$ and $R_2\to R_2-R_3$

$$\Rightarrow\begin{vmatrix}p-a & b-q & 0\\ 0 & q-b & c-r\\ a & b & r\end{vmatrix}=0$$

On expanding along C_1, we get : $(p-a)\{r(q-b)-b(c-r)\}+a\{(b-q)(c-r)\}=0$

$\Rightarrow(p-a)r(q-b)+(p-a)b(r-c)+a(q-b)(r-c)=0$

Dividing both sides by $(p-a)(q-b)(r-c)$, we get :

$$\Rightarrow\frac{r}{r-c}+\frac{b}{q-b}+\frac{a}{p-a}=0$$

$$\Rightarrow\frac{r}{r-c}+\frac{q+b-q}{q-b}+\frac{p+a-p}{p-a}=0$$

$$\Rightarrow \frac{r}{r-c}+\frac{q}{q-b}+\frac{b-q}{q-b}+\frac{p}{p-a}+\frac{a-p}{p-a}=0$$

$$\Rightarrow \frac{r}{r-c}+\frac{q}{q-b}-1+\frac{p}{p-a}-1=0$$

$$\therefore \frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=2.$$

OR Let $A=\begin{pmatrix}1 & 1 & 1\\ \alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\end{pmatrix}$ $\Rightarrow A^T=\begin{pmatrix}1 & \alpha & \alpha^2\\ 1 & \beta & \beta^2\\ 1 & \gamma & \gamma^2\end{pmatrix}$

As $AA^T=\begin{pmatrix}1 & 1 & 1\\ \alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\end{pmatrix}\begin{pmatrix}1 & \alpha & \alpha^2\\ 1 & \beta & \beta^2\\ 1 & \gamma & \gamma^2\end{pmatrix}=\begin{pmatrix}3 & \alpha+\beta+\gamma & \alpha^2+\beta^2+\gamma^2\\ \alpha+\beta+\gamma & \alpha^2+\beta^2+\gamma^2 & \alpha^3+\beta^3+\gamma^3\\ \alpha^2+\beta^2+\gamma^2 & \alpha^3+\beta^3+\gamma^3 & \alpha^4+\beta^4+\gamma^4\end{pmatrix}$

That is, $AA^T=\begin{pmatrix}S_0 & S_1 & S_2\\ S_1 & S_2 & S_3\\ S_2 & S_3 & S_4\end{pmatrix}$ $\therefore |AA^T|=|A||A^T|=\begin{vmatrix}S_0 & S_1 & S_2\\ S_1 & S_2 & S_3\\ S_2 & S_3 & S_4\end{vmatrix}$

$$\Rightarrow \begin{vmatrix}S_0 & S_1 & S_2\\ S_1 & S_2 & S_3\\ S_2 & S_3 & S_4\end{vmatrix}=|A||A|=|A|^2=\left[(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)\right]^2.$$

Q27. Max. P = 1950 at (15, 25).

Q29. (10, 14, 4).

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