**Series : PTS/12**

**Code No. 18/11/12**

**Candidates must write the Code on the title page of the answer-book.**

**Roll No.** | 2 | 1 | 8 | 1 | 2 | | |

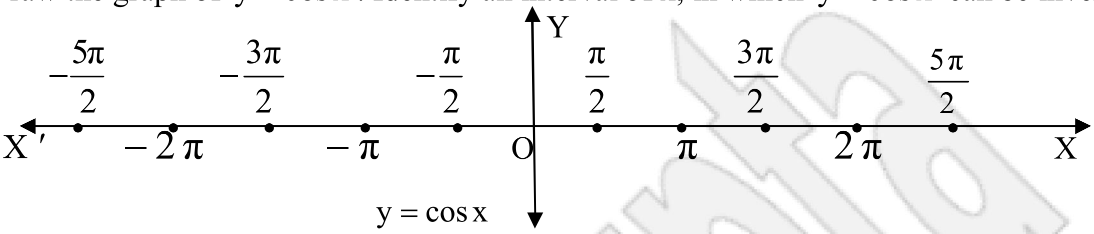
# PLEASURE TEST SERIES XII – 12

**A Compilation By : O.P. Gupta (WhatsApp @ +91 9650350480)**

**Time Allowed : 180 Minutes** **Max. Marks : 100**

## SECTION A

**Q01.** Draw the graph of $y = \cos x$. Identify an interval of x, in which $y = \cos x$ can be inverted.




**Q02.** If * is binary operation defined as a*b = GCF of a and b then, write the value of 13*19.

**Q03.** Write the sum of the order and degree of the differential equation representing the family of curves $y^2 = 4ax$, where 'a' is an arbitrary constant.

**Q04.** Evaluate : $\sin\left(\int_0^1 \frac{dx}{\sqrt{16-(1-x)^2}}\right)$. **OR** Find : $\int_{1/2}^{1} \frac{dx}{\sqrt{1-x^2}}$.

## SECTION B

**Q05.** If $\alpha \le 2\sin^{-1}x + \cos^{-1}x \le \beta$, then find the value of $\alpha$ and $\beta$.

**Q06.** Find a unit vector in XY-plane, making an angle of 30° with the positive direction of x-axis.

**OR** Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. Also let $c_1 = 1$ and $c_2 = 2$, find $c_3$, which makes $\vec{a}$, $\vec{b}$ and $\vec{c}$ coplanar.

**Q07.** Find the unit vector of the normal to the plane 2x – 3y + 3z = 5.

**OR** Prove that the line through A(0, –1, –1) and B(4, 5, 1) intersects the line through C(3, 9, 4) and D(–4, 4, 4).

**Q08.** Find the primitive of $\sin 2x \,.\, \text{cosec}^2 x$.

**Q09.** A swimming pool is to be drained for cleaning. If L represents the number of litres of water in the pool t seconds after the pool has been plugged off to drain and $L = 200\,(10 - t)^2$. How fast is the water running out at the end of 5 seconds? What is the average rate at which the water flows out during the first 5 seconds?

**OR** The pressure P and the volume V of a gas are connected by $PV^{1.4}$ = Constant. Find the % error in P corresponding to a decrease of 0.5% in V.

**Q10.** If curves $y = 3e^{2x}$ and $y = be^{-2x}$ cut each other orthogonally, then determine the value of b.

**Q11.** Show by an example that for matrices $A \ne O, B \ne O$, we may have $AB = O$.

**Q12.** A bag contains 4 balls. Two balls are drawn at random, and are found to be blue. What is the probability that 50% balls in the bag were blue?

## SECTION C

**Q13.** Find the intervals in which the value of the determinant of matrix $\begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$ lies.

**OR** If $A = diag\begin{pmatrix} a & b & c \end{pmatrix}$, show that $A^n = diag\begin{pmatrix} a^n & b^n & c^n \end{pmatrix}\ \forall\, n \in N$.

**Q14.** If $y=\sin^{-1}[\sqrt{x^4-x^6}+\sqrt{x^2-x^6}]$ then, prove that $\frac{dy}{dx}=\frac{2x}{\sqrt{1-x^4}}+\frac{1}{\sqrt{1-x^2}}$.

**Q15.** Evaluate : $\cot(45^\circ-2\cot^{-1}3)$ **OR** Evaluate : $\sin(2\tan^{-1}(1/3))+\cos(\tan^{-1}2\sqrt{2})$.

**Q16.** Find $\frac{dy}{dx}$, if $y=\sqrt{a^{\sin^{-1}t}}$ and, $x=\sqrt{a^{\cos^{-1}t}}$, where 'a' is a constant.

**OR** Find $\frac{dy}{dx}$, if $y=\sqrt{a+\sqrt{a+\sqrt{a+x^2}}}$, where 'a' is a constant.

**Q17.** Find the equation of normal and tangent to the curve $x=\sin 3t,\ y=\cos 2t$ at $t=\pi/4$.

**Q18.** Evaluate : $\int_0^\pi \frac{dx}{3+2\sin x+\cos x}$.

**Q19.** If the vectors $\vec{p}$ and $\vec{q}$ are the diagonals of a parallelogram with sides $\vec{a}$ and $\vec{b}$, find the area of parallelogram in terms of its diagonals. Hence find the area in terms of its sides.

**Q20.** Solve : $(1+y+x^2y)dx+(x+x^3)dy=0$, where $y=0$ when $x=1$.

**Q21.** A line makes angles α, β, γ and δ with the diagonals of a cube, then evaluate the following :

$$\sin^2\alpha+\sin^2\beta+\sin^2\gamma+\sin^2\delta.$$

**Q22.** A drunkard man takes a step forward with probability 0.6 and takes a step backward with probability 0.4. He takes 9 steps in all. Find the probability that he is just one step away from the initial point.

**Q23.** Two thirds of the students in a class are boys and the rest are girls. It is known that the probability of a girl getting a first class is 0.25 and that of a boy getting a first class is 0.28. Find the probability that a student chosen at random will get first class marks in the subject.

## SECTION D

**Q24.** If it is known that $R_1$ and $R_2$ are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.

**Q25.** Prove that : $\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix}=2s^3(s-a)(s-b)(s-c)$, where s represents the semi-perimeter of a triangle with side lengths a, b and c.

**OR** Using properties, prove that $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix}=(a+b+c)(a^2+b^2+c^2)$.

**Q26.** Find the area bounded by the curve x = y | y |, x-axis and the ordinates x = – 1 and x = 1.

**OR** Show that the curves $y=x^2$ and $x=y^2$ divide the square bounded by x = 0, y = 0, x = 1 and y = 1 into three parts that are equal in area. Also find the area of each equal part.

**Q27.** Evaluate : $\int \sin^{-1}x \log x\ dx$. **OR** Evaluate : $\int_{-1}^{1/2}\frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}dx$.

**Q28.** Find the equation(s) of line(s) through the origin intersecting the line $3x-9=6y-18=6z$ at an angle of $\pi/3$.

**Q29.** A village has 500 hectares of land to grow two types of plants X and Y. The contribution of total amount of oxygen produced by plant X and Y are 60% and 40% per hectare respectively. To control weeds, a liquid herbicide has to be used for the plants X and Y at the rate of 20 litres and 10 litres per hectare, respectively. Further no more than 8000 litres of herbicides should be used in order to protect aquatic animals in a pond which collects drainage from this land. How much land should allocated to each crop so as to maximize the total production of oxygen?

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# ANSWERS & HINTS For PTS – 12

## SECTION A

**Q01.** Draw the graph yourself. Also required intervals for existence of $\cos^{-1}:[-\pi,0],[0,\pi],[\pi,2\pi]$ etc.

**Q02.** 1 **Q03.** Obtain $2x\dfrac{dy}{dx}-y=0$. Then order + degree = 1 + 1 = 2.

**Q04.** $\sin\left(\int_0^1 \dfrac{dx}{\sqrt{16-(1-x)^2}}\right)=\sin\left(\left[\dfrac{1}{-1}\sin^{-1}\dfrac{1-x}{4}\right]_0^1\right)=\sin\left(\left[-\sin^{-1}0\right]-\left[-\sin^{-1}\dfrac{1}{4}\right]\right)=\sin\sin^{-1}\dfrac{1}{4}=\dfrac{1}{4}$.

**OR** $\int_{1/2}^{1}\dfrac{dx}{\sqrt{1-x^2}}=\left[\sin^{-1}x\right]_{1/2}^{1}=\sin^{-1}1-\sin^{-1}\left(\dfrac{1}{2}\right)=\dfrac{\pi}{2}-\left(\dfrac{\pi}{6}\right)=\dfrac{\pi}{3}$.

## SECTION B

**Q05.** As $-\dfrac{\pi}{2}\le\sin^{-1}x\le\dfrac{\pi}{2}$ $\Rightarrow -\dfrac{\pi}{2}+\dfrac{\pi}{2}\le\sin^{-1}x+\dfrac{\pi}{2}\le\dfrac{\pi}{2}+\dfrac{\pi}{2}$

$\Rightarrow 0\le\sin^{-1}x+(\sin^{-1}x+\cos^{-1}x)\le\pi \Rightarrow 0\le 2\sin^{-1}x+\cos^{-1}x\le\pi$

On comparing with $\alpha\le 2\sin^{-1}x+\cos^{-1}x\le\beta$, we get $\alpha=0,\ \beta=\pi$.

**Q06.** $\dfrac{\sqrt{3}}{2}\hat{i}+\dfrac{1}{2}\hat{j}$

**OR** If $\vec{a},\vec{b}$ and $\vec{c}$ are coplanar then, $[\vec{a}\ \vec{b}\ \vec{c}]=0$ i.e., $\begin{vmatrix}1&1&1\\1&0&0\\c_1&c_2&c_3\end{vmatrix}=0 \Rightarrow \begin{vmatrix}1&1&1\\1&0&0\\1&2&c_3\end{vmatrix}=0$

Expanding along $R_2$, we get : $c_3-2=0 \quad \therefore c_3=2$.

**Q07.** $\dfrac{2\hat{i}-3\hat{j}+3\hat{k}}{\sqrt{22}}$.

**OR** Eq. of line AB : $\vec{r}=-\hat{j}-\hat{k}+\lambda(4\hat{i}+6\hat{j}+2\hat{k})$ and,

Eq. of line CD : $\vec{r}=3\hat{i}+9\hat{j}+4\hat{k}+\mu(-7\hat{i}-5\hat{j})$.

Clearly, $\vec{a}_2-\vec{a}_1=3\hat{i}+10\hat{j}+5\hat{k}$, $\vec{b}_1=4\hat{i}+6\hat{j}+2\hat{k}$, $\vec{b}_2=-7\hat{i}-5\hat{j}$

Since $(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)=[\vec{a}_2-\vec{a}_1\ \ \vec{b}_1\ \ \vec{b}_2]=\begin{vmatrix}3&10&5\\4&6&2\\-7&-5&0\end{vmatrix}=3\times10-10\times14+5\times22=0$.

Hence the lines are in the same plane so, they must intersect each other.

**Q08.** As $\int\dfrac{f'(x)}{f(x)}dx=\log|f(x)|+C$. So, $\int\dfrac{\sin 2x}{\sin^2 x}dx=\log|\sin^2 x|+C=2\log|\sin x|+C$.

**Q09.** See NCERT Exemplar Solutions by O.P. Gupta (Ch 06 - Q09).

**OR** Given $\dfrac{\Delta V}{V}=-\dfrac{1}{2}\%$ $\Rightarrow \Delta V=\left(-\dfrac{1}{2}\%\right)V\approx dV$.

Now $PV^{1.4}=\text{Constant}$

On differentiating w. r. t. V, we get : $P(1.4V^{0.4})+V^{1.4}\dfrac{dP}{dV}=0$

$\Rightarrow\dfrac{dP}{dV}=-1.4\left(\dfrac{P}{V}\right) \quad \Rightarrow\dfrac{dP}{P}=-1.4\left(\dfrac{dV}{V}\right) \quad \Rightarrow\dfrac{dP}{P}=-1.4\left(\dfrac{-0.5V}{100V}\right)=0.7\%$.

**Q10.** 1/12 .

**Q12.** Let E : both the balls drawn were Blue. Let $E_1$, $E_2$, $E_3$, $E_4$ and, $E_5$ be the events that bag contains 0, 1, 2, 3 and, 4 Blue balls, respectively.
By Bayes' Theorem,

$$P(E_3|E)=\frac{P(E|E_3)P(E_3)}{P(E|E_1)P(E_1)+P(E|E_2)P(E_2)+P(E|E_3)P(E_3)+P(E|E_4)P(E_4)+P(E|E_5)P(E_5)}$$

$$\Rightarrow P(E_3|E)=\frac{\frac{{}^2C_2}{{}^4C_2}\times\frac{1}{5}}{0\times\frac{1}{5}+0\times\frac{1}{5}+\frac{{}^2C_2}{{}^4C_2}\times\frac{1}{5}+\frac{{}^3C_2}{{}^4C_2}\times\frac{1}{5}+\frac{{}^4C_2}{{}^4C_2}\times\frac{1}{5}}=\frac{\frac{1}{6}}{\frac{1}{6}+\frac{3}{6}+1}=\frac{1}{1+3+6}=\frac{1}{10}.$$

**SECTION C**

**Q13.** Obtain the value of $\Delta=2(1+\sin^2\theta)$ then, $\Delta\in[2,4]$.

**OR** **See O.P. Gupta's Mathematicia**.

**Q15.** LHS: Let $Y=\cot\left(\frac{\pi}{4}-2\cot^{-1}3\right)=\frac{1}{\tan\left(\frac{\pi}{4}-2\tan^{-1}\frac{1}{3}\right)}$

$$\Rightarrow\quad=\frac{1+\tan\frac{\pi}{4}\tan\left(2\tan^{-1}\frac{1}{3}\right)}{\tan\frac{\pi}{4}-\tan\left(2\tan^{-1}\frac{1}{3}\right)}=\frac{1+1.\frac{2\tan\tan^{-1}\frac{1}{3}}{1-\left(\tan\tan^{-1}\frac{1}{3}\right)^2}}{1-\frac{2\tan\tan^{-1}\frac{1}{3}}{1-\left(\tan\tan^{-1}\frac{1}{3}\right)^2}}$$

$$\Rightarrow\quad=\frac{1+\frac{\frac{2}{3}}{1-1/9}}{1-\frac{2/3}{1-1/9}}=\frac{14}{2}=7=\text{RHS}.$$

**OR** Let $Y=\sin 2\tan^{-1}\frac{1}{3}+\cos\tan^{-1}2\sqrt{2}$

$$=\sin\sin^{-1}\left(\frac{2.\frac{1}{3}}{1+\left(\frac{1}{3}\right)^2}\right)+\cos\cos^{-1}\frac{1}{3}$$

$\left[\text{Put } \tan^{-1}2\sqrt{2}=\theta\Rightarrow\tan\theta=2\sqrt{2}\right.$

$\left.\Rightarrow\cos\theta=\frac{1}{3}\Rightarrow\theta=\cos^{-1}\frac{1}{3}\right]$

$$=\frac{\frac{2}{3}}{\frac{10}{9}}+\frac{1}{3}=\frac{6}{10}+\frac{1}{3}=\frac{14}{15}.$$

**Q16.** Here $y=\sqrt{a^{\sin^{-1}t}}$ and, $x=\sqrt{a^{\cos^{-1}t}}$ s. t., $yx=\sqrt{a^{\sin^{-1}t}}\sqrt{a^{\cos^{-1}t}}$ $\Rightarrow xy=a^{\frac{\sin^{-1}t+\cos^{-1}t}{2}}$

$\Rightarrow xy=a^{\pi/4}$ $\quad\therefore x\frac{dy}{dx}+y.1=0$ *i.e.*, $\frac{dy}{dx}=-\frac{y}{x}$.

# Note that we may solve this question in many other ways too!

**OR** As $y=\sqrt{a+\sqrt{a+\sqrt{a+x^2}}}$ $\Rightarrow y^2-a=\sqrt{a+\sqrt{a+x^2}}$ $\Rightarrow(y^2-a)^2=a+\sqrt{a+x^2}$

$\therefore 2(y^2-a)\times 2yy'=0+\frac{x}{\sqrt{a+x^2}}$ $\quad\Rightarrow\frac{dy}{dx}=\frac{x}{4y(y^2-a)\sqrt{a+x^2}}$.

**Q17.** Eq. of normal at $t=\pi/4$ : $y-\cos 2(\pi/4)=\frac{3\cos 3(\pi/4)}{2\sin 2(\pi/4)}\left[x-\sin 3(\pi/4)\right]$

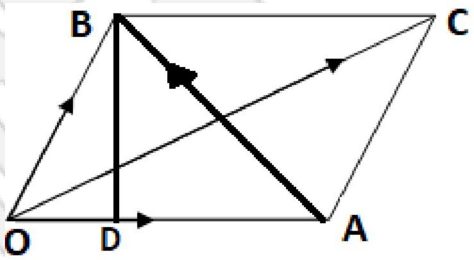
$$\Rightarrow y-0=\frac{-3\times\frac{1}{\sqrt{2}}}{2\times 1}\left[x-\frac{1}{\sqrt{2}}\right] \qquad \therefore 3\sqrt{2}\ x+4y=3.$$

Eq. of tangent at $t=\pi/4$ : $y-\cos 2(\pi/4)=\frac{-2\sin 2(\pi/4)}{3\cos 3(\pi/4)}\left[x-\sin 3(\pi/4)\right]$

$$\Rightarrow y-0=\frac{-2\times 1}{-3\times\frac{1}{\sqrt{2}}}\left[x-\frac{1}{\sqrt{2}}\right] \qquad \Rightarrow y=\frac{2\sqrt{2}}{3}\left[\frac{\sqrt{2}\,x-1}{\sqrt{2}}\right] \qquad \therefore 4x-3\sqrt{2}\,y=2\sqrt{2}.$$

**Q18.** Use formulas $\sin A=\frac{2\tan(A/2)}{1+\tan^2(A/2)}, \cos A=\frac{1-\tan^2(A/2)}{1+\tan^2(A/2)}$ $\quad\therefore I=\frac{\pi}{4}$.

**Q19.** Note that, the area of parallelogram $=$ Base $\times$ Height.
Consider the diagram shown.
Let $\overrightarrow{OA}=\vec{a}$ and $\overrightarrow{OB}=\vec{b}$. Also $\overrightarrow{OC}=\vec{p}=\vec{b}+\vec{a}$ and $\overrightarrow{AB}=\vec{q}=\vec{b}-\vec{a}$.
Let $\theta$ be the angle between sides OA and OB of $||^{gm}$.




In $\Delta$OBD, $BD=OB\sin\theta \Rightarrow |\overrightarrow{BD}|=|\overrightarrow{OB}|\sin\theta$

$\therefore$ Height $=|\overrightarrow{BD}|=|\vec{b}|\sin\theta$.

Now area of $||^{gm}=OA.BD=|\overrightarrow{OA}||\overrightarrow{BD}|$

$\Rightarrow \quad =|\vec{a}||\vec{b}|\sin\theta=|\vec{a}\times\vec{b}|$ ...(i)

Also, we know that diagonals of $||^{gm}$ are $\vec{p}=\vec{b}+\vec{a}$ and $\vec{q}=\vec{b}-\vec{a}$.

So, clearly $\vec{b}=\frac{\vec{p}+\vec{q}}{2}$ and, $\vec{a}=\frac{\vec{p}-\vec{q}}{2}$.

By (i), we have area of $||^{gm}=|\vec{a}\times\vec{b}|=\left|\left(\frac{\vec{p}-\vec{q}}{2}\right)\times\left(\frac{\vec{p}+\vec{q}}{2}\right)\right|=\left|\frac{\vec{p}\times\vec{p}+\vec{p}\times\vec{q}-\vec{q}\times\vec{p}-\vec{q}\times\vec{q}}{4}\right|$

$\Rightarrow \quad =\left|\frac{\vec{0}+\vec{p}\times\vec{q}+\vec{p}\times\vec{q}-\vec{0}}{4}\right|=\left|\frac{2\vec{p}\times\vec{q}}{4}\right|=\frac{1}{2}|\vec{p}\times\vec{q}|$ ...(ii)

The equation (ii) represents area of parallelogram in terms of its diagonals and, equation (i) represents the area in terms of its sides.

**Q20.** As $\frac{dy}{dx}+\frac{1+y+x^2y}{x+x^3}=0 \quad \Rightarrow \frac{dy}{dx}+\left(\frac{1+x^2}{x(1+x^2)}\right)y=-\frac{1}{x+x^3} \quad \Rightarrow \frac{dy}{dx}+\left(\frac{1}{x}\right)y=-\frac{1}{x+x^3}$

It's Linear Differential Equation in y. Proceed to complete, now. Ans. $4(xy+\tan^{-1}x)=\pi$.

**Q21.** See NCERT Page 494 Example 26 (Similar Question).

**Q22.** Given probability that the man takes a step forward = 0.6 = *p* so, *q* = 1 – p = 0.4.
Let $E_1$ be the event that out of 9 steps the man takes exactly 5 steps are forward and 4 steps backward.
Let $E_2$ be the event that out of 9 steps the man takes exactly 4 steps are forward and 5 steps backward.
Also let E be the event that at the end of 9 steps, the man is one step away from the starting point.

$\therefore P(E)=P(E_1)+P(E_2)=P(5)+P(4)={}^9C_5(0.6)^5(0.4)^4+{}^9C_4(0.6)^4(0.4)^5$

$\Rightarrow \quad =126(0.6)^5(0.4)^4+126(0.6)^4(0.4)^5=126(0.6)^4(0.4)^4(0.6+0.4)=126\times(0.24)^4$

**Q23.** Ans. 0.27. See **O.P. Gupta's Mathematicia** (Total Probability).

## SECTION D

**Q24.** Since $R_1$ and $R_2$ are equivalence relations, (a, a) ∈ $R_1$, and (a, a) ∈ $R_2$ ∀ a ∈ A.

This implies that $(a, a) \in R_1 \cap R_2, \forall$ a, showing $R_1 \cap R_2$ is reflexive.
Further, $(a, b) \in R_1 \cap R_2 \Rightarrow (a, b) \in R_1$ and $(a, b) \in R_2 \Rightarrow (b, a) \in R_1$ and $(b, a) \in R_2$
$\Rightarrow (b, a) \in R_1 \cap R_2$, hence, $R_1 \cap R_2$ is symmetric.
Similarly, $(a, b) \in R_1 \cap R_2$ and $(b, c) \in R_1 \cap R_2 \Rightarrow (a, c) \in R_1$ and $(a, c) \in R_2 \Rightarrow (a, c) \in R_1 \cap R_2$.
This shows that $R_1 \cap R_2$ is transitive.
Thus, $R_1 \cap R_2$ is an equivalence relation.

**Q25.** Let $s-a=p,\ s-b=q,\ s-c=r \quad \Rightarrow 3s-(a+b+c)=p+q+r \quad \Rightarrow p+q+r=3s-2s=s$

$\therefore a=s-p=(p+q+r)-p=q+r,\ b=r+p,\ c=p+q$

$$\text{Now let } \Delta=\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix}$$

$$\Rightarrow \Delta=\begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix}$$

Now proceed to complete it.

**OR** Let $\Delta=\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix}$ By $C_1 \to aC_1, C_2 \to bC_2, C_3 \to cC_3$

$\Rightarrow \Delta=\dfrac{1}{abc}\begin{vmatrix} a^2 & b^2-bc & c^2+bc \\ a^2+ac & b^2 & c^2-ac \\ a^2-ab & b^2+ab & c^2 \end{vmatrix}$ By $C_1 \to C_1+C_2+C_3$

$\Rightarrow \Delta=\dfrac{1}{abc}\begin{vmatrix} a^2+b^2+c^2 & b^2-bc & c^2+bc \\ a^2+b^2+c^2 & b^2 & c^2-ac \\ a^2+b^2+c^2 & b^2+ab & c^2 \end{vmatrix}$ Take $a^2+b^2+c^2$ common from $C_1$

$\Rightarrow \Delta=\dfrac{a^2+b^2+c^2}{abc}\begin{vmatrix} 1 & b^2-bc & c^2+bc \\ 1 & b^2 & c^2-ac \\ 1 & b^2+ab & c^2 \end{vmatrix}$ By $R_1 \to R_1-R_2,\ R_2 \to R_2-R_3$

$\Rightarrow \Delta=\dfrac{a^2+b^2+c^2}{abc}\begin{vmatrix} 0 & -bc & bc+ac \\ 0 & -ab & -ac \\ 1 & b^2+ab & c^2 \end{vmatrix}$ Expanding along $C_1$

$$\Rightarrow \Delta=\frac{a^2+b^2+c^2}{abc}\left[abc^2+ab^2c+a^2bc\right]$$

$$\Rightarrow \Delta=\frac{a^2+b^2+c^2}{abc}\times abc\left[c+b+a\right]$$

$\therefore \Delta=(a+b+c)(a^2+b^2+c^2)$.

**Q26.** Ans. 4/3 Sq. units.
**OR** **See similar question in NCERT Textbook Part II Chapter 08 Example 13**.
Here 1/3 Sq. units for each part.

**Q27.** Let $I=\int \sin^{-1}x \log x\ dx=\sin^{-1}x\int \log x\ dx-\int\left(\dfrac{d}{dx}\sin^{-1}x\int \log x\ dx\right)dx$

$\Rightarrow I=\sin^{-1}x\left[x(\log x-1)\right]+\int \dfrac{-x(\log x-1)}{\sqrt{1-x^2}}dx$ $\left[\begin{array}{l}\text{Put } 1-x^2=t^2 \Rightarrow 1-t^2=x^2 \\ \text{and, } -xdx=tdt\end{array}\right.$

$$\Rightarrow I=\sin^{-1}x\left[x(\log x-1)\right]+\int\frac{(\log\sqrt{1-t^2}-1)}{t}\times t\,dt$$

$$\Rightarrow I=\sin^{-1}x\left[x(\log x-1)\right]+\int\log\sqrt{1-t^2}\,dt-\int 1\,dt$$

$$\Rightarrow I=\sin^{-1}x\left[x(\log x-1)\right]+\log\sqrt{1-t^2}\int 1\,dt-\int\left(\frac{d}{dt}\log\sqrt{1-t^2}\int 1\,dt\right)dt-t$$

$$\Rightarrow I=\sin^{-1}x\left[x(\log x-1)\right]+t\times\log\sqrt{1-t^2}-\int\left(\frac{1}{\sqrt{1-t^2}}\times\frac{-t}{\sqrt{1-t^2}}\times t\right)dt-t$$

$$\Rightarrow I=\sin^{-1}x\left[x(\log x-1)\right]+t\times\log\sqrt{1-t^2}-\int\left(\frac{-t^2}{1-t^2}\right)dt-t$$

$$\Rightarrow I=\sin^{-1}x\left[x(\log x-1)\right]+t\times\log\sqrt{1-t^2}-\int\left(1-\frac{1}{1-t^2}\right)dt-t$$

$$\Rightarrow I=\sin^{-1}x\left[x(\log x-1)\right]+t\times\log\sqrt{1-t^2}-\left[t-\frac{1}{2}\log\left|\frac{1-t}{1+t}\right|\right]-t+C$$

$$\Rightarrow I=x(\log x-1)\sin^{-1}x+\sqrt{1-x^2}\times\log\sqrt{x^2}+\frac{1}{2}\log\left|\frac{1-\sqrt{1-x^2}}{1+\sqrt{1-x^2}}\right|-2\sqrt{1-x^2}+C$$

$$\therefore I=x(\log x-1)\sin^{-1}x+\sqrt{1-x^2}\times\log x+\frac{1}{2}\log\left|\frac{1-\sqrt{1-x^2}}{1+\sqrt{1-x^2}}\right|-2\sqrt{1-x^2}+C.$$

**OR** Consider $\frac{(2-x^2)}{(1-x)\sqrt{1-x^2}}=\frac{(1-x^2)}{(1-x)\sqrt{1-x^2}}+\frac{1}{(1-x)\sqrt{1-x^2}}$

$$\Rightarrow\quad=\frac{(1+x)}{\sqrt{1-x^2}}+\frac{1}{(1-x)\sqrt{1-x^2}}=\sqrt{\frac{1+x}{1-x}}+\frac{1}{(1-x)\sqrt{1-x^2}}$$

Using $\int e^x\left[f(x)+f'(x)\right]dx=e^xf(x)+C$, we have :

$$\int_{-1}^{1/2}e^x\left[\sqrt{\frac{1+x}{1-x}}+\frac{1}{(1-x)\sqrt{1-x^2}}\right]dx=\left[e^x\sqrt{\frac{1+x}{1-x}}\right]_{-1}^{1/2}=\left[e^{1/2}\sqrt{3}-0\right]=\sqrt{3e}.$$

**Q28.** Let the coordinates of any random point on the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ be M( $2\lambda+3,\lambda+3,\lambda$ ).

Since the required line passes through the origin O(0, 0, 0) so, d.r.'s of line OM : $2\lambda+3,\lambda+3,\lambda$.
As the angle between required line OM and given line is $\pi/3$ so,

$$\cos\frac{\pi}{3}=\frac{(2\lambda+3).2+(\lambda+3).1+\lambda.1}{\sqrt{(2\lambda+3)^2+(\lambda+3)^2+\lambda^2}\sqrt{2^2+1^2+1^2}}$$

$\Rightarrow\lambda=-2,-1$.

$\therefore$ d. r.'s of required lines are –1, 1,–2 or 1, 2,–1.

Hence the equations of required lines are : $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1};\ \frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$.

**Q29.** Let x and y hectares of land be allocated to plant X and Y respectively.

To maximize : $Z=\left(\frac{60}{100}x+\frac{40}{100}y\right)$

Subject to constraints : $x, y\ge 0,\ x+y\le 500,\ 20x+10y\le 8000$

Maximum value of Z = ₹260/- is attained at (300, 200).