Series : PTS/19

Code No. 18/12/19

Roll No. | 2 | 1 | 8 | 1 | 2 | | |

Candidates must write the Code on the title page of the answer-book.

PLEASURE TEST SERIES XII – 19

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Time Allowed : 180 Minutes **Max. Marks : 100**

SECTION A

Q01. If $f(x) = \{4-(x-7)^3\}$, then find $f^{-1}(x)$.

OR Check whether the Binary Operation * defined on the set **R** of real numbers by $a*b = 3ab/7$ is commutative or not.

Q02. Evaluate : $\cos\left[\sin^{-1}0.25+\sec^{-1}1.\overline{3}\right]$.

Q03. Differentiate $\sqrt{\tan\sqrt{x}}$ w.r.t. x.

Q04. Find the least value of the function $f(x) = ax + bx^{-1}$; $a > 0, b > 0, x > 0$.

SECTION B

Q05. Express the matrix $A=\begin{pmatrix}-1 & 2\\ 0 & 5\end{pmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Q06. Which is greater : $\tan 1$ or $\tan^{-1}1$? Give reason (s). **OR** Find : $\sin(2\cos^{-1}(-0.6))$.

Q07. Evaluate : $\int_2^1 e^{-\log x}dx$. **OR** Find : $\int\frac{x\sin(\tan^{-1}x^2)}{1+x^4}dx$.

Q08. Write the direction cosines of the line joining the points (2, 3, 4) to its image in XY-plane.

Q09. Find $[\vec{a}\ \vec{b}\ \vec{c}]$, if $[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}] = 72$.

OR If $|\vec{a}| = 4$ and $-3 \le \lambda \le 2$, then find the range of $|\lambda\vec{a}|$.

Q10. If $y = (\tan^{-1}x)^2$, then prove that $(x^2+1)^2y_2 + 2x(x^2+1)y_1 = 2$.

Q11. Evaluate the anti-derivative of $5^x + x^5 + 5x^{-1} + 0.2x$.

Q12. Two events E and F are independent. If P(E) = 0.3, $P(E\cup F) = 0.5$, then find $P(E|F) - P(F|E)$.

SECTION C

Q13. Find the value of κ so that $f(x)=\begin{cases}\dfrac{\sqrt{1+\kappa x}-\sqrt{1-\kappa x}}{x}, & \text{if } -1\le x<0\\ \dfrac{2x+1}{x-1}, & \text{if } 0\le x\le 1\end{cases}$ is continuous at x = 0.

Q14. Find the general solution of (1 + tan y) (dx – dy) + 2xdy = 0.

Q15. Solve for x : $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}(3x)$.

OR Solve : $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$.

Q16. Evaluate : $\int_0^{\pi/2}\frac{\sec^2 x}{(\sec x+\tan x)^n}dx$.

Q17. A manufacturer produces three products x, y, z which he sells in two markets.
Annual sales are indicated below :

Market	**Products**		
I	10000	2000	18000
II	6000	20000	8000

(a) If unit sale prices of x, y and z are ₹ 2.50, ₹ 1.50 and ₹ 1.00, respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are ₹ 2.00, ₹ 1.00 and ₹ 0.50 respectively. Find the gross profit.

Q18. Find the equation of tangent to an hyperbola having length of transverse and conjugate axes as 2a and 2b along x and y axes respectively at the point $(\sqrt{2}\,a, b)$. Also find the corresponding normal.

Q19. If $y\sqrt{1-x^2}=\sin^{-1}x$, show that $(1-x^2)y_2-3xy_1-y=0$.

OR Find $f'(x)$, if $f(x)=\cot^{-1}\left[\dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}\right];\ \dfrac{\pi}{2}<x<\dfrac{3\pi}{2}$.

Q20. Bag I contains 3 black and 2 white bulbs, bag II contains 2 black and 4 white bulbs. One bag is selected at random and a ball is drawn from it. Find the probability of drawing a black bulb.

Q21. Prove that $\vec{a}.(\vec{b}+\vec{c})\times(\vec{a}+2\vec{b}+3\vec{c})=\vec{a}.(\vec{b}\times\vec{c})$.

Q22. Show that the equation of the $\perp^{er}$ from the point (1, 6, 3) to the line $6x=3(y-1)=2(z-2)$ is $x=1,\ \dfrac{y-6}{-3}=\dfrac{z-3}{2}$ and the foot of $\perp^{er}$ is (1, 3, 5) and the length of the $\perp^{er}$ is $\sqrt{13}$ units.

Q23. Four cards are drawn successively with replacement from a well shuffled deck of 52 playing cards. What is the probability that (i) all the 4 cards are spades? (ii) only 2 cards are spades?

OR A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of number of successes. Hence find the mean of the distribution.

SECTION D

Q24. Show that the lines $5x-25=5y-35=-4z-12$ and $3x-24=21y-84=7z-35$ intersect each other. Also find their point of intersection.

OR Find the vector and Cartesian equations of the plane passing through the line of intersection of the planes $\vec{r}.(2\hat{i}+2\hat{j}-3\hat{k})=7,\ \vec{r}.(2\hat{i}+5\hat{j}+3\hat{k})=9$ such that the intercepts made by the plane on x-axis and z-axis are equal.

Q25. A manufacturer has three machines I, II and III installed in his factory. Machine I and II are capable of being operated for at most α hours and machine III must be operated for at least β hours a day, where $\alpha>\beta$ and both are the roots of $x^2-17x+60=0$. It produces only two items M and N each requiring the use of all three machines. The no. of hours required for producing a unit of each of M and N on the three machines are given as follow :

	Machines		
Items	I	II	III
M	1	2	1
N	2	1	5/4

It makes a profit of ₹600 and ₹400 on items M and N respectively. How many of each item should be produced so as to maximize his profit assuming that he can sell all the items that he produced? Also find the maximum profit.

Q26. Using properties of determinants, prove that $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}=0$.

Q27. Everyone wants to be a perfect ideal human being. Let us assume that dishonesty is one of the factors that affects our perfectness and perfectness has an inverse square relationship with dishonesty. For any value x of level of dishonesty, we have a unique value y of perfection.

(i) Write down the equation that relates y with x.

(ii) Does this relationship from $x\in(0,\infty)$ to $y\in(0,\infty)$, form a function?

(iii) For what level of dishonesty one can achieve $(1/4)^{th}$ level of perfection?

(iv) Write the change in level of perfection when the level of dishonesty changes from 4 to 2?

Q28. A poor deceased farmer had agriculture land bounded by the curve y = cos x, between x = 0 and the line x = 2π. He had two sons. They want to distribute this land in 2 parts as decided by their deceased father, such that both of them have equal share of land. Find the area of each part.

OR Find the area of the region $\{(x,y): y^2\le 4x,\ 4x^2+4y^2\le 9\}$, using integration.

Q29. Evaluate : $\int \tan^{2/3}x\,dx$. **OR** Evaluate : $\int \sin^3\sqrt{x}\,dx$. ◘

ANSWERS & HINTS For PTS-19

SECTION A

Q01. $7+\sqrt[3]{4-x}$

OR As $a*b=\frac{3ab}{7}$ and, $b*a=\frac{3ba}{7}=\frac{3ab}{7}=a*b$ so, clearly * is commutative.

Q02. $\cos\left[\sin^{-1}\frac{1}{4}+\sec^{-1}\frac{4}{3}\right]=\cos\left[\sin^{-1}\frac{1}{4}+\cos^{-1}\frac{3}{4}\right]$

$\Rightarrow \quad =\cos\left[\sin^{-1}\frac{1}{4}\right]\cos\left[\cos^{-1}\frac{3}{4}\right]-\sin\left[\sin^{-1}\frac{1}{4}\right]\sin\left[\cos^{-1}\frac{3}{4}\right]$

$\Rightarrow \quad =\cos\left[\cos^{-1}\frac{\sqrt{15}}{4}\right]\times\frac{3}{4}-\frac{1}{4}\sin\left[\sin^{-1}\frac{\sqrt{7}}{4}\right]=\frac{3\sqrt{15}-\sqrt{7}}{16}$.

Q03. $\frac{\sec^2\sqrt{x}}{4\sqrt{x\tan\sqrt{x}}}$. **Q04.** $2\sqrt{ab}$.

SECTION B

Q05. $A=\begin{pmatrix}-1 & 2\\ 0 & 5\end{pmatrix}=\begin{pmatrix}-1 & 1\\ 1 & 5\end{pmatrix}+\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$

Q06. See Exemplar Solutions by O.P. Gupta

OR Let $y=\sin\left(2\cos^{-1}\left(-\frac{3}{5}\right)\right)=2\sin\cos^{-1}\left(-\frac{3}{5}\right)\cos\cos^{-1}\left(-\frac{3}{5}\right)$

$\Rightarrow y=2\sqrt{1-\left\{\cos\cos^{-1}\left(-\frac{3}{5}\right)\right\}^2}\times\left(-\frac{3}{5}\right) \qquad \Rightarrow y=2\sqrt{1-\frac{9}{25}}\times\left(-\frac{3}{5}\right)=-\frac{6}{5}\times\frac{4}{5}=-\frac{24}{25}$.

Q07. Let $I=\int_2^1 e^{-\log x}dx \qquad \Rightarrow I=\int_2^1 e^{\log x^{-1}}dx \qquad \Rightarrow I=\int_2^1 x^{-1}dx$

$\Rightarrow I=\int_2^1\frac{1}{x}dx \qquad \Rightarrow I=\left[\log|x|\right]_2^1 \qquad \Rightarrow I=\log|1|-\log|2|$.

Hence, $I=\log 1-\log 2=-\log 2$ or, $\log\left(\frac{1}{2}\right)$.

OR Let $I=\int\frac{x\sin(\tan^{-1}x^2)}{1+x^4}dx$ [Put $\tan^{-1}x^2=t\Rightarrow\frac{x}{1+x^4}dx=\frac{1}{2}dt$.

$\therefore I=\frac{1}{2}\int\sin t\,dt=\frac{1}{2}(-\cos t)+k \qquad \Rightarrow I=-\frac{1}{2}\cos\left(\tan^{-1}x^2\right)+k$

Q08. $0,0,\pm1$.

Q09. As $[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]=2[\vec{a}\ \vec{b}\ \vec{c}] \qquad \Rightarrow 72=2[\vec{a}\ \vec{b}\ \vec{c}] \qquad \therefore[\vec{a}\ \vec{b}\ \vec{c}]=36$.

(First of all show that, $[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]=(\vec{a}+\vec{b}).\{(\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\}=2[\vec{a}\ \vec{b}\ \vec{c}]$.

OR The smallest value of $|\lambda\vec{a}|$ will exist at numerically smallest value of λ, i.e., at $\lambda=0$, which gives $|\lambda\vec{a}|=|\lambda||\vec{a}|=0\times4=0$.

Also, numerically greatest value of λ is 3, at which $|\lambda\vec{a}|=|\lambda||\vec{a}|=3\times4=12$.

Hence the range of $|\lambda\vec{a}|$ is [0, 12].

Q11. $\frac{5^x}{\log 5}+\frac{x^6}{6}+5\log|x|+\frac{x^2}{10}+C$ **Q12.** 1/70.

SECTION C

Q13. $\kappa=-1$ **Q14.** See Mathematicia by O.P. Gupta

Q15. $\tan^{-1}(x-1)+\tan^{-1}(x+1)=\tan^{-1}3x-\tan^{-1}x$

$\Rightarrow \tan^{-1}\frac{(x-1)+(x+1)}{1-(x-1)(x+1)}=\tan^{-1}\frac{3x-x}{1+(3x)x} \Rightarrow x=0,\pm\frac{1}{2}$

OR $\sin^{-1}x+\cos^{-1}x+\sin^{-1}(1-x)=2\cos^{-1}x$ $\Rightarrow \frac{\pi}{2}+\sin^{-1}(1-x)=2\cos^{-1}x$

$\Rightarrow \cos\left(\frac{\pi}{2}+\sin^{-1}(1-x)\right)=\cos\left(2\cos^{-1}x\right)$ $\Rightarrow -\sin\sin^{-1}(1-x)=2\left(\cos\cos^{-1}x\right)^2-1$

$\Rightarrow -(1-x)=2x^2-1 \quad \Rightarrow x=0, 1/2$.

Q16. Ans. $\frac{n}{1-n^2}$. See Mathematicia by O.P. Gupta

Q17. See NCERT Miscellaneous Exercise Chapter 03 Q10.
Ans. (a) 46000, 53000 (in ₹) (b) 15000, 17000 (in ₹).

Q18. Eq. of hyperbola : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ $\therefore \frac{dy}{dx}=\frac{b^2}{a^2}\times\frac{x}{y}$ $\Rightarrow \left.\frac{dy}{dx}\right]_{at\,(\sqrt{2}a,\,b)}=\frac{b^2}{a^2}\times\frac{\sqrt{2}a}{b}=\frac{b\sqrt{2}}{a}=m_T$

Also, $m_N=-\frac{a}{b\sqrt{2}}$.

Then, eq. of tangent : $y-b=\frac{b\sqrt{2}}{a}(x-\sqrt{2}a) \Rightarrow \frac{y-b}{b\sqrt{2}}=\frac{x-\sqrt{2}a}{a}$ $\Rightarrow b\sqrt{2}x-ay=ab$

And, eq. of normal : $y-b=-\frac{a}{b\sqrt{2}}(x-\sqrt{2}a)$ $\Rightarrow ax+b\sqrt{2}y+\sqrt{2}(a^2+b^2)=0$.

Q19. $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$ $\Rightarrow y\sqrt{1-x^2}=\sin^{-1}x$. Now differentiate. **OR** $f'(x)=\frac{d}{dx}\left(\frac{x}{2}\right)=\frac{1}{2}$.

Q20. 7/15. (Total Probability)

Q21. LHS : $\vec{a}.(\vec{b}+\vec{c})\times(\vec{a}+2\vec{b}+3\vec{c})=\vec{a}.\left\{\vec{b}\times\vec{a}+\vec{b}\times(2\vec{b})+\vec{b}\times(3\vec{c})+\vec{c}\times\vec{a}+\vec{c}\times(2\vec{b})+\vec{c}\times(3\vec{c})\right\}$

$\Rightarrow \quad =\vec{a}.\left\{\vec{b}\times\vec{a}+\vec{0}+3\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+2\vec{c}\times\vec{b}+\vec{0}\right\}$

$\Rightarrow \quad =\vec{a}.\vec{b}\times\vec{a}+\vec{a}.3\vec{b}\times\vec{c}+\vec{a}.\vec{c}\times\vec{a}+\vec{a}.2\vec{c}\times\vec{b}$

$\Rightarrow \quad =[\vec{a}\ \vec{b}\ \vec{a}]+3[\vec{a}\ \vec{b}\ \vec{c}]+[\vec{a}\ \vec{c}\ \vec{a}]+2[\vec{a}\ \vec{c}\ \vec{b}]$

$\Rightarrow \quad =0+3[\vec{a}\ \vec{b}\ \vec{c}]+0-2[\vec{a}\ \vec{b}\ \vec{c}]=[\vec{a}\ \vec{b}\ \vec{c}]=\vec{a}.(\vec{b}\times\vec{c})=RHS$.

Q23. **(i)** $\left(\frac{13}{52}\right)^4=\frac{1}{256}$ **(ii)** ${}^4C_2\left(\frac{13}{52}\times\frac{13}{52}\times\frac{39}{52}\times\frac{39}{52}\right)=\frac{54}{256}$.

OR Mean $=\frac{864}{1296}$

SECTION D

Q24. (1,3,2) **OR** $\vec{r}.(12\hat{i}+27\hat{j}+12\hat{k})=52$, $12x+27y+12z=52$.

Q25. Let x and y be the no. of items M and N respectively.
To maximize : $Z=600x+400y$ in ₹
Subject to constraints : $x, y\ge 0;\ x+2y\le 12,\ 2x+y\le 12,\ 4x+5y\ge 20$.
Maximum profit ₹ 4000 at (4, 4).

Q27. (i) $y=\frac{1}{x^2}, x\ne 0$ (ii) Yes (iii) For $y=\frac{1}{4}$, we have $\frac{1}{4}=\frac{1}{x^2}$ $\Rightarrow x=2$ (iv) $\Delta y=\frac{1}{4}-\frac{1}{16}=\frac{3}{16}$

Q28. See Mathematicia *by* O.P. Gupta **OR** See Mathematicia *by* O.P. Gupta.

Q29. Let $I=\int \tan^{2/3}x\,dx$

Put $\tan x=y^3$ $\Rightarrow \sec^2 x dx=3y^2dy$ $\Rightarrow dx=\frac{3y^2dy}{1+y^6}$ $\therefore I=3\int\frac{y^4dy}{1+y^6}$

Consider $\frac{y^4}{1+y^6}=\frac{m^2}{1+m^3}=\frac{m^2}{(1+m)(1-m+m^2)}=\frac{A}{1+m}+\frac{B(2m-1)}{1-m+m^2}+\frac{C}{1-m+m^2}$, where $m=y^2$

$\Rightarrow m^2=A(1-m+m^2)+B(2m-1)(1+m)+C(1+m)$ $\therefore A=\frac{1}{3},\ B=\frac{1}{3},\ C=0$

Now $I=3\int\left(\frac{1}{3}\times\frac{1}{1+y^2}+\frac{1}{3}\times\frac{2y^2-1}{1-y^2+y^4}+\frac{0}{1-y^2+y^4}\right)dy$ $\Rightarrow I=\int\frac{1}{1+y^2}dy+\int\frac{2y^2-1}{1-y^2+y^4}dy$

$\Rightarrow I=\tan^{-1}y+I_1$...(i)

Now $I_1=\int\frac{2y^2-1}{1-y^2+y^4}dy=\int\frac{y^2}{1-y^2+y^4}dy+\int\frac{y^2-1}{1-y^2+y^4}dy$

$$\Rightarrow I_1=\int\frac{1}{\frac{1}{y^2}-1+y^2}dy+\int\frac{1-\frac{1}{y^2}}{\frac{1}{y^2}-1+y^2}dy \qquad \Rightarrow I_1=\frac{1}{2}\int\frac{\left(1+\frac{1}{y^2}\right)+\left(1-\frac{1}{y^2}\right)}{\frac{1}{y^2}-1+y^2}dy+\int\frac{1-\frac{1}{y^2}}{\frac{1}{y^2}-1+y^2}dy$$

$$\Rightarrow I_1=\frac{1}{2}\int\frac{\left(1+\frac{1}{y^2}\right)}{\frac{1}{y^2}-1+y^2}dy+\frac{1}{2}\int\frac{\left(1-\frac{1}{y^2}\right)}{\frac{1}{y^2}-1+y^2}dy+\int\frac{1-\frac{1}{y^2}}{\frac{1}{y^2}-1+y^2}dy$$

$$\Rightarrow I_1=\frac{1}{2}\int\frac{1+\frac{1}{y^2}}{\frac{1}{y^2}-1+y^2}dy+\frac{3}{2}\int\frac{1-\frac{1}{y^2}}{\frac{1}{y^2}-1+y^2}dy$$

$$\Rightarrow I_1=\frac{1}{2}\int\frac{1+\frac{1}{y^2}}{\left(y-\frac{1}{y}\right)^2+1^2}dy+\frac{3}{2}\int\frac{1-\frac{1}{y^2}}{\left(y+\frac{1}{y}\right)^2-(\sqrt{3})^2}dy$$

Put $y-\frac{1}{y}=p\Rightarrow\left(1+\frac{1}{y^2}\right)dy=dp$ in the first integral

and, $y+\frac{1}{y}=q\Rightarrow\left(1-\frac{1}{y^2}\right)dy=dq$ in the second integral

$$\therefore I_1=\frac{1}{2}\int\frac{dp}{p^2+1^2}+\frac{3}{2}\int\frac{dq}{q^2-(\sqrt{3})^2} \qquad \Rightarrow I_1=\frac{1}{2}\tan^{-1}p+\frac{3}{2}\times\frac{1}{2\sqrt{3}}\log\left|\frac{q-\sqrt{3}}{q+\sqrt{3}}\right|+C$$

$$\Rightarrow I_1=\frac{1}{2}\tan^{-1}\left(y-\frac{1}{y}\right)+\frac{\sqrt{3}}{4}\log\left|\frac{y^2-\sqrt{3}y+1}{y^2+\sqrt{3}y+1}\right|+C$$

By (i), $I=\tan^{-1}y+\frac{1}{2}\tan^{-1}\left(y-\frac{1}{y}\right)+\frac{\sqrt{3}}{4}\log\left|\frac{y^2-\sqrt{3}y+1}{y^2+\sqrt{3}y+1}\right|+C$

$$\therefore I=\tan^{-1}(\tan^{1/3}x)+\frac{1}{2}\tan^{-1}\left(\tan^{1/3}x-\cot^{1/3}x\right)+\frac{\sqrt{3}}{4}\log\left|\frac{\tan^{2/3}x-\sqrt{3}\tan^{1/3}x+1}{\tan^{2/3}x+\sqrt{3}\tan^{1/3}x+1}\right|+C.$$

OR See Mathematicia *by* O.P. Gupta.

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