Series : PTS/17

Code No. 18/12/17

Roll No. | 2 | 1 | 8 | 1 | 2 | | |

Candidates **must write the Code on the title page of the answer-book.**

PLEASURE TEST SERIES XII – 17

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Time Allowed : 180 Minutes **Max. Marks : 100**

SECTION A

Q01. If $\cos^{-1} : [-1,1] \to [-2\pi, -\pi]$ is a function, then write the value of $\cos^{-1}(-0.5)$.

Q02. Find the distance of (2, 3, 4) from y-axis.

Q03. Write the integral of $\left[4-\frac{1}{4}\right]+\left[-\frac{1}{4}\right]+[3]$ w.r.t. x between the limits $x = 0$ to $x = 2$. Here [p] means greatest integer less than or equal to p.

Q04. What is the value of $|\hat{i}-\hat{j}|^2$? **OR** Find a vector parallel to $\vec{a} = \hat{i} - 2\hat{j}$, having magnitude of 7.

SECTION B

Q05. If a line makes angles α, β, γ with the positive direction of coordinate axes, then write the integral value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ w.r.t. x. **OR** Evaluate : $\int_0^{\pi/6} \sin^4 x \cos^3 x \, dx$.

Q06. Solve for x : $\begin{bmatrix}2\\1\end{bmatrix} + x\begin{bmatrix}3\\5\end{bmatrix} + \begin{bmatrix}-8\\-11\end{bmatrix} = O$. **OR** If $[2x \;\; 3]\begin{bmatrix}1 & 2\\-3 & 0\end{bmatrix}\begin{bmatrix}x\\3\end{bmatrix} = O$, find x, $x \in Z$.

Q07. Form the differential equation satisfied by the equation $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, where 'a' is any arbitrary constant.

Q08. Find the range of $y = \cos^{-1} x + \sin^{-1} x + \tan^{-1} x$.

Q09. Vectors $\vec{a}$ and $\vec{b}$ are inclined at angle of $\theta = 120^\circ$. Also it is known that $|\vec{a}| = 1$ and $|\vec{b}| = 2$ then, what is the value of $[(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})]^2$?

Q10. Differentiate $\sin x^2$ w.r.t. $\cos 2x$.

Q11. If A is a 3rd order matrix & $|A| = 5$, find $|2AA^T|$.

Q12. If the radius of a circle is increasing at the rate of 0.7 cm/s then, find the rate of increase of its circumference? **OR** For what values of a, the curves $y = x^2 + ax + 25$ touches the axis of x?

SECTION C

Q13. Let $A = \begin{bmatrix}1 & -2 & 1\\-2 & 3 & 1\\1 & 1 & 5\end{bmatrix}$. Verify that A (adj. A) = |A| I.

Q14. It is given that for the function $f(x) = x^3 + bx^2 + ax + 5$ on [1, 3], Rolle's theorem holds with $c = 2 + 3^{-1/2}$. Find the values of a & b.

Q15. Integrate $(2\sin 2x - \cos x)\sqrt{6 - \cos^2 x - 4\sin x}$.

Q16. Find the values of p and q so that $f(x) = \begin{cases} x^2 + 3x + p, & \text{if } x \le 1 \\ qx + 2, & \text{if } x > 1 \end{cases}$ is differentiable at x = 1.

Q17. Consider an isosceles triangle ABC inscribed in a circle of radius r. If the vertical angle of ΔABC is 2θ, show that the maximum area of triangle is obtained when $\pi = 6\theta$.

Q18. Using properties to evaluate the integral $[|x-1| + |x-2| + |x-4|]$ w.r.t. x, $x \in [1,4]$.

Q19. Using vectors, prove that the parallelogram on the same base and between the same parallels are equal in area. **OR** Using vectors, prove that sin (A + B) = sin A cos B + sin B cos A.

Q20. Solve : $(1 + \tan y)(dx - dy) + 2xdy = 0$.

OR Solve : $[xe^{y/x} - y\sin(y/x)]dx + x\sin(y/x)dy = 0$, if $y = 0$, when $x = 1$.

Q21. Determine the distance of the point (2, 4,–1) from the line $\vec{r} = -5\hat{i} - 3\hat{j} + 6\hat{k} + \lambda(\hat{i} + 4\hat{j} - 9\hat{k})$.

Q22. Fifteen cards numbered from one to fifteen are placed in a box and a card is drawn at random from the box. If it is known that the number on the drawn card is more than 3 then, find the probability that it is an even number. If 50 times this probability be the number of students who suffered in a fire accident in a school building then, find the number of those students.

OR A die is thrown again and again until the number 6 is obtained three times. Find the probability that the third six comes in the seventh toss.

Q23. 40% students of a college reside in hostel and the remaining reside outside. At the end of year, 50% of the hosteliers got A grade while from outside students, only 30% got A grade in the examination. At the end of year, a student of the college was chosen at random and was found to get A grade. What is the probability that the selected student was a hostelier?

SECTION D

Q24. (a) If f (x) = sin x, g(x) = cos x and h(x) = 2x; f, g, h are real valued functions, check if ho(fog) and foh are equal or not.

(b) Let * be a binary operation defined on the set of integers as a * b = a + b – 1 for a, b $\in$ Z. Show that 1 is an identity element. Also, find the value of x such that 2 * (x * 3) = 10.

Q25. Using properties, prove that : $\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{a^2+c^2}{b} \end{vmatrix} = 4abc$.

OR

Two farmers Ramkrishna and Hari Prasad cultivated three varieties of rice namely Basmati, Permal and Naura. The sale (in Rupees) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices 'A' and 'B' :

	September Sales (in Rupees)				October Sales (in Rupees)			
	Basmati	Permal	Naura		Basmati	Permal	Naura	
A =	10000	20000	30000	Ramkrishna	5000	10000	6000	Ramkrishna
	50000	30000	10000	Hari Prasad	20000	10000	10000	Hari Prasad

and B =

(i) Find the combined sale in September and October for each farmer in each variety.

(ii) Find the decrease in sales from September to October.

(iii) If both farmers receive 2% profit on gross sales, compute the profit for each farmer and for each variety sold in October.

(iv) Which farmer gets more profit in the overall sales for both the months?

Q26. Find the area of the region $\{(x, y) : x^2 + y^2 \le 1 \le x + y\}$.

Q27. Evaluate : $\int_{-\pi/4}^{\pi/4} \log(\sin x + \cos x)\,dx$. **OR** Evaluate : $\int \frac{x^4}{x^4 + 81}\,dx$.

Q28. Avinash has been given two lists of problems from his mathematics teacher with the instructions to submit not more than 100 of them correctly solved for marks. The problems in the first list are worth 10 marks each and those in the second list are worth 5 marks each. He knows from past experience that he requires on an average of 4 minutes to solve a problem of 10 marks and 2 minutes to solve a problem of 5 marks. He has other subjects to worry about; he cannot devote more than 4 hours to his mathematics assignment. With reference to manage his time in best possible way, how many problems from each list shall he do to maximize his marks?

Q29. Find the distance of point P(3, 4, 5) from the plane x + y + z = 2 measured parallel to the line whose equation is given as $2x = y = z$.

OR Find the position vector of the image of $2\hat{i} + 3\hat{j} + 4\hat{k}$ in the plane $\vec{r}.(2\hat{i} + \hat{j} + 3\hat{k}) = 26$. ◘

ANSWERS & HINTS For PTS-17

SECTION A

Q01. $-4\pi/3$ **Q02.** $\sqrt{20}$ units.

Q03. Note that $\left[4-\frac{1}{4}\right]+\left[-\frac{1}{4}\right]+[3]=\left[\frac{15}{4}\right]+[-0.25]+3=3-1+3=5$ so clearly, $\int_0^2 5dx=[5x]_0^2=10$.

Q04. 2. **OR** $\frac{7}{\sqrt{5}}(\hat{i}-2\hat{j})$.

SECTION B

Q05. $2x+C$. **OR** $\frac{23}{4480}$.

Q06. We have $\begin{bmatrix}3x-6\\5x-10\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$ $\Rightarrow 3x-6=0$ and $5x-10=0$ *i.e.*, $x=2$. **OR** $x=0$.

Q07. Put $x=\sin\theta, y=\sin\beta \Rightarrow \frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

Q08. Here domain of $\cos^{-1}x$ and $\sin^{-1}x$ is $[-1, 1]$. Also domain of $\tan^{-1}x$ is R.
Hence the domain of function y is $x\in[-1, 1]$.
Therefore, at $x=-1$, $y=\pi+(-\pi/2)+(-\pi/4)=\pi/4$ and at $x=1$, $y=0+\pi/2+\pi/4=3\pi/4$.
$\therefore$ Range of y is $[\pi/4, 3\pi/4]$. **Q09.** 300.

Q10. Let $y=\sin x^2, z=\cos 2x \Rightarrow \frac{dy}{dx}=2x\cos x^2, \frac{dz}{dx}=-2\sin 2x$ $\therefore \frac{dy}{dz}=\frac{dy}{dx}\times\frac{dx}{dz}=-\frac{x\cos x^2}{\sin 2x}$.

Q11. $|2AA^T|=2^3|AA^T|=8|A||A^T|=8\times5\times5=200$. $\{\because |A|=|A^T|$

Q11. If A is a 3rd order matrix & $|A|=5$, find $|2AA^T|$.

Q12. 4.4cm/s. **OR** $a=\pm10$.

SECTION C

Q14. Given $f(x)=x^3+bx^2+ax+5$ on $[1, 3]$

$\Rightarrow f'(x)=3x^2+2bx+a \Rightarrow f'(c)=3c^2+2bc+a=0 \Rightarrow 3\left(2+\frac{1}{\sqrt{3}}\right)^2+2b\left(2+\frac{1}{\sqrt{3}}\right)+a=0\ldots(i)$

Also $f(1)=f(3)\Rightarrow b+a+6=32+9b+3a$ or, $a+4b=-13\ldots(ii)$

Solving (i) and (ii), we get : $a=11, b=-6$.

Q15. $\frac{4}{3}[\sin^2x-4\sin x+5]^{\frac{3}{2}}+\frac{7}{2}\left[(\sin x-2)\sqrt{\sin^2x-4\sin x+5}+\log|\sin x-2+\sqrt{\sin^2x-4\sin x+5}|\right]+C$

Q16. See solution of Q79 Ex 5.3 **NCERT Exemplar Problems Solutions by O.P. Gupta**

Q17. **See NECRT Exemplar Chapter 06 Example 18**

Q18. 23/2.

Q19. **See NCERT Exemplar Solutions by O.P. Gupta**

Q20. **See NECRT Exemplar Problems Solutions by O.P. Gupta** Ex9.3 Q26

OR Given differential equation is homogeneous.

$\therefore y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

So, $\frac{dy}{dx}=\frac{y\sin\left(\frac{y}{x}\right)-xe^{y/x}}{x\sin\left(\frac{y}{x}\right)}$ $\Rightarrow v+x\frac{dv}{dx}=\frac{vx\sin\left(\frac{vx}{x}\right)-xe^{vx/x}}{x\sin\left(\frac{vx}{x}\right)}=\frac{v\sin v-e^v}{\sin v}$

$\Rightarrow v + x\frac{dv}{dx} = v - \frac{e^v}{\sin v}$ or $x\frac{dv}{dx} = -\frac{e^v}{\sin v}$

$\therefore \int e^{-v}\sin v\,dv = -\int \frac{dx}{x} \Rightarrow I_1 = -\log x + C_1 ...(i)$

Now $I_1 = \int e^{-v}\sin v\,dv = \sin v\int e^{-v}dv + \int e^{-v}\cos v\,dv$

$\Rightarrow \quad = -e^{-v}\sin v - e^{-v}\cos v - \int e^{-v}\sin v\,dv$

$\Rightarrow I_1 = -\frac{1}{2}e^{-v}(\sin v + \cos v) + C_2$

Putting value of I_1 in (i), $-\frac{1}{2}e^{-v}(\sin v + \cos v) = -\log x + C_1 + C_2$

$e^{-y/x}\left(\sin\frac{y}{x} + \cos\frac{y}{x}\right) = \log x^2 + C$, where $C = -2C_1 - 2C_2$

As it is given that $y = 0,$ when $x = 1$, so $C = 1$

$\therefore$ the solution is $e^{-y/x}\left(\sin\frac{y}{x} + \cos\frac{y}{x}\right) = \log x^2 + 1$.

Q21. Foot of $\perp^{er}$: (–4, 1,–3) and distance : 7 units.

Q22. 1/2, 25.

OR If 3rd six comes in 7th throw then, the earlier six throws should have resulted in two 6.

So $n = 6,\ p = \frac{1}{6}$ and $q = \frac{5}{6}$.

$\therefore P(2) = {}^6C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{6-2} = \frac{6\times5}{2}\times\frac{5^4}{6^6}$.

Hence, P(3 sixes in 7 throws) $= P(2)\times\left(\frac{1}{6}\right) = \frac{3125}{2\times6^7}$.

Q23. Let E_1: Student resides in the hostel, E_2: Student resides outside the hostel, A: Getting A grade in the examination.

$P(E_1) = 40/100, P(E_2) = 3/5, P(A|E_1) = 50/100, P(A|E_2) = 30/100$.

By Bayes' theorem, $P(E_1|A) = \frac{P(A|E_1)P(E_1)}{P(A|E_1)P(E_1) + P(A|E_2)P(E_2)}$

$$\therefore P(E_1|A) = \frac{\frac{2}{5}\times\frac{1}{2}}{\frac{2}{5}\times\frac{1}{2} + \frac{3}{5}\times\frac{3}{10}} = \frac{10}{19}.$$

SECTION D

Q24. (a) Not equal (b) $x = 7$.

Q25. **OR**

(i)

$$A + B = \begin{matrix}\text{Basmati} & \text{Permal} & \text{Naura}\end{matrix}\begin{pmatrix}15000 & 30000 & 36000\\ 70000 & 40000 & 20000\end{pmatrix}\begin{matrix}\text{Ramkrishna}\\ \text{Hari Prasad}\end{matrix}$$

(ii)

$$A - B = \begin{matrix}\text{Basmati} & \text{Permal} & \text{Naura}\end{matrix}\begin{pmatrix}5000 & 10000 & 24000\\ 30000 & 20000 & 0\end{pmatrix}\begin{matrix}\text{Ramkrishna}\\ \text{Hari Prasad}\end{matrix}$$

(iii) 2% of B $= 0.02\times B$

$$\begin{matrix} & \text{Basmati} & \text{Permal} & \text{Naura} \\ = \begin{pmatrix} 100 & 200 & 120 \\ 400 & 200 & 200 \end{pmatrix} & \begin{matrix} \text{Ramkrishna} \\ \text{Hari Prasad} \end{matrix} \end{matrix}$$

Hence in October Ramkrishna receives ₹100, ₹200 & ₹120 as profit in the sale of each variety of rice respectively. Also Hari Prasad receives profit of ₹400, ₹200 & ₹200 in the sale of each variety of rice, respectively.

(iv) Hari Prasad gets more profit on sales of both varieties of rice for both the months.

Q26. $\left(\frac{\pi}{4}-\frac{1}{2}\right)$ sq. units.

Q27. See **NCERT Exemplar Problems Solutions by O.P. Gupta**.

OR $x-\frac{3}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-9}{3\sqrt{2}x}\right)-\frac{3}{4\sqrt{2}}\log\left|\frac{x^2+3\sqrt{2}x+9}{x^2-3\sqrt{2}x+9}\right|+C$.

Q28. To maximize : $Z=10x+5y$

Subject to constraints : $x\geq 0,\ y\geq 0,\ x+y\leq 100,\ 4x+2y\leq 240$ where x and y represent the no. of questions solved by Avinash from 1st and 2nd list respectively.

Maximum marks of 600 is obtained at all the points on the line segment joining the points (60, 0) and (20, 80).

Q29. 6 units. **OR** $4\hat{i}+4\hat{j}+7\hat{k}$.