Series : PTS/15

Code No. 18/11/15

Roll No. | 2 | 1 | 8 | 1 | 2 | | |

Candidates must write the Code on the title page of the answer-book.

PLEASURE TEST SERIES XII – 15

A Compilation By : O.P. Gupta (WhatsApp @ +91 9650350480)

Time Allowed : 180 Minutes **Max. Marks : 100**

SECTION A

Q01. For the set A = {1, 2, 3}, define a relation R in the set A as : R = {(1, 1), (2, 2), (3, 3), (1, 3)}. Write the ordered pairs to be added to R to make it the smallest equivalence relation.

Q02. For what value of p, the matrix $\begin{pmatrix} 2p+3 & 4 & 5 \\ -4 & 4p+6 & -6 \\ -5 & 6 & -2p-3 \end{pmatrix}$ is a skew symmetric matrix?

OR A and B are square matrices of order 3 each, $|A| = 2$ and $|B| = 3$. Find $|3AB|$.

Q03. What is the distance of the point (p, q, r) from the x-axis?

Q04. Find the maximum and minimum values (if any) of $f(x) = -|x+1| + 3$ on R.

SECTION B

Q05. Let $f : R \to R$ and $g : R \to R$ be defined by $f(x) = 3x^2 - 5$ and $g(x) = \dfrac{x}{x^2+1}$. Find gof.

OR How many equivalence relations on the set {1, 2, 3} containing (1, 2) and (2, 1) are there in all? Justify your answer.

Q06. Write the differential equation of the family of circles with fixed radius 5 units and centre on the straight line $y = 2$.

Q07. A fair coin is tossed 10 times. What is the probability of getting head as many times in the first six throws as in the last four?

Q08. Solve : $\tan^{-1}\left(\dfrac{2x}{1-x^2}\right) + \cot^{-1}\left(\dfrac{1-x^2}{2x}\right) = \dfrac{\pi}{3}$, $x > 0$.

OR Solve : $\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$.

Q09. Let l_i, m_i, n_i ; $i = 1, 2, 3$ denote the direction cosines of three mutually perpendicular vectors in the space. Show that $AA^T = I_3$ where $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$.

Q10. Find $f'(x)$ if, $f(x) = |x|, x \neq 0$. **OR** If $e^y(x+1) = 1$, show that $dy/dx = -e^y$.

Q11. Find the Cartesian and vector equations of the line which passes through the point (–2, 4, –5) and parallel to the line given by $10(x+3) = 6(y-4) = 5(z-8)$.

Q12. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which its area increases, when side is 10 cm long.

SECTION C

Q13. Find the intervals of increasing & decreasing for the function $2x^2 - \log|x|$.

Q14. Solve : $\operatorname{cosec} x \log y \, dy + x^2y^2 \, dx = 0$.

OR If y(t) is a solution of $(1+t)\dfrac{dy}{dt} - ty = 1$ and $y(0) = -1$ then show that $y(1) = -1/2$.

Q15. Prove that : $\cos^{-1}\left(\dfrac{\cos\alpha+\cos\beta}{1+\cos\alpha\cos\beta}\right)=2\tan^{-1}\left(\tan\dfrac{\alpha}{2}\tan\dfrac{\beta}{2}\right)$.

Q16. Find $\displaystyle\int\frac{x^2}{x^4-3x^2+16}dx$. **OR** Evaluate $\displaystyle\int_1^2(4-e^x+x^2)\,dx$ as the limits of sums.

Q17. If $A=\begin{bmatrix}1&2&0\\-2&-1&-2\\0&-1&1\end{bmatrix}$, find A^{-1}. Using A^{-1}, solve : $x-2y=10,\ 2x-y-z=8,\ -2y+z=7$.

Q18. Of all the closed right circular cylindrical cans of volume $128\pi\ \text{cm}^3$, find the dimension of the can which has minimum surface area.

Q19. If $y=\log(x^{\sin 3x}+\cot^2 2x)$ then, find dy/dx.

OR If $y=x^x$, show that : $y_2-y^{-1}(y_1)^2-x^{-1}y=0$.

Q20. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3/5 be the probability that he knows the answer and 2/5 be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability 1/3, what is the probability that the student knows the answer given that he answered it correctly?

Q21. Using vectors, prove that in any $\Delta ABC,\ 2bc\cos A=b^2+c^2-a^2$ where a, b, c are the magnitudes of the sides opposite to the vertices A, B, C, respectively.

Q22. Find the equations of the perpendicular drawn from the point (2, 4, –1) to the line $x+5=\dfrac{1}{4}(y+3)=-\dfrac{1}{9}(z-6)$ and hence obtain the coordinates of the foot of this perpendicular.

Q23. Two biased dice are thrown together. For the first die P(6) = 1/2, the other scores being equally likely while for the second die, P(1) = 2/5 and the other scores are equally likely. Find the probability distribution of 'the number of ones seen'.

SECTION D

Q24. Find the coordinates of the point where the line $4x-8=3y+3=6z-12$ and plane $x-y+z=5$ intersect. Also find the angle between the line and the plane.

OR $\overrightarrow{AB}=3\hat{i}-\hat{j}+\hat{k}$ and $\overrightarrow{CD}=-3\hat{i}+2\hat{j}+4\hat{k}$ are two vectors. Position vectors of the points A and C are $6\hat{i}+7\hat{j}+4\hat{k}$ and $-9\hat{j}+2\hat{k}$, respectively. Find the position vector of a point P on the line AB and a point Q on the line CD such that $\overrightarrow{PQ}$ is perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{CD}$ both.

Q25. A dietician wants to develop a special diet using two foods X and Y. Each packet (contains 30 g) of food X contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Y contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. Make an LPP to find how many packets of each food should be used to minimise the amount of vitamin A in the diet, and solve it graphically.

Q26. Using properties, find the value of $\Delta=\begin{vmatrix}(x-2)^2&(x-1)^2&x^2\\(x-1)^2&x^2&(x+1)^2\\x^2&(x+1)^2&(x+2)^2\end{vmatrix}$.

Q27. Let A and B be two sets. Show that $f:A\times B\to B\times A$ such that $f(a,b)=(b,a)$ is a bijective function.

Q28. Find the area bounded by $x^2+y^2=25,\ 4y=|4-x^2|,\ x=0$ which is lying above the x-axis.

OR Find the area of the region for $\{(x,y):x^2+y^2\le 2ax,\ y^2\ge ax,\ x\ge 0, y\ge 0\}$.

Q29. Evaluate : $\displaystyle\int_0^{\pi}\frac{x}{1+\sin x}dx$. **OR** Evaluate : $\displaystyle\int\sin\sqrt[3]{x}\,dx$. ◘

ANSWERS & HINTS For PTS-15

SECTION A

Q01. (3, 1).

Q02. For skew-symmetric matrix, we have $a_{ii} = 0 \ \therefore p = -3/2$.

OR $|3AB| = 3^3 |A||B| = 27 \times 2 \times 3 = 162$.

Q03. Distance of the point (p, q, r) from the x-axis is same as the distance of (p, q, r) from (p, 0, 0). So, the required distance is $\sqrt{0^2 + q^2 + r^2}$ i.e., $\sqrt{q^2 + r^2}$.

Q04. Maximum value = 3 and minimum value isn't defined.

SECTION B

Q05. $gof(x) = g(f(x)) = g(3x^2 - 5) = \dfrac{3x^2 - 5}{(3x^2 - 5)^2 + 1} = \dfrac{3x^2 - 5}{9x^4 - 30x^2 + 26}$.

OR Following equivalence relations can be possible in the given conditions these are, $\{(1,1),(2,2),(3,3),(1,2),(2,1)\}$ and $\{(1,1),(2,2),(3,3),(1,2),(1,3),(2,1),(2,3),(3,1),(3,2)\}$. Clearly, only two equivalence relations are there.

Q06. $(y-2)^2 y_1^2 = 25 - (y-2)^2$.

Q07. Let E : getting head as many times in the first six throws as in the last four.

$$\therefore P(E) = {}^6C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^6 \times {}^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4 + {}^6C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^5 \times {}^4C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3$$

$$+ {}^6C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^4 \times {}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 + {}^6C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3 \times {}^4C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1$$

$$+ {}^6C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^2 \times {}^4C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0 \qquad \Rightarrow P(E) = \frac{210}{1024}.$$

Q08. $\tan^{-1}\left(\dfrac{2x}{1-x^2}\right) + \cot^{-1}\left(\dfrac{1-x^2}{2x}\right) = \dfrac{\pi}{3} \qquad \Rightarrow 2\tan^{-1}x + \tan^{-1}\left(\dfrac{2x}{1-x^2}\right) = \dfrac{\pi}{3}$

$\Rightarrow 2\tan^{-1}x + 2\tan^{-1}x = \dfrac{\pi}{3} \qquad \Rightarrow \tan^{-1}x = \dfrac{\pi}{12} \qquad \therefore x = \tan\dfrac{\pi}{12} = 2 - \sqrt{3}$.

OR $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x \ \Rightarrow 2\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x + \sin^{-1}x$

$\Rightarrow \sin^{-1}(1-x) = \dfrac{\pi}{2} - 2\sin^{-1}x \ \Rightarrow \sin\sin^{-1}(1-x) = \sin\left(\dfrac{\pi}{2} - 2\sin^{-1}x\right) \quad \Rightarrow 1 - x = \cos(2\sin^{-1}x)$

$\Rightarrow 1 - x = 1 - 2(\sin\sin^{-1}x)^2 \quad \Rightarrow x = 2x^2 \quad \Rightarrow x(1-2x) = 0 \qquad \therefore x = 0, 1/2$.

Q09. Given $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$

$\Rightarrow A^T = \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix}$

Now $AA^T = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}\begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix}$

$$\Rightarrow AA^T = \begin{bmatrix} l_1^2+m_1^2+n_1^2 & l_1l_2+m_1m_2+n_1n_2 & l_1l_3+m_1m_3+n_1n_3 \\ l_2l_1+m_2m_1+n_2n_1 & l_2^2+m_2^2+n_2^2 & l_2l_3+m_2m_3+n_2n_3 \\ l_3l_1+m_3m_1+n_3n_1 & l_3l_2+m_3m_2+n_3n_2 & l_3^2+m_3^2+n_3^2 \end{bmatrix}$$

Since lines are perpendicular so, $AA^T = \begin{bmatrix} l_1^2+m_1^2+n_1^2 & 0 & 0 \\ 0 & l_2^2+m_2^2+n_2^2 & 0 \\ 0 & 0 & l_3^2+m_3^2+n_3^2 \end{bmatrix}$

$\Rightarrow AA^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$. Hence Proved.

Q10. $f'(x) = \begin{cases} 1, \text{ if } x > 0 \\ -1, \text{ if } x < 0 \end{cases}$

OR Here $e^y(x+1) = 1$ $\Rightarrow e^y + (x+1)e^y \dfrac{dy}{dx} = 0$

$\Rightarrow e^y + 1 \times \dfrac{dy}{dx} = 0$ $\therefore \dfrac{dy}{dx} = -e^y$.

Q11. The d.r.'s of the line $\dfrac{x+3}{3} = \dfrac{y-4}{5} = \dfrac{8-z}{-6}$ are 3, 5, 6.

Therefore the Cartesian eq. of required line : $\dfrac{x+2}{3} = \dfrac{y-4}{5} = \dfrac{z+5}{6}$.

Also the vector eq. : $\vec{r} = -2\hat{i} + 4\hat{j} - 5\hat{k} + \lambda(3\hat{i} + 5\hat{j} + 6\hat{k})$.

Q12. Here, area $A = \dfrac{\sqrt{3}}{4} x^2$, where x is side of the equilateral triangle.

So, $\dfrac{dA}{dt} = \dfrac{\sqrt{3}}{4} \times 2x \dfrac{dx}{dt} = \dfrac{\sqrt{3}\,x}{2} \times \dfrac{dx}{dt}$ $\therefore \left.\dfrac{dA}{dt}\right]_{\text{at } x=10\text{cm}} = \dfrac{\sqrt{3} \times 10}{2} \times 2 = 10\sqrt{3} \text{ cm}^2/\text{sec}$.

SECTION C

Q13. Increasing : $\left[-\dfrac{1}{2}, 0\right) \cup \left[\dfrac{1}{2}, \infty\right)$; Decreasing : $\left(-\infty, -\dfrac{1}{2}\right] \cup \left(0, \dfrac{1}{2}\right]$.

Q14. $1 + \log y = y\left(2\cos x + 2x\sin x - x^2 \cos x\right) + C\,y$

OR See O.P. Gupta's Exemplar Solutions.

Q15. See Mathematicia by O.P. Gupta.

Q16. See Mathematicia by O.P. Gupta **OR** $\dfrac{19}{3} + e - e^2$.

Q17. $A^{-1} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$; $x = 0,\ y = -5,\ z = -3$.

Q18. Radius and height are respectively 4 and 8 (both in cm)

Q19. $\dfrac{x^{\sin 3x}\left(\dfrac{\sin 3x}{x} + 3\log x \cos 3x\right) - 4\cot 2x \operatorname{cosec}^2 2x}{x^{\sin 3x} + \cot^2 2x}$.

OR See Mathematicia by O.P. Gupta (Example).

Q20. 9/11.

Q21. See O.P. Gupta's Exemplar Solutions

Q22. (–4, 1, –3), $\frac{x-2}{6}=\frac{y-4}{3}=\frac{z+1}{2}$.

Q23. See Exemplar Solutions by O.P. Gupta.

Ans. Following table represents the Probability distribution :

X	0	1	2
P(X)	0.54	0.42	0.04

SECTION D

Q24. (2, –1, 2), $\sin^{-1}\frac{1}{\sqrt{87}}$ **OR** $\overrightarrow{OP}=3\hat{i}+8\hat{j}+3\hat{k}$ and, $\overrightarrow{OQ}=-3\hat{i}-7\hat{j}+6\hat{k}$.

Q25. 150 units at (15, 20).

Q26. $\Delta=-8$ (See O.P. Gupta's Mathematicia)

Q27. See O.P. Gupta's Mathematicia

Q28. $\frac{25}{2}\sin^{-1}\left(\frac{4}{5}\right)+2$ sq.units. **OR** $\left(\frac{3\pi-8}{12}\right)a^2$ sq.units .

Q29. Let $I=\int_0^{\pi}\frac{x}{1+\sin x}dx \Rightarrow I=\int_0^{\pi}\frac{\pi-x}{1+\sin(\pi-x)}dx=\int_0^{\pi}\frac{\pi-x}{1+\sin x}dx=\int_0^{\pi}\frac{\pi}{1+\sin x}dx-\int_0^{\pi}\frac{x}{1+\sin x}dx$

$\Rightarrow I=\int_0^{\pi}\frac{\pi}{1+\sin x}dx-I \Rightarrow 2I=\pi\int_0^{\pi}\frac{1}{1+\sin x}dx \Rightarrow 2I=\pi\int_0^{\pi}\frac{1}{1+\sin x}dx$

$\Rightarrow I=\frac{\pi}{2}\int_0^{\pi}\frac{1}{1+\cos\left(\frac{\pi}{2}-x\right)}dx=\frac{\pi}{4}\int_0^{\pi}\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)dx \Rightarrow I=-\frac{\pi}{2}\left[\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]_0^{\pi}$

$\Rightarrow I=-\frac{\pi}{2}\left[\tan\left(\frac{\pi}{4}-\frac{\pi}{2}\right)-\tan\left(\frac{\pi}{4}-0\right)\right]$

$\Rightarrow I=-\frac{\pi}{2}[-1-1]=\pi$.

OR See O.P. Gupta's Mathematicia.