Chapter 02

INVERSE TRIGONOMETRIC FUNCTIONS ✍

IMPORTANT TERMS, DEFINITIONS & RESULTS

A function $f : A \to B$ is said to be invertible if f is bijective (i.e., one-one and onto). The inverse of f is denoted by $f : B \to A$ such that $f^{-1}(y) = x$ if $f(x) = y,\ \forall\, x \in A,\ y \in B$.

As trigonometric functions are many-one so, their inverse doesn't exist. But they become one-one onto by restricting their domains. Therefore, inverse of trigonometric functions are defined with restricted domains. In fact, in the discussion below we have used all the restrictions required so that the inverse of the concerned trigonometric functions do exist. If these restrictions are removed, the terms will represent **inverse trigonometric relations** and not the functions. Note that the inverse trigonometric functions are also called as **Inverse Circular Functions**.

01. List of Formulae and their proofs for Inverse Trigonometric Functions :

I.

a) $\sin^{-1}(x) = \text{cosec}^{-1}\left(\frac{1}{x}\right),\ x \in [-1,1]$

b) $\text{cosec}^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right),\ x \in (-\infty,-1] \cup [1,\infty)$

c) $\cos^{-1}(x) = \sec^{-1}\left(\frac{1}{x}\right),\ x \in [-1,1]$

d) $\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right),\ x \in (-\infty,-1] \cup [1,\infty)$

e) $\tan^{-1}(x) = \begin{cases} \cot^{-1}\left(\frac{1}{x}\right), & x > 0 \\ -\pi + \cot^{-1}\left(\frac{1}{x}\right), & x < 0 \end{cases}$

f) $\cot^{-1}(x) = \begin{cases} \tan^{-1}\left(\frac{1}{x}\right), & x > 0 \\ \pi + \tan^{-1}\left(\frac{1}{x}\right), & x < 0 \end{cases}$

II.

a) $\sin^{-1}(-x) = -\sin^{-1} x,\ x \in [-1, 1]$

b) $\cos^{-1}(-x) = \pi - \cos^{-1} x,\ x \in [-1, 1]$

c) $\tan^{-1}(-x) = -\tan^{-1} x,\ x \in R$

d) $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1} x,\ |x| \geq 1$

e) $\sec^{-1}(-x) = \pi - \sec^{-1} x,\ |x| \geq 1$

f) $\cot^{-1}(-x) = \pi - \cot^{-1} x,\ x \in R$

III.

a) $\sin^{-1}(\sin x) = x,\ -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

b) $\cos^{-1}(\cos x) = x,\ 0 \leq x \leq \pi$

c) $\tan^{-1}(\tan x) = x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$

d) $\text{cosec}^{-1}(\text{cosec}\, x) = x,\ -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}, x \neq 0$

e) $\sec^{-1}(\sec x) = x,\ 0 \leq x \leq \pi,\ x \neq \frac{\pi}{2}$

f) $\cot^{-1}(\cot x) = x,\ 0 < x < \pi$

IV.

a) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2},\ x \in [-1,1]$

b) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2},\ x \in R$

c) $\operatorname{cosec}^{-1}x+\sec^{-1}x=\frac{\pi}{2}$, $|x|\ge 1$ i.e., $x\le -1$ or $x\ge 1$ i.e., $x\in R-(-1,1)$

V. **a)** $\sin^{-1}x\pm\sin^{-1}y=\sin^{-1}\left[x\sqrt{1-y^2}\pm y\sqrt{1-x^2}\right]$

b) $\cos^{-1}x\pm\cos^{-1}y=\cos^{-1}\left[xy\mp\sqrt{1-x^2}\sqrt{1-y^2}\right]$

c) $$\tan^{-1}x+\tan^{-1}y=\begin{cases}\tan^{-1}\left(\frac{x+y}{1-xy}\right), & xy<1\\ \pi+\tan^{-1}\left(\frac{x+y}{1-xy}\right), & x>0,\ y>0,\ xy>1\\ -\pi+\tan^{-1}\left(\frac{x+y}{1-xy}\right), & x<0,\ y<0,\ xy>1\end{cases}$$

d) $$\tan^{-1}x-\tan^{-1}y=\begin{cases}\tan^{-1}\left(\frac{x-y}{1+xy}\right), & xy>-1\\ \pi+\tan^{-1}\left(\frac{x-y}{1+xy}\right), & x>0,\ y<0,\ xy<-1\\ -\pi+\tan^{-1}\left(\frac{x-y}{1+xy}\right), & x<0,\ y>0,\ xy<-1\end{cases}$$

e) $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$

VI. **a)** $2\tan^{-1}x=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, $|x|\le 1$

b) $2\tan^{-1}x=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $x\ge 0$

c) $2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, $-1<x<1$

02. Principal Value :
Numerically smallest angle is known as the principal value.

Finding the principal value: For finding the principal value, following algorithm can be followed–

STEP1- Firstly, draw a trigonometric circle and mark the quadrant in which the angle may lie.
STEP2- Select anticlockwise direction for 1st and 2nd quadrants and clockwise direction for 3rd and 4th quadrants.
STEP3- Find the angles in the first rotation.
STEP4- Select the numerically least (magnitude wise) angle among these two values. The angle thus found will be the principal value.
STEP5- In case, two angles one with positive sign and the other with the negative sign qualify for the numerically least angle then, it is the convention to select the angle with positive sign as principal value.

*The principal value is **never** numerically greater than π .*

04. Table demonstrating domains and ranges of Inverse Trigonometric functions :

Inverse Trigonometric Functions *i.e.,* $f(x)$	Domain/ Values of x	Range/ Values of $f(x)$
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\text{cosec}^{-1} x$	$R-(-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]-\{0\}$
$\sec^{-1} x$	$R-(-1, 1)$	$[0, \pi]-\left\{\frac{\pi}{2}\right\}$
$\tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1} x$	R	$(0, \pi)$

Discussion about the range of inverse circular functions other than their respective principal value branch

We know that the domain of sine function is the set of real numbers and range is the closed interval [–1, 1]. If we restrict its domain to $\left[-\frac{3\pi}{2}, -\frac{\pi}{2}\right]$, $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ etc. then, it becomes bijective with the range [–1, 1]. So, we can define the inverse of sine function in each of these intervals. Hence, all the intervals of $\sin^{-1}$ function, **except principal value branch** (here except of $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ for $\sin^{-1}$ function) are known as the **range of $\sin^{-1}$ other than its principal value branch**. The same discussion can be extended for other inverse circular functions. (Refer Q16 in the Exercise of MATHEMATICIA Vol.1)

05. To simplify inverse trigonometrical expressions, following substitutions can be considered :

Expression	Substitution
a^2+x^2 or $\sqrt{a^2+x^2}$	$x=a\tan\theta$ or $x=a\cot\theta$
a^2-x^2 or $\sqrt{a^2-x^2}$	$x=a\sin\theta$ or $x=a\cos\theta$
x^2-a^2 or $\sqrt{x^2-a^2}$	$x=a\sec\theta$ or $x=a\ \text{cosec}\theta$
$\sqrt{\frac{a-x}{a+x}}$ or $\sqrt{\frac{a+x}{a-x}}$	$x=a\cos 2\theta$
$\sqrt{\frac{a^2-x^2}{a^2+x^2}}$ or $\sqrt{\frac{a^2+x^2}{a^2-x^2}}$	$x^2=a^2\cos 2\theta$
$\sqrt{\frac{x}{a-x}}$ or $\sqrt{\frac{a-x}{x}}$	$x=a\sin^2\theta$ or $x=a\cos^2\theta$
$\sqrt{\frac{x}{a+x}}$ or $\sqrt{\frac{a+x}{x}}$	$x=a\tan^2\theta$ or $x=a\cot^2\theta$

☞ Note the followings and keep them in mind

- The symbol $\sin^{-1}x$ is used to denote the **smallest angle** whether positive or negative, such that the sine of this angle will give us x. Similarly $\cos^{-1}x$, $\tan^{-1}x$, $\text{cosec}^{-1}x$, $\sec^{-1}x$ and $\cot^{-1}x$ are defined.
- You should note that $\sin^{-1}x$ can be written as ***arc*sinx**. Similarly other Inverse Trigonometric Functions can also be written as *arc*cosx, *arc*tanx, *arcsecx* etc.
- Also **note that $\sin^{-1}x$** (and similarly other Inverse Trigonometric Functions) **is entirely different from $(\sin x)^{-1}$**. In fact, $\sin^{-1}x$ is the **measure of an angle in Radians** whose sine is x whereas $(\sin x)^{-1}$ is $\frac{1}{\sin x}$ (which is obvious as per the **laws of exponents**).
- Keep in mind that these inverse trigonometric relations are **true only in their domains** i.e., they are valid only for some values of 'x' for which inverse trigonometric functions are well defined!

☞ **Graphs of Inverse Trigonometric Functions should be learnt from the NCERT Textbook.**

EXERCISE FOR PRACTICE

VERY SHORT ANSWER TYPE QUESTIONS – I ◘ **1 Mark**

Q01. Evaluate : $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$. **[SP 2014, 2012 AI**

Q02. Using principal values, write the value of $2\cos^{-1}\frac{1}{2}+3\sin^{-1}\frac{1}{2}$. **[SP 2013**

Q03. Evaluate: $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$. **[SP 2013**

Q04. Write the value of : $\tan^{-1}\left(\frac{a}{b}\right)-\tan^{-1}\left(\frac{a-b}{a+b}\right)$. **[Compt. 2013**

Q05. Write the principal value of $\left[\cos^{-1}\frac{\sqrt{3}}{2}+\cos^{-1}\left(-\frac{1}{2}\right)\right]$. **[Compt. 2013**

Q06. Write the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$. **[Compt. 2013**

Q07. If $2\tan^{-1}\frac{3}{4}=\tan^{-1}x$, write the value of x. **[Compt. 2013**

Q08. Write the principal value of $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)$. **[2013 D**

Q09. Write the value of $\tan\left(2\tan^{-1}\frac{1}{5}\right)$. **[2013 D**

Q10. If $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{4}$, where $xy<1$, find the value of $x+y+xy$. **[Compt. 2012**

Q11. Write the principal value of $\cos^{-1}[\cos(680^o)]$. **[Compt. 2014 D**

Q12. Write the value of $\cot\left(\tan^{-1}a+\cot^{-1}a\right)$. **[2012 F**

Q13. Write the principal value of $\cos^{-1}\left(\frac{1}{2}\right)-2\sin^{-1}\left(-\frac{1}{2}\right)$. **[2012 D**

Q14. What is the principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$? **[2011 AI**

Q15. Write the value of $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$. **[2011, 08 D**

Q16. Write the principal value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$. **[2009 AI, '11 D**

Q17. Write the principal value of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$. **[2009 F, '11 D**

Q18. If $\sin^{-1}\frac{1}{3}+\cos^{-1}x=\frac{\pi}{2}$, then find x. **[Compt. 2010**

Q19. Find the value of $\sin^{-1}\left(\sin\frac{4\pi}{5}\right)$. **[2010 AI**

Q20. What is the principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$? **[2010 D**

Q21. What is the principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$? **[2010 D**

Q22. Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)+\cos^{-1}\left(-\frac{1}{2}\right)$. **[2010 D**

Q23. Using principal value, evaluate $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$. **[2009 D**

Q24. Write the range of one branch of $\cos^{-1}x$, other than the principal branch. **[Exemplar**

Q25. Evaluate $\tan\tan^{-1}(-4)$. **[Exemplar**

Q26. Write the principal value of $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$. **[Compt. 2014 AI**

Q27. Find the value of the following : $\cot\left(\frac{\pi}{2}-2\cot^{-1}\sqrt{3}\right)$. **[Compt. 2014 AI**

Q28. What is the principle value of $\tan^{-1}\left(\tan(2\pi/3)\right)$? **[SP 2017**

VERY SHORT ANSWER TYPE QUESTIONS – II ◘ 2 Marks

Q01. Simplify $\cot^{-1}\frac{1}{\sqrt{x^2-1}}$ for $x<-1$. **[SP 2017**

Q02. If $4\sin^{-1}x+\cos^{-1}x=\pi$, then find the value of x. **[SP 2018**

Q03. Prove that if $\frac{1}{2}\le x\le 1$ then, $\cos^{-1}x+\cos^{-1}\left[\frac{x}{2}+\frac{\sqrt{3-3x^2}}{2}\right]=\frac{\pi}{3}$. **[SP 2018**

SHORT ANSWER TYPE QUESTIONS ◘ 4 Marks

Q01. Prove that : $\cot^{-1}7+\cot^{-1}8+\cot^{-1}18=\cot^{-1}3$. **[SP 2014**

Q02. Find the greatest and least value of $(\sin^{-1}x)^2+(\cos^{-1}x)^2$. **[SP 2014**

Q03. Show that: $\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}]=\sqrt{\dfrac{x^2+1}{x^2+2}}$. **[SP 2013**

Q04. Solve for x : $3\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)-4\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)+2\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)=\dfrac{\pi}{3}$. **[SP 2013**

Q05. Prove that : $\sin^{-1}\dfrac{8}{17}+\sin^{-1}\dfrac{3}{5}=\tan^{-1}\dfrac{77}{36}$. **[Compt. 2013**

Q06. Solve : $\tan^{-1}2x+\tan^{-1}3x=\pi/4$. **[Compt. 2013, '08, '09 D**

Q07. Prove that : $\dfrac{9\pi}{8}-\dfrac{9}{4}\sin^{-1}\dfrac{1}{3}=\dfrac{9}{4}\tan^{-1}2\sqrt{2}$. **[Compt. 2013**

Q08. Solve for x : $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$, $x\neq 0$. **[C' 2014 D, C' 13, '09 AI**

Q09. Solve for x: $2\tan^{-1}(\sin x)=\tan^{-1}(2\sec x)$, $x\neq\pi/2$. **[Compt. 2010, '12 F**

Q10. Find the value of : $\tan\dfrac{1}{2}\left[\sin^{-1}\dfrac{2x}{1+x^2}+\cos^{-1}\dfrac{1-y^2}{1+y^2}\right]$, $|x|<1, y>0$ and $xy<1$. **[2013 D**

Q11. Prove that : $\tan^{-1}\left(\dfrac{1}{2}\right)+\tan^{-1}\left(\dfrac{1}{5}\right)+\tan^{-1}\left(\dfrac{1}{8}\right)=\dfrac{\pi}{4}$. **[2013 D, Compt. 2012**

Q12. Solve for x: $\tan^{-1}\dfrac{x-1}{x-2}+\tan^{-1}\dfrac{x+1}{x+2}=\dfrac{\pi}{4}$. **[Compt. 2012, '10 D, '08 AI**

Q13. Prove that : $\cos^{-1}(x)+\cos^{-1}\left(\dfrac{x}{2}+\dfrac{\sqrt{3-3x^2}}{2}\right)=\dfrac{\pi}{3}$. **[Compt. 2014 AI**

Q14. Prove that: $\sin^{-1}\dfrac{3}{5}+\cos^{-1}\dfrac{12}{13}=\sin^{-1}\dfrac{56}{65}$. **[2012 AI, 2010 D, Compt. 2010**

Q15. Prove that: $\sin^{-1}\left(\dfrac{63}{65}\right)=\sin^{-1}\left(\dfrac{5}{13}\right)+\cos^{-1}\left(\dfrac{3}{5}\right)$. **[2012 F**

Q16. Prove that $\tan^{-1}\left(\dfrac{\cos x}{1+\sin x}\right)=\dfrac{\pi}{4}-\dfrac{x}{2}$, $x\in\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$. **[2012 D**

Q17. Show that $\sin^{-1}\left(\dfrac{8}{17}\right)+\sin^{-1}\left(\dfrac{3}{5}\right)=\cos^{-1}\left(\dfrac{36}{85}\right)$. **[2012 D, Compt. 2010, 2014 C AI**

Q18. Prove that : $2\tan^{-1}\dfrac{1}{2}+\tan^{-1}\dfrac{1}{7}=\tan^{-1}\dfrac{31}{17}$. **[2011 AI**

Q19. Prove that $\tan^{-1}\left(\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\dfrac{\pi}{4}-\dfrac{1}{2}\cos^{-1}x, -\dfrac{1}{\sqrt{2}}\leq x\leq 1$. **[2011 AI, 2017 C**

Q20. Prove that $\dfrac{9\pi}{8}-\dfrac{9}{4}\sin^{-1}\dfrac{1}{3}=\dfrac{9}{4}\sin^{-1}\dfrac{2\sqrt{2}}{3}$. **[2011 F**

Q21. Solve for x : $\tan^{-1}\left(\dfrac{1-x}{1+x}\right)=\dfrac{1}{2}\tan^{-1}(x)$, $[x>0]$. **[2011 F, '08 AI, 2014 C AI**

Q22. Prove that : $\cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]=\dfrac{x}{2}$, $x\in\left(0,\dfrac{\pi}{4}\right)$. **[2011 D, Compt. 2014 D**

OR Prove that : $\cot^{-1}\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}=\dfrac{x}{2}$, $0<x<\dfrac{\pi}{2}$. **[2016 F**

Q23. Find the value of : $\tan^{-1}\left(\dfrac{x}{y}\right)-\tan^{-1}\left(\dfrac{x-y}{x+y}\right)$. **[2011 D**

Q24. Prove that: $\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$. **[Compt. 2010**

Q25. Solve for x: $\tan^{-1}(x+2)+\tan^{-1}(x-2)=\tan^{-1}\left(\frac{8}{79}\right);\ x>0$. **[Compt. 2010**

Q26. Solve for x : $\tan^{-1}x+2\cot^{-1}x=2\pi/3$. **[Compt. 2014 AI**

Q27. Prove that : $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)=\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$. **[2009 F**

Q28. Solve for x: $\tan^{-1}\frac{x}{2}+\tan^{-1}\frac{x}{3}=\frac{\pi}{4};\ \ 0<x<\sqrt{6}$. **[Compt. 2010**

Q29. Prove that : $\tan^{-1}x+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$. **[2010 AI**

Q30. Prove that : $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]=\sqrt{\frac{1+x^2}{2+x^2}}$. **[2010 AI**

Q31. Prove that : $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$. **[2008 D, '10 D**

Q32. Prove that: $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right),\ x\in(0,1)$. **[2010 D**

Q33. Prove that: $\tan^{-1}(1)+\tan^{-1}(2)+\tan^{-1}(3)=\pi$. **[2010 D**

Q34. Prove that: $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$. **['10 F, '08 AI, '17 D**

Q35. Solve the equation for x : $\cos(\tan^{-1}x)=\sin\left(\cot^{-1}\frac{3}{4}\right)$. **[2016 AI, '17 D**

Q36. If $\sin[\cot^{-1}(x+1)]=\cos(\tan^{-1}x)$, then find x. **[2015 Delhi, AI**

Q37. If $(\tan^{-1}x)^2+(\cot^{-1}x)^2=\frac{5\pi^2}{8}$, then find x. **[Delhi 2015**

Q38. If $2\tan^{-1}(\cos\theta)=\tan^{-1}(2\operatorname{cosec}\theta),\ (\theta\neq 0)$, then find the value of θ. **[2015 F, 2016 D**

Q39. If $\tan^{-1}\left(\frac{1}{1+1.2}\right)+\tan^{-1}\left(\frac{1}{1+2.3}\right)+...+\tan^{-1}\left(\frac{1}{1+n.(n+1)}\right)=\tan^{-1}\theta$, then find the value of θ. **[2015 F**

Q40. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2};\ x,y,z>0$, then find the value of $xy+yz+zx$. **[2015 AI**

Q41. Evaluate : $\tan\left\{2\tan^{-1}\left(\frac{1}{5}\right)+\frac{\pi}{4}\right\}$. **[2015 AI**

Q42. Prove that $\cot^{-1}\left(\frac{xy+1}{x-y}\right)+\cot^{-1}\left(\frac{yz+1}{y-z}\right)+\cot^{-1}\left(\frac{zx+1}{z-x}\right)=0;\ (0<xy,\ yz,\ zx<1)$. **[2015 AI**

Q43. Solve for x : $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$. **[2015 AI**

Q44. Prove that : $2\sin^{-1}\frac{3}{5}-\tan^{-1}\frac{17}{31}=\frac{\pi}{4}$. **[2015, 16 AI**

Q45. Solve for x : $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$. **[2015 AI**

Q46. Prove that $\sin\left[\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]=1,\ 0<x<1$. **[2015 AI**

Q47. If $\tan^{-1}\left(\frac{x-5}{x-6}\right)+\tan^{-1}\left(\frac{x+5}{x+6}\right)=\frac{\pi}{4}$, then find the value of x. **[2015 AI**

Q48. Solve for x : $\tan^{-1}\left(\frac{x-2}{x-1}\right)+\tan^{-1}\left(\frac{x+2}{x+1}\right)=\frac{\pi}{4}$. **[2016 F, '16 Compt. Delhi**

Q49. Solve for x : $\tan^{-1}\left(\frac{x-2}{x-3}\right)+\tan^{-1}\left(\frac{x+2}{x+3}\right)=\frac{\pi}{4}$, $|x|<1$. **[2015 AI**

Q50. Prove that : $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$. **[2016 D**

Q51. Prove that : $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)=\cos^{-1}\left(\frac{a\cos x+b}{a+b\cos x}\right)$. **[2015 AI**

Q52. Solve for x : $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$. **[2016 AI**

Q53. Prove that : $\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right)-\tan^{-1}\left(\frac{4x}{1-4x^2}\right)=\tan^{-1}2x;\ |2x|<\frac{1}{\sqrt{3}}$. **[2016 AI**

Q54. Solve the equation for x : $\sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x$. **[2016 AI**

Q55. If $\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\alpha$, prove that $\frac{x^2}{a^2}-2\frac{xy}{ab}\cos\alpha+\frac{y^2}{b^2}=\sin^2\alpha$. **[2016 AI**

Q56. Solve for x : $\tan^{-1}\left(\frac{2-x}{2+x}\right)=\frac{1}{2}\tan^{-1}\frac{x}{2}$, $x>0$. **[2016 AI**

Q57. Prove that $\tan^{-1}\left\{\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right\}=\frac{\pi}{4}+\frac{x}{2}$, if $0<x<\frac{\pi}{2}$. **[2016 D Compt.**

Q58. If $\tan^{-1}\frac{x-3}{x-4}+\tan^{-1}\frac{x+3}{x+4}=\frac{\pi}{4}$, then find the value of x. **[2017 AI**

LONG ANSWER TYPE QUESTIONS ◘ 6 Marks

Q01. Does the equation $\tan^{-1}\frac{x+1}{x-1}+\tan^{-1}\frac{x-1}{x}=-\tan^{-1}7$ have any solutions? If yes, then solve. **[SP 2017**

ANSWERS

CHAPTER 02

(Very Short Answer I)

Q01.	$-\pi/3$	**Q02.**	$7\pi/6$	**Q03.**	$\pi/4$	**Q04.**	$\pi/4$
Q05.	$5\pi/6$	**Q06.**	$\pi/6$	**Q07.**	$24/7$	**Q08.**	$11\pi/12$
Q09.	$5/12$	**Q10.**	1	**Q11.**	40^{o}	**Q12.**	0
Q13.	$2\pi/3$	**Q14.**	π	**Q15.**	1	**Q16.**	$5\pi/6$
Q17.	$-\pi/4$	**Q18.**	$1/3$	**Q19.**	$\pi/5$	**Q20.**	$-\pi/3$
Q21.	$5\pi/6$	**Q22.**	$\pi/2$	**Q23.**	$2\pi/5$	**Q24.**	$[\pi,2\pi],[-\pi,0]$ etc.
Q25.	-4	**Q26.**	$-\pi/4$	**Q27.**	$\sqrt{3}$	**Q28.**	$-\pi/3$

(Very Short Answer II)

Q01. $\pi-\sec^{-1}x$ **Q02.** $1/2$

(Short Answer)

Q02. Least value $=\dfrac{\pi^2}{8}$ & greatest value $=\dfrac{5\pi^2}{4}$ **Q04.** $\dfrac{1}{\sqrt{3}}$ **Q06.** $\dfrac{1}{6}$

Q08. $\pi/4$ **Q09.** $\pi/4$ **Q10.** $\dfrac{x+y}{1-xy}$ **Q12.** $\pm\dfrac{1}{\sqrt{2}}$

Q13. Substitute $\cos^{-1}x=\theta \Rightarrow \cos\theta = x$ in LHS **Q21.** $\dfrac{1}{\sqrt{3}}$ **Q23.** $\dfrac{\pi}{4}$

Q25. $\dfrac{1}{4}$ **Q26.** $x=\sqrt{3}$ **Q28.** 1 **Q35.** $x=\pm 3/4$

Q36. $x=-\dfrac{1}{2}$ **Q37.** $x=-1$ **Q38.** $\dfrac{\pi}{4}$ **Q39.** $\dfrac{n}{n+2}$

Q40. 1 **Q41.** $17/7$ **Q43.** $1/4$ **Q45.** $x=0$

Q47. $x=\pm\dfrac{7}{\sqrt{2}}$ **Q48.** $x=\pm\sqrt{\dfrac{7}{2}}$

Q49. Here $x=\pm\sqrt{\dfrac{7}{2}}$ but $|x|<1$ so, the given equation has no solution.

Q52. $x=0,\pm\dfrac{1}{2}$ **Q54.** $x=0,1/2$ **Q56.** $x=\sqrt{2}$ **Q58.** $\pm\dfrac{\sqrt{34}}{2}$

(Long Answer)

Q01. No solution.