Series : PTS/10

Code No. 18/10/10

Roll No. 2 1 8 1 2

Candidates must write the Code on the title page of the answer-book.

PLEASURE TEST SERIES XII – 10

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Time Allowed : 180 Minutes **Max. Marks : 100**

SECTION A

Q01. For a non-singular matrix A, find $|adj(A^T)|$ if $A^{-1}=\begin{bmatrix}1/5 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$.

OR If matrix $A=[a_{ij}]_{2\times 2}$ where $a_{ij}=2$ if $i\neq j$ and $a_{ij}=0$ if $i=j$, write the matrix A.

Q02. Fill in the blanks : If A and B′ are independent events, then $P(A'\cup B)=1-$__________.

Q03. If $f(x)=\int_0^x t\sin t\,dt$, then write the value of $f'(x)$.

Q04. Find the value of μ where it is given that $\mu=\hat{i}.(\hat{j}\times\hat{k})+\hat{j}.(\hat{k}\times\hat{i})+\hat{k}.(\hat{i}\times\hat{j})$.

SECTION B

Q05. Find the matrix A if, $\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}A\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}-2 & 4\\ 3 & -1\end{bmatrix}$.

OR Find the maximum and minimum value of $\begin{vmatrix}1 & 1\\ 1 & 1+\cos^2\alpha\end{vmatrix}$.

Q06. What are the points at which the function $f(x)=\big||x|-1\big|$ is not differentiable?

OR Find the differential equation for all the lines, which are at a unit distance from (0, 0).

Q07. Let f be a function defined as $f(x)=\dfrac{1}{2-\sin 3x}$. Write the range of $f(x)$.

Q08. Prove that : $2\tan^{-1}\left(\dfrac{1}{2}\right)+\tan^{-1}\left(\dfrac{1}{7}\right)=\sin^{-1}\left(\dfrac{31}{25\sqrt{2}}\right)$.

Q09. Prove that $[\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{d}]=[\vec{a}\ \ \vec{b}\ \ \vec{d}]+[\vec{a}\ \ \vec{c}\ \ \vec{d}]$.

OR If $\vec{a}$ and $\vec{b}$ denote the position vectors of points A and B respectively and C is a point on AB such that $AC=2CB$, then write the position vector of C.

Q10. Evaluate : $\int_{-\pi}^{\pi}\dfrac{\cos^2 x}{1+a^x}dx$.

Q11. If A = {1, 2, 3} and B = {a, b}, write the total number of function from A to B.

Q12. From a set of 100 cards numbered 1 to 100, one card is drawn at random. Find the probability that the number on the card is divisible by 6 or 8, but not by 24.

SECTION C

Q13. If $y=\tan^{-1}\left(\dfrac{x}{a}\right)+\log\sqrt{\dfrac{x-a}{x+a}}$; prove that $\dfrac{dy}{dx}=\dfrac{2ax^2}{x^4-a^4}$

Q14. An open box with square base is to be made out of a given iron sheet of area 27 sq.metres, show that the maximum value of the volume of the box is 13.5 cubic metres.

OR Find the point (s) on $y=\dfrac{x}{1+x^2}$, where the tangent to the curve has the greatest slope.

Q15. For what value of λ the function defined by $f(x)=\begin{cases}\lambda(x^2+2), & \text{if } x\le 0\\ 4x+6, & \text{if } x>0\end{cases}$ is continuous at $x=0$?
Hence check the differentiability of $f(x)$ at $x=0$.

Q16. Find $\int \frac{\sec x(2+\sec x)}{(1+2\sec x)^2}dx$. **OR** Evaluate : $\int \frac{\cos^2 x\ dx}{1+\tan x}$.

Q17. A trust caring for handicapped children gets ₹30000 every month from its donors. The trust spends half of the funds received for medical and educational care of the children and for that it charges 2% of the spent amount from them, and deposits the balance amount in a private bank to get the money multiplied so that in future the trust goes on functioning regularly. What percent of interest should the trust get from the bank to get a total of ₹1800 every month?
Use matrix method, to find the rate of interest.

Q18. Evaluate : $\int_0^{\pi/2} \frac{\tan x\ dx}{1+m^2\tan^2 x}$.

Q19. Which equation of curve would satisfy $\frac{dy}{dx}=\sin(10x+6y)$ such that it passes through origin?

Q20. In a game, a man wins ₹10 for a number more than 4 and loses ₹3 for any other number, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he wins. Find the expected value of the amount he wins/loses.

Q21. Show that $[\vec{a}\ \vec{b}\ \vec{c}]^2=\begin{vmatrix}\vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c}\\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c}\\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c}\end{vmatrix}$.

Q22. Find the equation of a plane passing through the line of intersection of the planes x + 2y + 3z = 2 and x – y + z = 3 and at a distance of $\frac{2}{\sqrt{3}}$ units from the point (3, 1, –1).

OR Let P(3, 2, 6) be a point in the space and Q be a point on the line $\vec{r}=(\hat{i}-\hat{j}+2\hat{k})+\mu(-3\hat{i}+\hat{j}+5\hat{k})$, then find the value of μ for which the vector $\overline{PQ}$ is parallel to the plane $x-4y+3z=1$.

Q23. A bag contains 5 balls. Two balls are drawn at random from the bag and are found to be white. What is the probability that 4 balls in the bag are white and 1 is non-white?

SECTION D

Q24. Find the line of intersection of the planes $\vec{r}.(\hat{i}+2\hat{j}+3\hat{k})=0$ and $\vec{r}.(3\hat{i}+2\hat{j}+\hat{k})=0$. Show that this line is equally inclined to $\hat{i}$ and $\hat{k}$ and makes an angle $\frac{1}{2}\sec^{-1}(3)$ with $\hat{j}$.

Q25. A farmer decides to plant upto 10 hectares with cabbages and potatoes. He decides to grow at least 2 but not more than 8 hectares of cabbages and at least 1 but not more than 6 hectares of potatoes. He can make a profit of ₹ 1500 per hectare on cabbages and ₹ 2000 per hectare on potatoes. How should he plan his farming so as to maximize his profit.

Q26. If $x+y+z=0$, prove that $\begin{vmatrix}xa & yb & zc\\ yc & za & xb\\ zb & xc & ya\end{vmatrix}=xyz\begin{vmatrix}a & b & c\\ c & a & b\\ b & c & a\end{vmatrix}$.

OR For positive numbers x, y and z, find the numerical value of : $\begin{vmatrix}1 & \log_x y & \log_x z\\ \log_y x & 1 & \log_y z\\ \log_z x & \log_z y & 1\end{vmatrix}$.

Q27. Let C be the set of complex nos. Show that the mapping $f: C\to R$ given by $f(z)=|z|\ \forall z\in C$, is neither one-one nor onto.

Q28. Find the area of the region bounded by $y = 6x - x^2$ and $y = x^2 - 2x$.

OR Using integration, find the area bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$.

Q29. Evaluate : $\int_0^{\pi} \frac{1}{5 + 4\cos x} dx$. **OR** Evaluate : $\int_0^{\pi/2} \frac{x + \sin x}{1 + \cos x} dx$.

ANSWERS & SOLUTIONS For PTS-10

SECTION A

Q01. We have $A^{-1}=\begin{bmatrix}1/5 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ $\Rightarrow |A^{-1}|=\begin{vmatrix}1/5 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix}=\frac{1}{5}$

$\therefore |A|=\frac{1}{|A^{-1}|}=5=|A^T|$. So $|\text{adj}A^T|=|A^T|^{3-1}=5^2=25$.

OR Let $A=\begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix}$ $\therefore A=\begin{pmatrix}0 & 2\\ 2 & 0\end{pmatrix}$.

Q02. $P(A\cap B')$ or $P(A)P(B')$

Q03. $x\sin x$

Q04. 3.

SECTION B

Q05. Use $PAQ=R \Rightarrow A=P^{-1}RQ^{-1}$. See **O.P. Gupta's MATHEMATICIA**.

OR Let $\Delta=\begin{vmatrix}1 & 1\\ 1 & 1+\cos^2\alpha\end{vmatrix}$ $\Rightarrow \Delta=1+\cos^2\alpha-1=\cos^2\alpha$

As $-1\le\cos\alpha\le 1$ for all $\alpha\in R$

So, $0\le\cos^2\alpha\le 1$ i.e., $0\le\Delta\le 1$.

Therefore, the maximum value of Δ is 1 and minimum value of Δ is 0.

Q06. The function $f(x)=\big||x|-1\big|$ is not differentiable at x = 0. Also for $x\ne 0$, we have $f(x)=|x-1|$ if x > 0 and $f(x)=|-x-1|$ if x < 0 which reflects their nature of non-differentiability at x = 1, –1 respectively. So, the function $f(x)$ is not differentiable at x = –1, 0, 1.

OR Let the equation of the line be $y=mx+c$ …(i)

Since the line is at unit distance from the origin i.e., $1=\frac{|m\times 0+c|}{\sqrt{1+m^2}}$ $\Rightarrow c=\sqrt{1+m^2}$

That implies, equation (i) becomes $y=mx+\sqrt{1+m^2}$ …(ii) $\Rightarrow \frac{dy}{dx}=m$

Replacing value of m in (ii), we get $y=\left(\frac{dy}{dx}\right)x+\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

On simplification, we get the required differential equation as $\left(\frac{dy}{dx}\right)^2=\left[y-x\left(\frac{dy}{dx}\right)\right]^2-1$.

Q07. [1/3, 1]

Q08. **LHS** : Let Let $y=2\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{7}\right)$

$$\Rightarrow y=\tan^{-1}\frac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2}+\tan^{-1}\left(\frac{1}{7}\right)=\tan^{-1}\left(\frac{4}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)$$

$$\Rightarrow y=\tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}\right)=\tan^{-1}\frac{31}{17}$$

$\Rightarrow \tan y = \frac{31}{17} \quad \Rightarrow \sec y = \sqrt{1+\left(\frac{31}{17}\right)^2} = \sqrt{\frac{289+961}{289}} = \sqrt{\frac{1250}{289}}$

$\Rightarrow \cos y = \frac{17}{25\sqrt{2}} \quad \Rightarrow \sin y = \sqrt{1-\left(\frac{17}{25\sqrt{2}}\right)^2} = \sqrt{\frac{961}{1250}} = \frac{31}{25\sqrt{2}}$

$\Rightarrow y = \sin^{-1}\left(\frac{31}{25\sqrt{2}}\right) =$ **RHS**.

Q09. **LHS :** $[\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{d}] = \{\vec{a}.(\vec{b}+\vec{c})\}\times\vec{d}$ $\quad [\because$ by definition $[\vec{a}\ \ \vec{b}\ \ \vec{c}] = \vec{a}.\vec{b}\times\vec{c}$

$\Rightarrow \quad = \{\vec{a}.\vec{b}+\vec{a}.\vec{c}\}\times\vec{d} = \vec{a}.\vec{b}\times\vec{d}+\vec{a}.\vec{c}\times\vec{d}$

$\Rightarrow \quad = [\vec{a}\ \ \vec{b}\ \ \vec{d}]+[\vec{a}\ \ \vec{c}\ \ \vec{d}] =$ **RHS**.

OR Clearly here C divides AB in 2 : 1 so, position vector of $C = \frac{2\vec{b}+\vec{a}}{3}$.

Q10. Let $I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}dx$ …(i). Use $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ to get, $I = \int_{-\pi}^{\pi} \frac{\cos^2(-x)}{1+a^{-x}}dx$ …(ii)

Adding (i) & (ii), we have : $I = \frac{1}{2}\int_{-\pi}^{\pi} \cos^2 x\,dx = \frac{1}{2}\times 2\int_0^{\pi} \cos^2 x\,dx = \int_0^{\pi}\left[\frac{1+\cos 2x}{2}\right]dx = \frac{\pi}{2}$.

Q11. Total number of function from A to B is $2^3 = 8$.

Q12. Total numbers which are divisible by 6 from 1 to 100 are 16, nos. which are divisible by 8 are 12 and the nos. which are divisible by 24 are 4.

$\therefore$ P(number is divisible by 6 but not by 24) $= \frac{16}{100}-\frac{4}{100} = \frac{12}{100}$

and P(number is divisible by 8 but not by 24) $= \frac{12}{100}-\frac{4}{100} = \frac{8}{100}$

Therefore, required probability $= \frac{12}{100}+\frac{8}{100} = \frac{20}{100}$ or $\frac{1}{5}$.

Alternatively, Let A : the no. is divisible by 6 and B : the no. is divisible by 8.

$\therefore A\cap B$: the no. is divisible by 6 and 8 both i.e., the no. is divisible by 24.

Now $P(A\cup B) = P(A)+P(B)-P(A\cap B) = \frac{16}{100}+\frac{12}{100}-\frac{4}{100} = \frac{24}{100} = P\begin{pmatrix}\text{no. is divisible by} \\ \text{6 or 8 or both}\end{pmatrix}$

Required probability $= P(A\cup B) - P(\text{no. is divisible by 24}) = \frac{24}{100}-\frac{4}{100} = \frac{20}{100}$.

SECTION C

Q13. We have $y = \tan^{-1}\left(\frac{x}{a}\right) + \log\sqrt{\frac{x-a}{x+a}}$ $\quad \Rightarrow y = \tan^{-1}\frac{x}{a} + \frac{1}{2}[\log(x-a) - \log(x+a)]$

On differentiating w.r.t. x both sides, we get : $\frac{dy}{dx} = \frac{1}{1+\frac{x^2}{a^2}}.\frac{1}{a} + \frac{1}{2}\left[\frac{1}{x-a} - \frac{1}{x+a}\right]$

$\Rightarrow \frac{dy}{dx} = \frac{a}{a^2+x^2} + \frac{1}{2}\left[\frac{2a}{x^2-a^2}\right] = \frac{a}{x^2+a^2} + \frac{a}{x^2-a^2}$ $\quad \therefore \frac{dy}{dx} = \frac{2ax^2}{x^4-a^4}$.

Q14. Let base of box be of x metres and height be h metres.

Therefore the surface area of the box is $x^2+4hx = 27$ …(i)

Also the volume of box is, $V = x\times x\times h = \frac{1}{4}x(27-x^2)$. Now proceed to complete.

OR Given curve is $y=\dfrac{x}{1+x^2}$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{(1+x^2).1-x(0+2x)}{(1+x^2)^2}=\dfrac{1-x^2}{(1+x^2)^2}$

Let $f(x)=\dfrac{1-x^2}{(1+x^2)^2}$ represents the slope of the tangent to the curve y at (x, y).

$\therefore f'(x)=\dfrac{(1+x^2)^2(-2x)-2(1-x^2)(1+x^2)(2x)}{(1+x^2)^4}=\dfrac{2x(1+x^2)(x^2-3)}{(1+x^2)^4}=\dfrac{2x(x^2-3)}{(1+x^2)^3}$

For local points of maxima and/or minima, $f'(x)=0$ $\Rightarrow \dfrac{2x(x^2-3)}{(1+x^2)^3}=0$

$\therefore x=0$ or $\pm\sqrt{3}$

And, $f''(x)=\dfrac{(1+x^2)^3(6x^2-6)-3(2x^3-6x)(1+x^2)^2(2x)}{(1+x^2)^6}$

Since $f''(0)=\dfrac{(1+0^2)^3(6\times0^2-6)-3(2\times0^3-6\times0)(1+0^2)^2(2\times0)}{(1+0^2)^6}<0$

& $f''(\pm\sqrt{3})=\dfrac{(1+(\pm\sqrt{3})^2)^3(6\times(\pm\sqrt{3})^2-6)-3(2\times(\pm\sqrt{3})^3-6\times(\pm\sqrt{3}))(1+(\pm\sqrt{3})^2)^2(2\times(\pm\sqrt{3}))}{(1+(\pm\sqrt{3})^2)^6}$

$\Rightarrow f''(\pm\sqrt{3})>0$

$\therefore x=0$ is point of local maxima. (Note that $x=\pm\sqrt{3}$ are points of local minima).

Replacing value of x in the given curve, $y=\dfrac{0}{1+0^2}=0$

Therefore, the required point is $(0,0)$.

Q15. Given function is $f(x)=\begin{cases}\lambda(x^2+2), \text{ if } x\le 0\\ 4x+6, \text{ if } x>0\end{cases}$

We have $f(0)=\lambda(0^2+2)=2\lambda$ and RHL (at x = 0) : $\lim\limits_{x\to0^+} f(x)=\lim\limits_{x\to0^+} 4x+6=4\times0+6=6$

Since f is continuous at x = 0 so, $\lim\limits_{x\to0^+} f(x)=\lim\limits_{x\to0^-} f(x)=f(0)\Rightarrow 6=2\lambda\Rightarrow\lambda=3$.

$\therefore f(x)=\begin{cases}3(x^2+2), \text{ if } x\le 0\\ 4x+6, \text{ if } x>0\end{cases}$ $\Rightarrow f(x)=\begin{cases}3x^2+6, \text{ if } x\le 0\\ 4x+6, \text{ if } x>0\end{cases}$

Differentiability at x = 0 :

LHD (at x = 0) : $\lim\limits_{x\to0^-}\dfrac{f(x)-f(0)}{x-0}=\lim\limits_{x\to0^-}\dfrac{3x^2+6-6}{x}=\lim\limits_{x\to0^-}3x=3\times0=0$,

RHD (at x = 0) : $\lim\limits_{x\to0^+}\dfrac{f(x)-f(0)}{x-0}=\lim\limits_{x\to0^+}\dfrac{4x+6-6}{x}=\lim\limits_{x\to0^+}\dfrac{4x}{x}=4\ne$ LHD (at $x=0$)

Hence $f(x)$ isn't differentiable at x = 0.

Q16. See **O. P. Gupta's Mathematicia**.

OR See **O. P. Gupta's Mathematicia**.

Q17. Let the rate of interest at which the half of the amount ₹30000 is deposited in the bank be x %.

$\therefore \begin{bmatrix}2\% & x\%\end{bmatrix}\begin{bmatrix}15000\\15000\end{bmatrix}=\begin{bmatrix}1800\end{bmatrix}$ $\Rightarrow\left[15000\times\dfrac{2}{100}+15000\times\dfrac{x}{100}\right]=\begin{bmatrix}1800\end{bmatrix}$

$\Rightarrow\begin{bmatrix}300+150x\end{bmatrix}=\begin{bmatrix}1800\end{bmatrix}$ $\Rightarrow 300+150x=1800$ [By equality of matrices

$\Rightarrow 150x=1800-300$ $\Rightarrow x=\dfrac{1500}{150}=10$

Hence the required rate of interest is 10%.

Q18. See **NCERT Exemplar Problems** Chapter 07. Ans. $\frac{1}{m^2-1}\log m$

Q19. $y=\frac{1}{3}\tan^{-1}\left(\frac{5\tan 4x}{4-3\tan 4x}\right)-\frac{5x}{3}$

Q20.

X	10	7	4	–9
P(X)	9/27	6/27	4/27	8/27

Expected amount = ₹$\frac{76}{27}$.

Q21. See **O. P. Gupta's MATHEMATICIA**.

Q22. The equation of a plane passing through the line of intersection of the planes x + 2y + 3z = 2 and x – y + z = 3 is $x+2y+3z-2+\lambda(x-y+z-3)=0$ i.e.,

$x(1+\lambda)+y(2-\lambda)+z(3+\lambda)-2-3\lambda=0\ldots$(i)

Since (i) is at a distance of $\frac{2}{\sqrt{3}}$ units from the point (3, 1,–1) therefore,

$$\frac{|3(1+\lambda)+1(2-\lambda)-1(3+\lambda)-2-3\lambda|}{\sqrt{(1+\lambda)^2+(2-\lambda)^2+(3+\lambda)^2}}=\frac{2}{\sqrt{3}}\Rightarrow\lambda=-\frac{7}{2}$$

Substituting the value of λ in (i), we get the required equation of plane : $5x-11y+z=17$.

OR As Q lies on $\vec{r}=(\hat{i}-\hat{j}+2\hat{k})+\mu(-3\hat{i}+\hat{j}+5\hat{k})$ so, coordinates of Q $(-3\mu+1,\mu-1,5\mu+2)$.

The direction ratios of line PQ : $-3\mu-2,\ \mu-3,\ 5\mu-4$.

$\because\overrightarrow{PQ}$ is parallel to $x-4y+3z=1$ so, $1(-3\mu-2)-4(\mu-3)+3(5\mu-4)=0$

$\Rightarrow\mu=\frac{1}{4}$.

Q23. Required probability $=\frac{3}{10}$.

SECTION D

Q24. See **O. P. Gupta's MATHEMATICIA**

Q25. Let the land used for growing cabbages and potatoes is respectively x and y hectares.

To maximize : Z = ₹ (1500x + 2000y)

Subject to constraints : $x\ge 0,\ y\ge 0,\ x+y\le 10,\ 2\le x\le 8,\ 1\le y\le 6$.

Q26. **LHS :** $\begin{vmatrix} xa & yb & zc\\ yc & za & xb\\ zb & xc & ya\end{vmatrix}$ Expanding along R_1, we get :

$=xa(yza^2-bcx^2)-yb(acy^2-zxb^2)+zc(xyc^2-abz^2)$

$=xyza^3-abcx^3-abcy^3+xyzb^3+xyzc^3-abcz^3$

$=xyz(a^3+b^3+c^3)-abc(x^3+y^3+z^3)$

$=xyz(a^3+b^3+c^3)-abc\left[(x+y+z)(x^2+y^2+z^2-xy-yz-zx)+3xyz\right]$

$=xyz(a^3+b^3+c^3)-abc\left[(0)(x^2+y^2+z^2-xy-yz-zx)+3xyz\right]$

$=xyz(a^3+b^3+c^3-3abc)\ldots$(i)

RHS : $xyz\begin{vmatrix} a & b & c\\ c & a & b\\ b & c & a\end{vmatrix}$ Expanding along R_1, we get :

$=xyz\left[a(a^2-bc)-b(ac-b^2)+c(c^2-ab)\right]$

$=xyz\left[a^3-abc-abc+b^3+c^3-abc\right]$

$= xyz(a^3 + b^3 + c^3 - 3abc) \ldots(ii)$

By (i) and (ii), we have : LHS = RHS.

OR **Method 1 :** $\Delta = \begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$

$\Rightarrow \Delta = \begin{vmatrix} \frac{\log x}{\log x} & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & \frac{\log y}{\log y} & \frac{\log z}{\log y} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & \frac{\log z}{\log z} \end{vmatrix}$ [Using $\frac{\log_b p}{\log_b a} = \log_a p$

Take log x, log y and log z common from C_1, C_2 and C_3 respectively. We have :

$\Delta = \log x \log y \log z \begin{vmatrix} \frac{1}{\log x} & \frac{1}{\log x} & \frac{1}{\log x} \\ \frac{1}{\log y} & \frac{1}{\log y} & \frac{1}{\log y} \\ \frac{1}{\log z} & \frac{1}{\log z} & \frac{1}{\log z} \end{vmatrix}$

Again take $\frac{1}{\log x}, \frac{1}{\log y}$ and $\frac{1}{\log z}$ common from R_1, R_2 and R_3 respectively. We've :

$\Delta = \frac{\log x \log y \log z}{\log x \log y \log z} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$.

Method 2 : We have $\Delta = \begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$

$\Delta = 1(1 - \log_z y \log_y z) - \log_x y(\log_y x - \log_z x \log_y z) + \log_x z(\log_y x \log_z y - \log_z x)$

$= (1-1) - \log_x y \log_y x + \log_z x \log_x y \log_y z + \log_y x \log_z y \log_x z - \log_x z \log_z x$

$= -\log_x y\left(\frac{1}{\log_x y}\right) + \log_z x\left(\frac{\log_y z}{\log_y x}\right) + \log_z y\left(\frac{\log_x z}{\log_x y}\right) - \log_x z\left(\frac{1}{\log_x z}\right)$

$= -1 + \log_z x \log_x z + \log_z y \log_y z - 1$

$\Rightarrow \Delta = 0$ [$\because \log_a b = \frac{1}{\log_b a}, \frac{\log_b p}{\log_b a} = \log_a p$

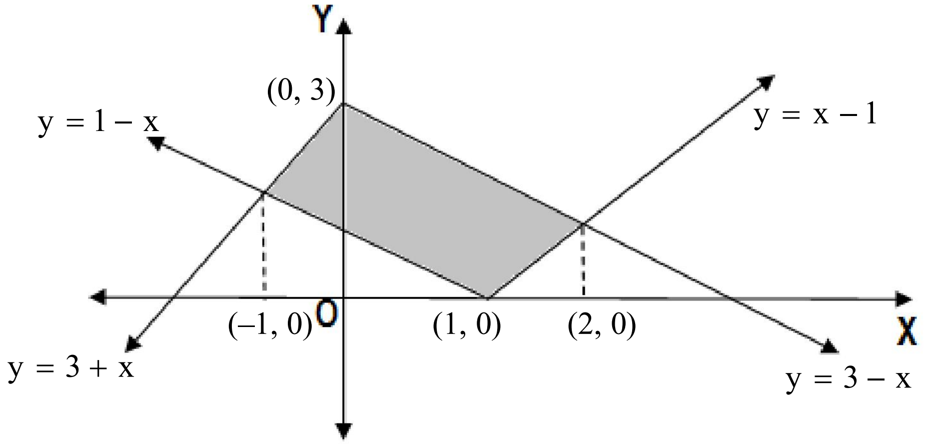
Q27. See **O.P. Gupta's Exemplar Solutions**.

Q28. See **O.P. Gupta's Mathematicia**

OR Given curves are $y = |x-1| = \begin{cases} x-1, \text{ if } x \ge 1 \\ -x+1, \text{ if } x < 1 \end{cases}$ ...(i) and $y = \begin{cases} 3-x, \text{ if } x \ge 0 \\ 3+x, \text{ if } x < 0 \end{cases}$ ...(ii)

When curves (i) and (ii) intersect each other in I quadrant, $x - 1 = 3 - x \Rightarrow x = 2 \therefore (2, 1)$

When curves (i) and (ii) intersect each other in II quadrant, $1 - x = 3 + x \Rightarrow x = -1 \therefore (-1, 2)$.



So, required area of shaded region $= \int_{-1}^{2} [y_{(ii)} - y_{(i)}]dx$

$\Rightarrow \quad = \int_{-1}^{0} (3+x)dx + \int_{0}^{2} (3-x)dx - \int_{-1}^{1} (1-x)dx - \int_{1}^{2} (x-1)dx$

$\Rightarrow \quad = \left[\frac{(3+x)^2}{2}\right]_{-1}^{0} + \left[\frac{(3-x)^2}{-2}\right]_{0}^{2} - \left[\frac{(1-x)^2}{-2}\right]_{-1}^{1} - \left[\frac{(x-1)^2}{2}\right]_{1}^{2}$

$\Rightarrow \quad = \left[\frac{9}{2} - 2\right] - \left[\frac{1}{2} - \frac{9}{2}\right] + \frac{1}{2}[0-4] - \frac{1}{2}[1-0]$

$\Rightarrow \quad = 4 - 2 + 4 - 2 = 4\,\text{sq.units}$.

Q29. $\frac{\pi}{3}$ **OR** $\frac{\pi}{2}$.

■