Series : PTS/8 **Code No. 18/10/8**

Roll No. | 2 | 1 | 8 | 1 | 2 | | |

Candidates must write the Code on the title page of the answer-book.

PLEASURE TEST SERIES XII – 08

A Compilation By : O.P. Gupta (WhatsApp @ +91 9650350480)

Time Allowed : 180 Minutes **Max. Marks : 100**

SECTION – A

Q01. Find the value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$.

Q02. If $A=\begin{pmatrix}1 & 2 & 2\\ 2 & 1 & x\\ -2 & 2 & -1\end{pmatrix}$ is matrix satisfying $AA'=9I$, find x.

OR If $A=\begin{pmatrix}1 & 2\\ 3 & -1\end{pmatrix}$ and $B=\begin{pmatrix}1 & 3\\ -1 & 1\end{pmatrix}$, write the value of $|AB|$.

Q03. Find the value of $[\hat{i}, \hat{k}, \hat{j}]$.

Q04. Find the identity element in the set Q^+ of all positive rational numbers for the operation * defined by $a*b=(3/2)ab$ for all $a,b\in Q^+$.

SECTION – B

Q05. Prove that $3\cos^{-1}x=\cos^{-1}(4x^3-3x),\ x\in[1/2,1]$.

Q06. If $A=\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$ be such that $A^{-1}=kA$, then find the value of k.

Q07. Differentiate $\tan^{-1}\left(\dfrac{\cos x-\sin x}{\cos x+\sin x}\right)$ with respect to x.

Q08. The total revenue received from the sale of x units of a product is given by $R(x)=3x^2+36x+5$ in rupees. Find the marginal revenue when $x=5$, where by marginal revenue we mean the rate of change of total revenue with respect to the number of items sold at an instant.

Q09. Find : $\int\dfrac{3-5\sin x}{\cos^2 x}dx$. **OR** Evaluate : $\int_1^2 x^{2x}(1+\log x)dx$.

Q10. Solve the differential equation $\cos\left(\dfrac{dy}{dx}\right)=a,\ (a\in\mathbb{R})$.

Q11. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=5,\ |\vec{b}|=6$ and $|\vec{c}|=9$, then find the angle between $\vec{a}$ and $\vec{b}$.

OR If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ so that $|\sqrt{2}\vec{a}-\vec{b}|=1$?

Q12. Evaluate $P(A\cup B)$, if $2P(A)=P(B)=5/13$ and $P(A|B)=2/5$.

OR Let A and B be two events such that P (A) = 0.6, P (B) = 0.2, and P (A | B) = 0.5. Then write the value of P (A′ | B′).

SECTION – C

Q13. Using properties of determinants, prove that

$$\begin{vmatrix}5a & -2a+b & -2a+c\\ -2b+a & 5b & -2b+c\\ -2c+a & -2c+b & 5c\end{vmatrix}=12(a+b+c)(ab+bc+ca).$$

Q14. If $\sin y = x\cos(a+y)$, then show that $y' = \cos^2(a+y)\sec a$.
Also, show that $dy/dx = \cos a$, when $x = 0$.

Q15. If $x = a\sec^3\theta$ and $y = a\tan^3\theta$, then find second order derivative of y w.r.t. x at $\theta = \pi/3$.

OR If $y = e^{\tan^{-1}x}$, prove that $(1+x^2)y'' + (2x-1)y' = 0$.

Q16. Find the angle of intersection of the curves $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$, at the point in the first quadrant.

OR Find the intervals in which the function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is
(i) Strictly increasing (ii) Strictly decreasing.

Q17. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 metres. Find the dimensions of the window to admit maximum light through the whole opening. How having large windows help us in saving electricity and conserving environment?

Q18. Find $\int \frac{4}{(x-2)(x^2+4)}dx$.

Q19. Solve the differential equation $(x^2 - y^2)dx + 2xydy = 0$.

OR Find the particular solution of the differential equation $(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$, given that $y = 0$ when $x = 1$.

Q20. Find x such that the four points A(4, 4, 4), B(5, x, 8), C(5, 4, 1) and D(7, 7, 2) are coplanar.

Q21. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

Q22. Two groups are competing for the position of the Board of Directors of a corporation. The probabilities that the first and second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

Q23. From a lot of 20 bulbs which include 5 defectives, a sample of 3 bulbs is drawn at random, one by one with replacement. Find the probability distribution of the number of defective bulbs. Also, find the mean of the distribution.

SECTION – D

Q24. Show that the relation R on the set **Z** of all integers defined by $(x, y) \in R \Leftrightarrow (x-y)$ is divisible by 3, is an equivalence relation.

OR A binary operation * on the set A = {0, 1, 2, 3, 4, 5} is defined as

$$a * b = \begin{cases} a+b, \text{ if } a+b<6 \\ a+b-6, \text{ if } a+b \geq 6 \end{cases}.$$

Write the operation table for a * b in A.
Show that zero is the identity for this operation * and each element ‘a’ $\neq 0$ of the set is invertible with 6 – a, being the inverse of ‘a’.

Q25. Given $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, compute $(AB)^{-1}$.

OR Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ by using elementary row transformation.

Q26. Using integration, find the area of region : $\{(x, y) : 0 \leq 2y \leq x^2, 0 \leq y \leq x, 0 \leq x \leq 3\}$.

Q27. Evaluate $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$. **OR** Evaluate $\int_1^3 (3x^2 + 2x + 1) dx$, as the limit of a sum.

Q28. Find the vector equation of the line passing through (1, 2, 3) and parallel to each of the planes $\vec{r}.(\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r}.(3\hat{i} + \hat{j} + \hat{k}) = 6$. Also find the point of intersection of the line thus obtained with the plane $\vec{r}.(2\hat{i} + \hat{j} + \hat{k}) = 4$.

Q29. A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold while that of B requires 1 g of silver and 2 g of gold. The company can use at most 9 g of silver and 8 g of gold. If each unit of type A brings a profit of ₹ 40 and that of type B ₹ 50, find the number of units of each type that the company should produce to maximize the profit. Formulate and solve graphically the LPP and find the maximum profit. ◘

SOLUTIONS For PTS – 08

Dear Scholars!

This PTS-08 is **Exactly Same Paper*** of CBSE 2018 (Compartment Exams).
So, you can **browse its Solution** at the following Links :

https://theopgupta.com/2-cbse-solved-papers-xii/

https://theopgupta.com/wp-content/uploads/2018/10/CBSE-2018-Compt.-Exam.pdf

If any Error is found, do let us know on **WhatsApp @ +91 9650350480**
Or, you can write to us at **THEOPGUPTA@GMAIL.COM**

- O.P. GUPTA, Your Math Mentor

***Though we have added Some More Questions in the Section A and B to make this paper in accordance with the Latest Pattern of CBSE 2019 Exams.**

Q02. $|AB| = |A||B| = \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}\begin{vmatrix} 1 & 3 \\ -1 & 1 \end{vmatrix} = (-1-6)(1+3) = -28$.

Q09. Let $I = \int_1^2 x^{2x}(1+\log x)dx \quad \Rightarrow I = \int_1^2 x^x[x^x(1+\log x)]dx$.

Substitute $x^x = t$ and proceed to get : $I = \frac{15}{2}$.

Q11. $|\sqrt{2}\vec{a} - \vec{b}| = 1 \quad \Rightarrow |\sqrt{2}\vec{a} - \vec{b}|^2 = 1 \quad \Rightarrow (\sqrt{2}\vec{a} - \vec{b}).(\sqrt{2}\vec{a} - \vec{b}) = 1$

$\Rightarrow 2a^2 - 2\sqrt{2}\vec{a}.\vec{b} + b^2 = 1 \quad \Rightarrow 2\times 1^2 - 2\sqrt{2}ab\cos\theta + 1^2 = 1$

$\Rightarrow 3 - 2\sqrt{2}\times 1\times 1\cos\theta = 1 \quad \Rightarrow -2\sqrt{2}\cos\theta = -2 \quad \Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \quad \therefore \theta = \frac{\pi}{4}$.

Q12. As $P(A|B) = \frac{P(A\cap B)}{P(B)} \quad \Rightarrow 0.5\times 0.2 = P(A\cap B) \quad \therefore P(A\cap B) = 0.1$

Also $P(A\cup B) = P(A) + P(B) - P(A\cap B) = 0.6 + 0.2 - 0.1 = 0.7$

Now $P(A'|B') = \frac{P(A'\cap B')}{P(B')} = \frac{P(A\cup B)'}{1-P(B)} = \frac{1-P(A\cup B)}{1-P(B)} = \frac{1-0.7}{1-0.2} = \frac{3}{8}$.