Series : PTS/4

Code No. 18/9/4

Roll No. | 2 | 1 | 8 | 1 | 2 | | |

Candidates must write the Code on the title page of the answer-book.

PLEASURE TEST SERIES XII – 04

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Time Allowed : 180 Minutes **Max. Marks : 100**

SECTION A

Q01. Write the sum of order and degree of the differential equation $1+\left(\frac{dy}{dx}\right)^4=7\left(\frac{d^2y}{dx^2}\right)^3$.

Q02. If $A=\begin{pmatrix}1 & 2\\ 3 & -1\end{pmatrix}$ and $B=\begin{pmatrix}1 & 3\\ -1 & 1\end{pmatrix}$, write the value of $|AB|$.

Q03. Write the sum of intercepts cut off by $\vec{r}.(2\hat{i}+\hat{j}-\hat{k})=5$ on the axes.

Q04. If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ so that $|\sqrt{2}\vec{a}-\vec{b}|=1$?

OR Find a vector in the direction of $\vec{a}=\hat{i}-2\hat{j}$ that has magnitude 7 units.

SECTION B

Q05. Write the differential equation obtained by eliminating the arbitrary constant C in the equation representing the family of curves $xy = C\cos x$.

Q06. If a line makes angles α, β, γ with the positive direction of coordinate axes, then write the integral value of $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$ w.r.t. x.

Q07. For what values of k, the system of equations $x+y+z=2,\ 2x+y-z=3,\ 3x+2y+kz=4$ has a unique solution?

OR If $[2x\ \ 3]\begin{bmatrix}1 & 2\\ -3 & 0\end{bmatrix}\begin{bmatrix}x\\ 3\end{bmatrix}=O$, find x.

Q08. Prove that $\tan^{-1}(63/16)=\sin^{-1}(5/13)+\cos^{-1}(3/5)$.

OR Prove that : $\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right)-\tan^{-1}\left(\frac{4x}{1-4x^2}\right)=\tan^{-1}2x;\ |2x|<\frac{1}{\sqrt{3}}$.

Q09. Find the area of a parallelogram having diagonals $3\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}-3\hat{j}-4\hat{k}$.

Q10. Set A has 3 elements and the set B has 4 elements. Then find the number of injective mappings that can be defined from A to B.

Q11. Let A and B be two events such that P (A) = 0.6, P (B) = 0.2, and P (A | B) = 0.5. Then write the value of P (A′ | B′).

OR Two cards are drawn from a well shuffled deck of 52 playing cards with replacement. Find the probability, that both cards are queens.

Q12. For what values of a, the curves $y = x^2 + ax + 25$ touches the axis of x?

SECTION C

Q13. If $y=\frac{x\cos^{-1}x}{\sqrt{1-x^2}}-\log\sqrt{1-x^2}$, then prove that $\frac{dy}{dx}=\frac{\cos^{-1}x}{(1-x^2)^{3/2}}$.

Q14. A tank with rectangular base and rectangular sides open at the top is to be constructed so that its depth is 3 m and volume is 75 m^3. If building of tank costs ₹100 per square metre for the base and ₹50 per square metres for the sides, find the cost of least expensive tank.

OR If $f(x) = x^3 + ax^2 + bx + 5c$ has a maxima at $x = -1$ and minima at $x = 3$, then find the value of a, b and c.

Q15. If $x = a\sec^3\theta,\ y = a\tan^3\theta$, find $D^2(y)$ at $\theta = \pi/4$. **Q16.** Find $\int \frac{(x^2+1)e^x}{(x+1)^2}dx$.

Q17. The monthly incomes of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5 : 7. If each saves ₹15,000 per month, find their monthly incomes using matrix method.

OR Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$. Use transformations.

Q18. **(a)** Evaluate : $\int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}}$. **(b)** Find $\int \frac{1}{(1+\sqrt{x})\sqrt{x-x^2}}dx$.

OR Find $\int \frac{x}{(x-1)^2(x+2)}dx$.

Q19. Solve : $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$.

Q20. From a lot of 15 bulbs which include 5 defectives, a sample of 2 bulbs is drawn at random (without replacement). Find the probability distribution of the number of defective bulbs.

Q21. Find the value of λ so that the four points A, B, C and D with position vectors $4\hat{i}+5\hat{j}+\hat{k}$, $-\hat{j}-\hat{k}$, $3\hat{i}+\lambda\hat{j}+4\hat{k}$ and $-4\hat{i}+4\hat{j}+4\hat{k}$ respectively are coplanar.

Q22. Find the shortest distance between the lines $\vec{r} = (\hat{i}+2\hat{j}+\hat{k}) + \lambda(\hat{i}-\hat{j}+\hat{k})$ and $\vec{r} = (2\hat{i}-\hat{j}-\hat{k}) + \mu\,(2\hat{i}+\hat{j}+2\hat{k})$.

Q23. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and two balls are drawn without replacement from the bag and are found to be both red. Find the probability that the balls are drawn from the first bag.

SECTION D

Q24. Find the equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$ which is perpendicular to the plane $x-y+z=0$. Then find the distances of plane thus obtained from the point A(1, 3, 6).

Q25. In order to supplement daily diet, a person wishes to take some X and some wishes Y tablets. The contents of iron, calcium and vitamins in X and Y (in milligrams per tablet) are given as :

Tablets	Iron	Calcium	Vitamin
X	6	3	2
Y	2	3	4

The person needs at least 18 milligrams of iron, 21 milligrams of calcium and 16 milligram of vitamins. The price of each tablet of X and Y is ₹2 and ₹1 respectively. How many tablets of each should the person take in order to satisfy the above requirement at the minimum cost?

Q26. Using properties of determinants, prove that $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2$.

OR If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that $A^3 - 4A^2 - 3A + 11I = O$. Hence, find A^{-1}.

Q27. Find the particular solution of the differential equation $x\frac{dy}{dx} + y - x + xy\cot x = 0;\ x \neq 0$ given that when $x = \pi/2,\ y = 0$.

OR Solve : $x^2dy + (xy + y^2)dx = 0$ given that $y = 1$ when $x = 1$.

Q28. Find the area enclosed by $x^2 + y^2 - 6x - 4y + 12 \le 0,\ y \le x$ and $x \le 5/2$.

OR Evaluate $\int_0^{\pi/2} \sin x \, dx$ as limit of sums.

Q29. Let $A = \mathbb{R} \times \mathbb{R}$ and * be the binary operation on A defined by (a, b) * (c, d) = (a + c, b + d). Show that * is commutative and associative. Find the identity element for * on A.

ANSWERS & HINTS For PTS – 04

SECTION A

Q01. Order is 2 and degree is 3 so, their sum is 5.

Q02. As $|AB| = |A||B| = \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}\begin{vmatrix} 1 & 3 \\ -1 & 1 \end{vmatrix} = (-7)(4) = -28$.

Q03. Here $\vec{r}.(2\hat{i}+\hat{j}-\hat{k})-5=0 \quad \Rightarrow 2x+y-z=5 \quad \Rightarrow \dfrac{x}{5/2}+\dfrac{y}{5}+\dfrac{z}{-5}=1 \quad \left[\because \dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1\right.$

Sum of intercepts $=\dfrac{5}{2}+5+(-5)=\dfrac{5}{2}$.

Q04. As $\left|\sqrt{2}\vec{a}-\vec{b}\right|^2 = (\sqrt{2}\vec{a}-\vec{b}).(\sqrt{2}\vec{a}-\vec{b}) = 2a^2+b^2-2\sqrt{2}\vec{a}.\vec{b}=1$

If α is the angle between the unit vectors $\vec{a}$ and $\vec{b}$, so, $2a^2+b^2-2\sqrt{2}ab\cos\alpha=1$

That is, $2+1-2\sqrt{2}.1.1\cos\alpha=1 \qquad \Rightarrow \cos\alpha=\dfrac{1}{\sqrt{2}} \quad \therefore \alpha=\dfrac{\pi}{4}$.

OR As required vector is $=7\hat{a}=7\left(\dfrac{\vec{a}}{|\vec{a}|}\right)=7\left(\dfrac{\hat{i}-2\hat{j}}{\sqrt{1+4}}\right)=\dfrac{7}{\sqrt{5}}(\hat{i}-2\hat{j})$.

SECTION B

Q05. Re-writing the equation of curve, we have : $xy\sec x = C$

Now differentiating the curve w.r.t. x, we get : $xy(\sec x\tan x)+\sec x(xy'+y.1)=0$

That is, $xy\tan x+xy'+y=0$ is the required differential equation.

Q06. As $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1 \quad \therefore \sin^2\alpha+\sin^2\beta+\sin^2\gamma=3-(\cos^2\alpha+\cos^2\beta+\cos^2\gamma)=2$

(Since $\cos^2\theta=1-\sin^2\theta$)

Now, $\int(\sin^2\alpha+\sin^2\beta+\sin^2\gamma)dx=\int 2dx=2x+C$.

Q07. Let $A=\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{pmatrix}$, which is matrix formed by the coefficients of x, y & z in the given system of equations.

For unique solution, $|A|\neq 0$ so, $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix}\neq 0 \qquad \Rightarrow 1(k+2)-1(2k+3)+1(4-3)\neq 0$

$\Rightarrow k+2-2k-3+1\neq 0 \qquad \therefore k\neq 0$. So, k can be any real number other than zero.

OR As $[2x \quad 3]\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 3 \end{bmatrix}=O \qquad \Rightarrow [2x-9 \quad 4x+0]\begin{bmatrix} x \\ 3 \end{bmatrix}=O \qquad \Rightarrow [2x^2+3x]=[0]$

By equality of matrices, we get : $2x^2+3x=0 \quad \Rightarrow x(2x+3)=0 \qquad \therefore x=0, -3/2$.

Q08. Note that $\sin^{-1}(5/13)+\cos^{-1}(3/5)=\tan^{-1}(5/12)+\tan^{-1}(4/3)$

$$=\tan^{-1}\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\times\frac{4}{3}}\right)=\tan^{-1}\frac{63}{16}.$$

OR LHS : Let $y=\tan^{-1}\left(\dfrac{6x-8x^3}{1-12x^2}\right)-\tan^{-1}\left(\dfrac{4x}{1-4x^2}\right)$ Put $2x=\tan\theta \Rightarrow \theta=\tan^{-1}2x$...(i)

$\therefore y=\tan^{-1}\left(\dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)-\tan^{-1}\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right) \qquad \Rightarrow y=\tan^{-1}(\tan 3\theta)-\tan^{-1}(\tan 2\theta)$

$\Rightarrow y = 3\theta - 2\theta = \theta = \tan^{-1} 2x = \text{RHS}$. [By (i)

Note that as $|2x| < \frac{1}{\sqrt{3}}$ implies, $-\frac{1}{\sqrt{3}} < \tan\theta < \frac{1}{\sqrt{3}}$ i.e., $-\frac{\pi}{6} < \theta < \frac{\pi}{6}$ i.e., $-\frac{\pi}{2} < 3\theta < \frac{\pi}{2}$ and $-\frac{\pi}{3} < 2\theta < \frac{\pi}{3}$.

Q09. Recall that area of parallelogram with diagonals $\vec{a}$ and $\vec{b}$, is $\frac{1}{2}|\vec{a}\times\vec{b}|$, so area is $5\sqrt{3}$ Sq.units.

Q10. The total number of injective mappings from the set containing 3 elements into the set containing 4 elements is ${}^4P_3 = 4! = 24$.
 # Don't forget to check these kind of questions (and the related formulas) where you have to find no. of functions, one-one functions, onto functions, binary operations etc.
☞☞ Go through **Mathematicia by O.P. Gupta.**

Q11. Note that, $P(A'|B') = \frac{P(A'\cap B')}{P(B')} = \frac{P(A\cup B)'}{1-P(B)} = \frac{1-P(A\cup B)}{1-P(B)} \ldots$(i)

Also $P(A|B) = \frac{P(A\cap B)}{P(B)} \Rightarrow P(A\cap B) = P(A|B).P(B) \ldots$(ii)

By (i) and (ii), we get : $P(A'|B') = \frac{1-\{P(A|B).P(B)\}}{1-P(B)}$

Replace the given values in the above expression to get : $P(A' | B') = 3/8$.

OR $P(Q_1 \cap Q_2) = P(Q_1).P(Q_2 | Q_1) = \frac{4}{52}\times\frac{4}{52} = \frac{1}{13}\times\frac{1}{13} = \frac{1}{169}$.

Q12. Let the curve touches x-axis at $(\alpha, 0)$ so, we have $0 = \alpha^2 + a\alpha + 25$...(i)
Also note that the curve touches the x-axis so, x-axis will be the tangent to the curve, having slope of 0. (Apart from that, recall that dy/dx gives a general form of slope of curve y = f (x) at any random point.)

Now, $\frac{dy}{dx} = 2x + a \quad \therefore \left.\frac{dy}{dx}\right]_{at\,(\alpha,0)} = 2\alpha + a = 0 \quad \Rightarrow a = -2\alpha \ldots$(ii)

By (i) and (ii), we get : $\alpha^2 + (-2\alpha)\alpha + 25 = 0 \quad \Rightarrow \alpha = \pm 5 \quad \therefore a = \pm 10$.

SECTION C

Q13. **Check Mathematicia (Ch 03 – Miscellaneous) by O.P. Gupta.**

Q14. **See Mathematicia (Ch 04 – Maxima & Minimima) by O.P. Gupta** for both the sums**.**
Ans. ₹5500/- **OR** Ans. a = –3, b = –9, c∈R.

Q15. $\frac{dx}{d\theta} = 3a\sec^3\theta\tan\theta, \frac{dy}{d\theta} = 3a\tan^2\theta\sec^2\theta \quad \therefore \frac{dy}{dx} = \frac{3a\tan^2\theta\sec^2\theta}{3a\sec^3\theta\tan\theta} = \sin\theta$

Now $\frac{d^2y}{dx^2} = \cos\theta \times \frac{d\theta}{dx} = \cos\theta \times \frac{1}{3a\sec^3\theta\tan\theta}$

So, $D^2(\pi/4) = \cos\frac{\pi}{4} \times \frac{1}{3a\sec^3\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{4}\right)} = \frac{1}{12a}$.

Q16. **See Mathematicia (Ch 05 – Type F) by O.P. Gupta.** Ans. $e^x\left(\frac{x-1}{x+1}\right) + C$.

Q17. **Check Mathematicia (Ch 01 – Miscellaneous) by O.P. Gupta.**
Ans. Monthly income for Aryan and Babban are ₹90,000 and ₹1,20,000.

OR Ans. $\begin{pmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$.

Q18. **(a)** As $\int_0^{\pi/2}\frac{dx}{1+\sqrt{\tan x}}=\int_0^{\pi/2}\frac{dx}{1+\sqrt{\tan\left(\frac{\pi}{2}-x\right)}}=\int_0^{\pi/2}\frac{dx}{1+\sqrt{\cot x}}=\int_0^{\pi/2}\frac{\sqrt{\tan x}\,dx}{\sqrt{\tan x}+1}=\int_0^{\pi/2}1dx-\int_0^{\pi/2}\frac{1dx}{\sqrt{\tan x}+1}$

$\Rightarrow 2\int_0^{\pi/2}\frac{dx}{1+\sqrt{\tan x}}=[x]_0^{\pi/2}$ $\quad\therefore \int_0^{\pi/2}\frac{dx}{1+\sqrt{\tan x}}=\frac{\pi}{4}$.

(b) Check **YouTube.com/OPGuptaMathsGuru** for a video of Similar sum.

Substitute $x=\sin^2\theta$ or, $x=\cos^2\theta$. Ans. $2\left(\frac{\sqrt{x}-1}{\sqrt{1-x}}\right)+C$, if $x=\sin^2\theta$.

Remember, your **Answer may vary**, depending upon your **choice of method**.

OR Use Partial Fraction method of Integrals. Ans. $\frac{2}{9}\log\left|\frac{x-1}{x+2}\right|-\frac{1}{3(x-1)}+C$.

Q19. Use formulas of $2\tan^{-1}x$ in terms of sine, cosine and tangent inverse x. Ans. $\frac{1}{\sqrt{3}}$.

Q20. See **Mathematicia by O.P. Gupta (Ch 12 Probability Distribution – Similar sum : Q29).**

Q21. Obtain the vectors $\overrightarrow{AB}=\vec{a}$, $\overrightarrow{AC}=\vec{b}$ and $\overrightarrow{AD}=\vec{c}$ and hence, use $[\vec{a}\ \vec{b}\ \vec{c}]=0$ for coplanarity of vectors. Ans. $\lambda=9$.

Q22. Use the formula of S.D. between two skew lines i.e., $p=\frac{\left|(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$. Ans. $\frac{3\sqrt{2}}{2}$ units.

Q23. Based on Bayes' Theorem. Ans. $\frac{6}{7}$.

SECTION D

Q24. Required plane is $x+y+z-1+\lambda(2x+3y+4z-5)=0\ \ldots(i)$

The d.r.'s of normal to the plane (i) are $1+2\lambda,\ 1+3\lambda,\ 1+4\lambda$.

As (i) is perpendicular to $x-y+z=0$ so, $1.(1+2\lambda)-1.(1+3\lambda)+1.(1+4\lambda)=0$

Now solve to get value of λ and put it in (i) to get the required eq. of plane.

Now use the formula $p=\frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$ to get distance of Point from the plane (i).

Ans. $x-z+2=0,\ \frac{3}{\sqrt{2}}$ units.

Q25. Let no. of tablest of X and Y type are x and y respectively.

To minimize : $Z=(2x+y)$ in ₹

Subject to constraints : $x\ge 0,\ y\ge 0,$

$6x+2y\ge 18,\ 3x+3y\ge 21,\ 2x+4y\ge 16$.

Ans. Minimum value of Z is ₹8 at (1, 6).

Q26. LHS : Let $\Delta=\begin{vmatrix}(a+1)(a+2) & a+2 & 1\\(a+2)(a+3) & a+3 & 1\\(a+3)(a+4) & a+4 & 1\end{vmatrix}$

By applying $R_2\to R_2-R_1,\ R_3\to R_3-R_2$, we get :

$\Rightarrow \Delta=\begin{vmatrix}(a+1)(a+2) & a+2 & 1\\2(a+2) & 1 & 0\\2(a+3) & 1 & 0\end{vmatrix}$

Expanding along C_3, we get :

$\therefore \Delta=2(a+2)-2(a+3)=-2=\text{RHS}$.

OR First of all show that $A^3 - 4A^2 - 3A + 11I = O$, by replacing values of A^3, A^2, A and I.
Hence, pre-multiply both sides by A^{-1} to get : $A^{-1}AA^2 - 4A^{-1}AA - 3A^{-1}A + 11A^{-1}I = A^{-1}O$
That is, $IA^2 - 4IA - 3I + 11A^{-1} = O \quad \therefore 11A^{-1} = 3I + 4A - A^2$.

Ans. $A^{-1} = \frac{1}{11}\begin{pmatrix} -2 & 5 & 3 \\ 7 & -1 & -5 \\ -4 & -1 & 6 \end{pmatrix}$.

Q27. As $\frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right)y = 1$ is a linear diff. eq. of the form $\frac{dy}{dx} + P(x)y = Q(x)$.
So, find $I.F. = e^{\int (1/x + \cot x)dx} = x\sin x$. Now proceed. Ans. $xy\sin x + x\cos x - \sin x + 1 = 0$.

OR Note that, it is homogeneous so, write $\frac{dy}{dx} = -\frac{xy + y^2}{x^2}$. Now put $y = vx$ and proceed.
Ans. $3x^2y - y - 2x = 0$.

Q28. **Check Mathematicia (Ch 07) by O.P. Gupta.** Ans. $\left(\frac{\sqrt{3}+1}{8} - \frac{\pi}{6}\right)$ sq.units.

OR **Check Mathematicia (Ch 06 - Example) by O.P. Gupta.** Ans. 1.

Q29. I'm leaving proof part of commutativity and associativity, intentionally.
Let (e, f) be the identity for * on set A.
Therefore, (a, b) * (e, f) = (a, b) = (e, f) * (a, b).
Consider (a, b) * (e, f) = (a, b) which implies, (a + e, b + f) = (a, b)
i.e., $a + e = a,\ b + f = b \quad \Rightarrow e = 0,\ f = 0$
Again, consider (e, f) * (a, b) = (a, b) which implies, (e + a, f + b) = (a, b)
i.e., $e + a = a,\ f + b = b \quad \Rightarrow e = 0,\ f = 0$
$\therefore$ Clearly, (0, 0) is the identity element for * on set A.