❖ A Help Guide On ❖
Vector Algebra & 3 Dimensional Geometry

A Help Guide By OP Gupta (Indira Award Winner)

Q01. If $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = 6\hat{i} + \lambda\hat{j} + 9\hat{k}$ and $\vec{a} \parallel \vec{b}$, then find the value of λ.

Sol. Since $\vec{a} \parallel \vec{b}$ so, the d.r.'s will be proportional i.e., $\frac{2}{6} = \frac{-1}{\lambda} = \frac{3}{9} \quad \Rightarrow \lambda = 3$.

Q02. Find the projection of $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ on $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

Sol. The projection of $\vec{a}$ on $\vec{b}$ is $\frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{(2\hat{i} - \hat{j} + \hat{k}).(\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{1^2 + (-2)^2 + 1^2}} = \frac{5}{\sqrt{6}}$ units.

Q03. Determine the direction cosines of a line whose direction ratios are 2, – 6, 3.

Sol. Direction cosines of line are $\pm\frac{2}{\sqrt{2^2 + (-6)^2 + 3^2}}, \pm\frac{-6}{\sqrt{4 + 36 + 9}}, \pm\frac{3}{7}$ i.e., $\pm\frac{2}{7}, \mp\frac{6}{7}, \pm\frac{3}{7}$.

Q04. Find $|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 15$.

Sol. Given $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 15 \quad \Rightarrow \vec{x}.\vec{x} + \vec{x}.\vec{a} - \vec{a}.\vec{x} - \vec{a}.\vec{a} = 15 \quad \Rightarrow |\vec{x}|^2 + \vec{x}.\vec{a} - \vec{x}.\vec{a} - |\vec{a}|^2 = 15$

$\Rightarrow |\vec{x}|^2 - (1)^2 = 15 \qquad \therefore |\vec{x}| = 4$.

Q05. For any three vectors $\vec{a}, \vec{b}$ and $\vec{c}$, prove that: $[\vec{a} + \vec{b} \;\; \vec{b} + \vec{c} \;\; \vec{c} + \vec{a}] = 2[\vec{a} \;\; \vec{b} \;\; \vec{c}]$.

Sol. LHS: $[\vec{a} + \vec{b} \;\; \vec{b} + \vec{c} \;\; \vec{c} + \vec{a}] = \left[(\vec{a} + \vec{b}) \times (\vec{b} + \vec{c})\right].(\vec{c} + \vec{a})$

$$\Rightarrow \quad = \left[\vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{c}\right].(\vec{c} + \vec{a})$$

$$\Rightarrow \quad = (\vec{a} \times \vec{b}).\vec{c} + (\vec{a} \times \vec{b}).\vec{a} + (\vec{a} \times \vec{c}).\vec{c} + (\vec{a} \times \vec{c}).\vec{a} + (\vec{b} \times \vec{c}).\vec{c} + (\vec{b} \times \vec{c}).\vec{a}$$

$$\Rightarrow \quad = (\vec{a} \times \vec{b}).\vec{c} + (\vec{b} \times \vec{c}).\vec{a}$$

$$\Rightarrow \quad = [\vec{a} \;\; \vec{b} \;\; \vec{c}] + [\vec{b} \;\; \vec{c} \;\; \vec{a}]$$

$$\Rightarrow \quad = [\vec{a} \;\; \vec{b} \;\; \vec{c}] + [\vec{a} \;\; \vec{b} \;\; \vec{c}]$$

$$\Rightarrow \quad = 2[\vec{a} \;\; \vec{b} \;\; \vec{c}] = \text{RHS.}$$

Q06. Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.(\hat{i} + 2\hat{j} + 3\hat{k}) = 4$, $\vec{r}.(2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r}.(5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$.

Sol. The plane passing through the intersection of given planes is,

$$\vec{r}.(\hat{i} + 2\hat{j} + 3\hat{k}) - 4 + \lambda[\vec{r}.(2\hat{i} + \hat{j} - \hat{k}) + 5] = 0$$

$$\Rightarrow \vec{r}.[(1 + 2\lambda)\hat{i} + (2 + \lambda)\hat{j} + (3 - \lambda)\hat{k}] - 4 + 5\lambda = 0 \ldots\text{(i)}$$

As (i) is perpendicular to $\vec{r}.(5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$, then

$5(1 + 2\lambda) + 3(2 + \lambda) - 6(3 - \lambda) = 0 \Rightarrow \lambda = \frac{7}{19}$ [Using $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Substituting the values of λ in (i), we get : $\vec{r}.(33\hat{i} + 45\hat{j} + 50\hat{k}) = 41$.

Q07. Find the equation of the plane passing through the intersection of the planes $2x + 3y - z + 1 = 0$, $x + y - 2z + 3 = 0$ and perpendicular to the plane $3x - y - 2z = 4$.

Sol. Equation of plane passing through the intersection of the planes $2x + 3y - z + 1 = 0$ and $x + y - 2z + 3 = 0$ is, $2x + 3y - z + 1 + \lambda(x + y - 2z + 3) = 0$

i.e., $x(2 + \lambda) + y(3 + \lambda) + z(-1 - 2\lambda) + 1 + 3\lambda = 0 \ldots$(i)

Also (i) is perpendicular to $3x - y - 2z = 4$ so, by using $a_1a_2 + b_1b_2 + c_1c_2 = 0$ we have :

$3(2+\lambda)+(-1)(3+\lambda)+(-2)(-1-2\lambda)=0\Rightarrow\lambda=-\dfrac{5}{6}$.

Substituting the value of λ in (i), we get : $7x+13y+4z=9$.

Q08. Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}$, $\vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d}=15$.

Sol. As $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$ so, $\vec{d}=\lambda(\vec{a}\times\vec{b})$ where $\lambda\in R-0$

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 4 & 2\\ 3 & -2 & 7\end{vmatrix}=32\hat{i}-\hat{j}-14\hat{k}$. So, $\vec{d}=\lambda(32\hat{i}-\hat{j}-14\hat{k})$.

Given that $\vec{c}.\vec{d}=15$ so, $(2\hat{i}-\hat{j}+4\hat{k}).[\lambda(32\hat{i}-\hat{j}-14\hat{k})]=0\Rightarrow\lambda=\dfrac{5}{3}$

Hence, $\vec{d}=\dfrac{5}{3}(32\hat{i}-\hat{j}-14\hat{k})=\dfrac{1}{3}(160\hat{i}-5\hat{j}-70\hat{k})$.

Q09. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$, find a vector $\vec{c}$ such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}.\vec{c}=3$.

Sol. Let $\vec{c}=x\hat{i}+y\hat{j}+z\hat{k}$.

$\therefore\vec{a}\times\vec{c}=(\hat{i}+\hat{j}+\hat{k})\times(x\hat{i}+y\hat{j}+z\hat{k})=(z-y)\hat{i}+(x-z)\hat{j}+(y-x)\hat{k}$

As $\vec{a}\times\vec{c}=\vec{b}$ so, $(z-y)\hat{i}+(x-z)\hat{j}+(y-x)\hat{k}=\hat{j}-\hat{k}$

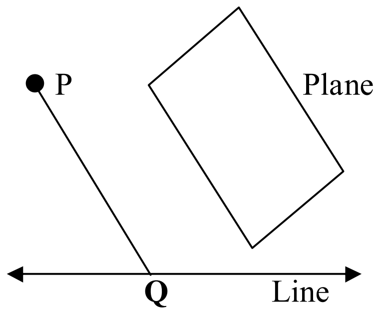
By equality of vectors, $z-y=0, x-z=1, y-x=-1$ …(i)

Also $\vec{a}.\vec{c}=3\Rightarrow x+y+z=3$ …(ii)

By (i) & (ii), we get $x=\dfrac{5}{3}, y=z=\dfrac{2}{3}$ $\Rightarrow\vec{c}=\dfrac{5}{3}\hat{i}+\dfrac{2}{3}\hat{j}+\dfrac{2}{3}\hat{k}$.

Q10. Find the distance of the point (–2, 3,–4) from the line $\dfrac{x+2}{3}=\dfrac{2y+3}{4}=\dfrac{3z+4}{5}$, measured parallel to the plane $4x+12y-3z+1=0$.

Sol. Let the line through P(–2,3,–4) parallel to the given plane meets the given line at point Q. So, the coordinates of point Q on the line



$\dfrac{x+2}{3}=\dfrac{2y+3}{4}=\dfrac{3z+4}{5}$ are $Q\left(9\lambda-2, 6\lambda-\dfrac{3}{2}, 5\lambda-\dfrac{4}{3}\right)$.

The d.r.'s of PQ are $9\lambda, 6\lambda-\dfrac{9}{2}, 5\lambda+\dfrac{8}{3}$.

As the line along PQ is parallel to the plane $4x+12y-3z+1=0$, therefore

$4(9\lambda)+12\left(6\lambda-\dfrac{9}{2}\right)-3\left(5\lambda+\dfrac{8}{3}\right)=0\Rightarrow\lambda=\dfrac{2}{3}$. [PQ will be $\perp^{er}$ to normal to plane

So the coordinates of point Q are given as, $Q\left(4,\dfrac{5}{2},2\right)$.

Required distance, $PQ=\sqrt{(4+2)^2+\left(\dfrac{5}{2}-3\right)^2+(2+4)^2}=\dfrac{17}{2}$ units.

Q11. Show that the points A, B and C with position vectors $\vec{a}=3\hat{i}-4\hat{j}-4\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-3\hat{j}-5\hat{k}$, respectively form the vertices of a right angled triangle.

Sol. We have $\overrightarrow{AB}=(2\hat{i}-\hat{j}+\hat{k})-(3\hat{i}-4\hat{j}-4\hat{k})=-\hat{i}+3\hat{j}+5\hat{k}$,

$\overrightarrow{BC}=(\hat{i}-3\hat{j}-5\hat{k})-(2\hat{i}-\hat{j}+\hat{k})=-\hat{i}-2\hat{j}-6\hat{k}$ and,

$\overrightarrow{CA}=(3\hat{i}-4\hat{j}-4\hat{k})-(\hat{i}-3\hat{j}-5\hat{k})=2\hat{i}-\hat{j}+\hat{k}$.

Since $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$ therefore, A, B and C are the vertices of a triangle.

Now, $\overrightarrow{AB}.\overrightarrow{CA}=(-\hat{i}+3\hat{j}+5\hat{k}).(2\hat{i}-\hat{j}+\hat{k})=0$ $\Rightarrow \overrightarrow{AB}\perp\overrightarrow{CA}$ $\Rightarrow \angle BAC=90^{o}$.

Hence the given points A, B and C are the vertices of a right angled triangle.

Q12. If the sum of two unit vectors is also a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

Sol. Let $\hat{a}$, $\hat{b}$ be the given unit vectors such that $\hat{a}+\hat{b}$ is also a unit vector.

So, $|\hat{a}+\hat{b}|^2=|\hat{a}|^2+2\hat{a}.\hat{b}+|\hat{b}|^2$ $\Rightarrow 2\hat{a}.\hat{b}=-1$ …(i) $[\because |\hat{a}|=|\hat{b}|=|\hat{a}+\hat{b}|=1$

Now, $|\hat{a}-\hat{b}|^2=|\hat{a}|^2-2\hat{a}.\hat{b}+|\hat{b}|^2=1-(-1)+1=3$ [Using (i)

$\therefore |\hat{a}-\hat{b}|=\sqrt{3}$.

Q13. Find the distance between the point P(6, 5, 9) and the plane determined by the points A(3,–1, 2), B(5, 2, 4) and C(–1,–1, 6).

Sol. Let the d.r.'s of normal to the plane be A, B, C.

The equation of plane passing through A(3,–1, 2) is A(x – 3) + B(y + 1) + C(z – 2) = 0…(i)

As (i) passes through the points B(5, 2, 4) and C(–1,–1, 6) so,

2A + 3B + 2C = 0 and

–4A + 0B + 4C = 0

Solving these equation by cross-multiplication method,

$\Rightarrow \dfrac{A}{12-0}=\dfrac{B}{-8-8}=\dfrac{C}{0+12}$ $\quad\therefore \dfrac{A}{3}=\dfrac{B}{-4}=\dfrac{C}{3}$

Replacing the proportionate values of d.r.'s A, B, C in (i), we get : $3x-4y+3z=19$.

Hence distance of P(6, 5, 9) from the plane is : $\dfrac{|3(6)-4(5)+3(9)-19|}{\sqrt{3^2+(-4)^2+3^2}}=\dfrac{6}{\sqrt{34}}$ units.

Q14. Show that the points having position vectors $\vec{a}-2\vec{b}+3\vec{c}$, $3\vec{b}-2\vec{a}+2\vec{c}$, $13\vec{b}-8\vec{a}$ are collinear, whatever $\vec{a}$, $\vec{b}$, $\vec{c}$ may be.

Sol. Let A, B, C be the points with position vectors $\vec{a}-2\vec{b}+3\vec{c}$, $3\vec{b}-2\vec{a}+2\vec{c}$, $13\vec{b}-8\vec{a}$ respectively.

Then $\overrightarrow{AB}=(3\vec{b}-2\vec{a}+2\vec{c})-(\vec{a}-2\vec{b}+3\vec{c})=-3\vec{a}+5\vec{b}-\vec{c}$,

and, $\overrightarrow{AC}=(13\vec{b}-8\vec{a})-(\vec{a}-2\vec{b}+3\vec{c})=-9\vec{a}+15\vec{b}-3\vec{c}$

Since $\overrightarrow{AC}=3\overrightarrow{AB}$ so, $\overrightarrow{AB}\parallel\overrightarrow{AC}$. But A is a common point.

Hence A, B, C are collinear points.

Q15. Find the direction cosines of the two lines which are connected by the relation $l-5m+3n=0$ and $7l^2+5m^2-3n^2=0$.

Sol. Given $l-5m+3n=0$ $l=5m-3n$ …(i) and, $7l^2+5m^2-3n^2=0$ …(ii)

By (i) & (ii), $7(5m-3n)^2+5m^2-3n^2=0$ $\Rightarrow 7(25m^2-30mn+9n^2)+5m^2-3n^2=0$

$\Rightarrow 180m^2-210mn+60n^2=0$ $\Rightarrow 6m^2-7mn+2n^2=0$

$\Rightarrow 6m^2-3mn-4mn+2n^2=0$ $\Rightarrow 3m(2m-n)-2n(2m-n)=0$

i.e., $(3m-2n)(2m-n)=0$ So, we get $m=\dfrac{2n}{3}$ or, $m=\dfrac{n}{2}$

If $m=\dfrac{2n}{3}$ then, by (i) we get : $l=\dfrac{n}{3}$ and, if $m=\dfrac{n}{2}$ then, by (i) we get : $l=-\dfrac{n}{2}$.

So, the d.r.'s of two lines are proportional to $\dfrac{n}{3},\dfrac{2n}{3},n$ i.e., 1, 2, 3 and $-\dfrac{n}{2},\dfrac{n}{2},n$ i.e., –1, 1, 2.

Q16. Find the shortest distance (S.D.) between the lines $\vec{r}=6\hat{i}+2\hat{j}+2\hat{k}+\lambda(\hat{i}-2\hat{j}+2\hat{k})$ and $\vec{r}=-4\hat{i}-\hat{k}+\mu(3\hat{i}-2\hat{j}-2\hat{k})$.

Sol. Let L_1 : $\vec{r}=6\hat{i}+2\hat{j}+2\hat{k}+\lambda(\hat{i}-2\hat{j}+2\hat{k})$ and L_2 : $\vec{r}=-4\hat{i}-\hat{k}+\mu(3\hat{i}-2\hat{j}-2\hat{k})$.

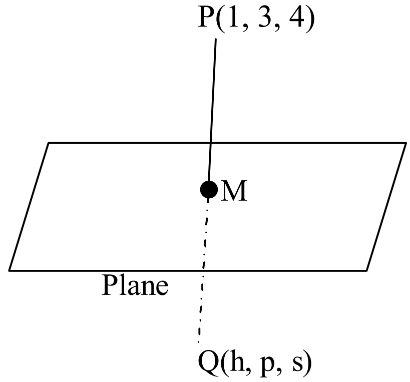
So, $\vec{a}_1=6\hat{i}+2\hat{j}+2\hat{k}, \vec{b}_1=\hat{i}-2\hat{j}+2\hat{k}, \vec{a}_2=-4\hat{i}-\hat{k}, \vec{b}_2=3\hat{i}-2\hat{j}-2\hat{k}$

$\Rightarrow \vec{a}_2-\vec{a}_1=-10\hat{i}-2\hat{j}-3\hat{k}, \vec{b}_1\times\vec{b}_2=8\hat{i}+8\hat{j}+4\hat{k}$ and $|\vec{b}_1\times\vec{b}_2|=\sqrt{8^2+8^2+4^2}=12$.

$$\therefore \text{S.D.}=\frac{|(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|}=\frac{|(-10\hat{i}-2\hat{j}-3\hat{k}).(8\hat{i}+8\hat{j}+4\hat{k})|}{12}=9\text{units}.$$

Q17. Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point (1, 3, 4) from the plane 2x – y + z + 3 = 0. Also find the image of the point in plane.

Sol. The d.r.'s of normal to the plane 2x – y + z + 3 = 0 are 2, –1, 1.



Let M be the foot of perpendicular from P(1, 3, 4) to the plane.

So the eq. of line PM is $\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-4}{1}$.

Hence $M(1+2\lambda, 3-\lambda, 4+\lambda)$ may be any arbitrary point on the line as well as the plane.

So M must satisfy the equation of plane i.e.

$2(1+2\lambda)-(3-\lambda)+(4+\lambda)+3=0 \Rightarrow \lambda=-1$.

$\therefore M(-1,4,3)$: Foot of perpendicular.

Perpendicular distance of P from plane is, $PM=\sqrt{(-1-1)^2+(4-3)^2+(3-4)^2}=\sqrt{6}$ units.

Now let Q(h, p, s) be the image of point P in the plane, then M must be the midpoint of PQ.

Therefore, $M(-1,4,3)=M\left(\frac{1+h}{2},\frac{3+p}{2},\frac{4+s}{2}\right) \Rightarrow Q(-3,5,2)$: Image of point P.

Q18. A variable plane which remains at a constant distance 3p units from the origin cut the coordinate axes at A, B and C respectively. Show that the locus of the centroid of the triangle ABC is $x^{-2}+y^{-2}+z^{-2}=p^{-2}$.

Sol. Let the equation of plane be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ ...(i)

Plane (i) meets the coordinate axes at A(a, 0, 0), B(0, b, 0) and C(0, 0, c) respectively.

Let (α,β,γ) be the coordinates of the centroid of the triangle ABC.

$$\therefore \alpha=\frac{a+0+0}{3}=\frac{a}{3}, \beta=\frac{0+b+0}{3}=\frac{b}{3}, \gamma=\frac{0+0+c}{3}=\frac{c}{3} \qquad \Rightarrow a=3\alpha, b=3\beta, c=3\gamma \text{ ...(ii)}$$

The plane (i) remains at 3p units away from the origin.

$$\therefore 3p=\frac{\left|\frac{0}{a}+\frac{0}{b}+\frac{0}{c}-1\right|}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}} \qquad \Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{9p^2}$$

By (ii), $\frac{1}{(3\alpha)^2}+\frac{1}{(3\beta)^2}+\frac{1}{(3\gamma)^2}=\frac{1}{9p^2} \Rightarrow \frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=\frac{1}{p^2}$

So the locus of (α,β,γ) is, $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$ i.e., $x^{-2}+y^{-2}+z^{-2}=p^{-2}$.

Q19. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitudes. Show that $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.

Sol. Let $|\vec{a}|=|\vec{b}|=|\vec{c}|=r$ ...(i)

Since $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors, therefore $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$ ...(ii)

Now $|\vec{a}+\vec{b}+\vec{c}|^2=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=r^2+r^2+r^2+2(0)$ [By (i) & (ii)

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|=\sqrt{3r^2}=\sqrt{3}r$ ...(iii)

Assume $\vec{a}+\vec{b}+\vec{c}$ makes angles α,β,γ with $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.

Then, $\cos\alpha=\frac{\vec{a}.(\vec{a}+\vec{b}+\vec{c})}{|\vec{a}|\,|\vec{a}+\vec{b}+\vec{c}|}=\frac{|\vec{a}|^2+\vec{a}.\vec{b}+\vec{a}.\vec{c}}{|\vec{a}|\,|\vec{a}+\vec{b}+\vec{c}|}=\frac{r^2}{r(\sqrt{3}r)}$ [Using (i), (ii) & (iii)

$\Rightarrow \alpha = \cos^{-1}\dfrac{1}{\sqrt{3}}$.

Similarly, $\cos\beta = \dfrac{\vec{b}.(\vec{a}+\vec{b}+\vec{c})}{|\vec{b}|\,|\vec{a}+\vec{b}+\vec{c}|} = \dfrac{\vec{b}.\vec{a}+|\vec{b}|^2+\vec{b}.\vec{c}}{|\vec{b}|\,|\vec{a}+\vec{b}+\vec{c}|} = \dfrac{r^2}{r(\sqrt{3}r)} \Rightarrow \beta = \cos^{-1}\dfrac{1}{\sqrt{3}}$

And, $\cos\gamma = \dfrac{\vec{c}.(\vec{a}+\vec{b}+\vec{c})}{|\vec{c}|\,|\vec{a}+\vec{b}+\vec{c}|} = \dfrac{\vec{c}.\vec{a}+\vec{c}.\vec{b}+|\vec{c}|^2}{|\vec{c}|\,|\vec{a}+\vec{b}+\vec{c}|} = \dfrac{r^2}{r(\sqrt{3}r)} \Rightarrow \gamma = \cos^{-1}\dfrac{1}{\sqrt{3}}$

$\because \alpha=\beta=\gamma=\cos^{-1}\dfrac{1}{\sqrt{3}}$, so $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.

Q20. Find the image of the point (1, 6, 3) in the line $6x = 3(y-1) = 2(z-2)$.

Sol. Let P(1, 6, 3) be the given point and M be the foot of perpendicular from P to the given line $6x = 3(y-1) = 2(z-2)$ i.e., $\dfrac{x}{1} = \dfrac{y-1}{2} = \dfrac{z-2}{3} = \lambda$ (say) …(i)

The coordinates of any random point on (i) is M($\lambda, 2\lambda+1, 3\lambda+2$).

The direction ratios of PM are $\lambda-1, 2\lambda-5, 3\lambda-1$

The d.r.'s of line (i) are 1, 2, 3.

Since PM is $\perp^{er}$ to line (i) so, using $a_1a_2+b_1b_2+c_1c_2=0$ we get :

$1(\lambda-1)+2(2\lambda-5)+3(3\lambda-1)=0 \Rightarrow \lambda=1$

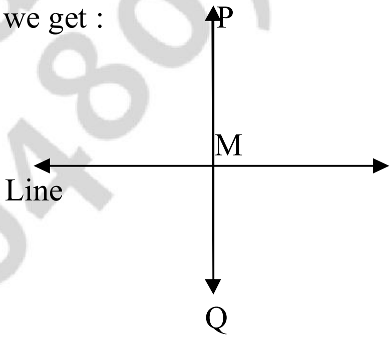
So, Foot of perpendicular : M(1, 3, 5)

Let Q(h, p, s) be the image of P in the line (i).

Then M will be the mid-point of PQ.

i.e., M(1, 3, 5) $= M\left(\dfrac{1+h}{2}, \dfrac{6+p}{2}, \dfrac{3+s}{2}\right)$

$\therefore$ Image of point P in line (i) is : Q(1, 0, 7).



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☞ For any clarification(s), please contact :

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