❖ A Help Guide On ❖
PROBABILITY-THEORY OF CHANCES

A Help Guide By OP Gupta (Indira Award Winner)

Q01. In a group of 200 people, 50% believe in that anger and violence will ruin the country, 30% do not believe in that anger and violence will ruin the country and 20% are not sure about anything. If 3 people are selected at random find the probability that 2 people believe and 1 does not believe that anger and violence will ruin the country. How do you consider that anger and violence will ruin the country?

Sol. Let A = those people who believe in that anger and violence will ruin the country,
B = those who do not believe in that anger and violence will ruin the country
and, C = those who are not sure about anything
$\therefore$ P(A)= 50/100; P(B) = 30/100; P(C) = 20/100,
So, P(2 people believe and 1 does not believe) $= {}^3C_2 \times P(A)\ P(A)\ P(B)$

$$\Rightarrow \quad = \frac{50}{100} \times \frac{50}{100} \times \frac{30}{100} \times 3 = \frac{9}{40}$$

Values : People in anger cannot use their presence of mind and become violent. They destroy public property in riots which is indirectly their own property.

Q02. Probability of winning by a cricket team are $\frac{1}{2}$ and $\frac{1}{3}$ respectively when batting coach A and bowling coach B work independently. If both try for the win independently, find the probability that there is a win. Will working independently may be effective? And why?

Sol. Given $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(\overline{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$, $P(\overline{B}) = 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore$ P(winning) = P(A) P($\overline{B}$) + P($\overline{A}$) P(B) + P(A) P(B)

$$= \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{3} = \frac{4}{6}$$

Values : Working independently may be effective as (i) ideas flow independently (ii) Hard work by both the coaches will pay success for the team.

Q03. A drunkard man takes a step forward with probability 0.4 and takes a step backward with probability 0.6. He takes 11 steps in all. Find the probability that he is just one step away from the initial point. Do you think drinking habit can ruin one's family life?

Sol. Since the man is just one step away from the initial point, he is either a step forward or a step backward from the initial point at the end of nine steps. **Case I :** If he is one step forward, then he must have taken five steps in forward direction while 4 steps in backward direction. **Case II :** If he is one step backward, then he must have taken four steps in forward and five steps in backward direction.

So required probability = P(X = 5 or 4) = P(5) + P(4) = $126 \times (0.6)^4 (0.4)^4 = 126 \times (0.24)^4$.

Values : Yes, addiction of wine or smoking is definitely harmful for a person and its family.

Q04. If group A contains the students who try to solve the problem by knowledge, Group B contains the students who guess to solve the problem Group C contains the students who give answer by cheating. Given that n(A) = 20, n(B) = 15, n(C) = 10. Two students are selected at random. Find the probability that they are from Group C. Do you think that cheating habit spoils the career?

Sol. Since n(A) = 20, n(B) = 15, n(C) = 10, $\therefore$ n(S) = 45

$$\text{Required probability} = \frac{{}^{10}C_2}{{}^{45}C_2} = \frac{1}{22}.$$

Values : Yes, cheating habit spoils the career because a cheater finds it difficult to do any work independently. It may benefit the cheater in present but, it is harmful in long run.

Q05. A clever student used a biased coin so that the head is 3 times as likely to occur as tail. If the coin tossed twice find the probability distribution and mean of numbers of tails. Is this a good tendency? Justify your answer.

Sol. Let P(H) = 3/4, P(T) = 1/4. Let X : no. of tails, $\therefore$ X : 0, 1, 2

X	0	1	2
P(X)	(3/4)(3/4) = 9/16	2(3/4)(1/4) = 6/16	(1/4)(1/4) = 1/16

$$\text{Mean} = \sum X\ P(X) = 0.P(0) + 1.P(1) + 2.P(2) = \frac{1}{2}.$$

Values : No, it's not a good practice. Honesty pays in a long run.

Q06. Out of a group of 8 highly qualified doctors in a hospital, 6 are very kind and cooperative with their patients and so are very popular, while the other two remain reserved. For a health camp, three doctors are selected at random. Find the probability distribution of the number of very popular doctors. What values are expected from the doctors?

Sol. Let X : number of very popular doctors. So, X = 1, 2, 3.

No. of kind, cooperative and popular doctors : 6, No. of doctors who remain reserved : 2.

X	1	2	3
P(X)	$\frac{{}^6C_1 \times {}^2C_2}{{}^8C_3} = \frac{3}{28}$	$\frac{{}^6C_2 \times {}^2C_1}{{}^8C_3} = \frac{15}{28}$	$\frac{{}^6C_3}{{}^8C_3} = \frac{10}{28}$

Values : A doctor is expected to be qualified, kind and cooperative with the patients.

Q07. There are 20 people in a group. Out of them 7 people are non-vegetarian, 2 people are selected randomly. Write the probability distribution of non-vegetarian people. Explain whether you would like to be vegetarian or non-vegetarian and why? Also keeping life of animals in mind how would you promote a person to be vegetarian?

Sol. P(vegetarian people) = 13/20, P(non-vegetarian) = 7/20

Let X : No. of non-vegetarian people s.t., X = 0, 1, 2.

X	0	1	2
P(X)	$^2C_0\left(\frac{13}{20}\times\frac{12}{19}\right) = \frac{156}{380}$	$^2C_1\left(\frac{13}{20}\times\frac{7}{19}\right) = \frac{182}{380}$	$^2C_1\left(\frac{7}{20}\times\frac{6}{19}\right) = \frac{42}{380}$

Values : I would like to be a vegetarian because vegetarian food is much easier to digest than non-vegetarian. Also for non-vegetarian food we have to kill animals this is not good thing because every living species has right to life.

Q08. A chairman is biased so that he selects his relatives for a job 3 times as likely as others. If there are 3 posts for a job, find the probability distribution for selection of persons other than their relatives. If the chairman is biased than which value of life will be demolished?

Sol. Let P(selecting relatives, i.e., biased) = 3/4, P(selecting non relatives, i.e., not biased) = 1/4,
Let X : Event of selecting non relatives so, X= 0, 1, 2, 3.

X	0	1	2	3
P(X)	$^3C_0\left(\frac{3}{4}\right)^3 = \frac{27}{64}$	$^3C_1\left(\frac{1}{4}\times\frac{3}{4}\times\frac{3}{4}\right) = \frac{27}{64}$	$^3C_2\left(\frac{1}{4}\times\frac{1}{4}\times\frac{3}{4}\right) = \frac{9}{64}$	$^3C_3\left(\frac{1}{4}\right)^3 = \frac{1}{64}$

Values lost by chairman are Honesty, Integrity.

Q09. In a group of 100 families, 30 families like male child, 25 families like female child and 45 families feel both children are equal. If two families are selected at random out of 100 families, find the probability distribution of the number of families feeling both children are equal. What is the importance in the society to develop the feeling that both the children, whether male or female, are equal?

Sol. Let P(families like male child) = 30/100, P(families like female child) = 25/100,
P(families like any one of the child) = 55/100 = 11/20,
P(families like both child) = 45/100 = 9/20
Let X : Event of liking both children so, X = 0, 1, 2.

X	0	1	2
P(X)	$^2C_0\left(\frac{11}{20}\times\frac{11}{20}\right)=\frac{121}{400}$	$^2C_1\left(\frac{11}{20}\times\frac{9}{20}\right)=\frac{198}{400}$	$^2C_2\left(\frac{9}{20}\times\frac{9}{20}\right)=\frac{81}{400}$

Values : In order to maintain the ratio of male and female equally, it's important to consider both children are equal.

Q10. A person has undertaken a construction job. The probabilities are 0.65 that there will be strike, 0.80 that the construction job will be completed on time if there is no strike and 0.32 that the construction job will be completed on time if there is strike. Determine the probability that the construction job will be completed on time. What values are driven by this question?

Sol. Let E_1 : there will be a strike, E_2 : there will be no strike, A : the construction job will be completed in time.

So, $P(E_1) = 0.65$, $P(A|E_1) = 0.32$, $P(E_2) = 0.35$, $P(A|E_2) = 0.80$.

$\therefore \quad P(A) = P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)$

$\Rightarrow \qquad = (0.65)(0.32) + (0.35)(0.80) = 0.488$

Values : Peace is better than strike. As the probability of completion of job on time if there is strike is less then 0.5 . So it is clear that the production rises in the time of peaceful work.

[This Q is based on Total Probability and is an example from NCERT Textbook. For more such kind of questions, you may Buy Study Package.]

Q11. A man is known to speak truth 5 out of 6 times. He draws a ball from the bag containing 4 white and 6 black balls and reports that it is white. Find the probability that it is actually white? Do you think that speaking truth is always good?

Sol. Let E_1 : man is speaking truth, E_2 : man is not speaking truth, A : he reports the ball as white.

$\therefore$ $P(E_1) = 5/6$, $P(A|E_1) = 4/10$, $P(E_2) = 1/6$, $P(A|E_2) = 6/10$.

By Bayes' Theorem, $P(E_1 \mid A) = \dfrac{P(E_1)P(A \mid E_1)}{P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)}$

$$\Rightarrow \qquad = \frac{\frac{5}{6}\times\frac{4}{10}}{\frac{5}{6}\times\frac{4}{10}+\frac{1}{6}\times\frac{6}{10}} = \frac{20}{26}.$$

Values : Speaking truth pays in the long run. Although speaking lie sometimes for a good cause is not bad.

Q12. In a school, 30% of the student has 100% attendance. Previous year result report tells that 70% of all students having 100% attendance attain A grade and 10% of remaining students attain A grade in their annual examination. At the end of the year, One student is chosen at random and he has an A grade. What is the probability that the student has 100% attendance? Also state the factors which affect the result of a student in the examination.

Sol. Let E_1 : students having 100% attendance, E_2 : students not having 100% attendance and, A : getting A grade by the students.

So, $P(E_1) = \dfrac{30}{100}$, $P(A|E_1) = \dfrac{70}{100}$, $P(E_2) = \dfrac{70}{100}$, $P(A|E_1) = \dfrac{10}{100}$.

By Bayes' Theorem, $P(E_1|A) = \dfrac{P(E_1)P(A \mid E_1)}{P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)}$

$$\Rightarrow \qquad = \frac{\frac{30}{100}\times\frac{70}{100}}{\frac{30}{100}\times\frac{70}{100}+\frac{70}{100}\times\frac{10}{100}} = \frac{21}{28}$$

Values : Factors affecting the result of a student are (i) Regular study (ii) Hard work (iii) Well time management (iv) Writing skills etc.

Q13. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is six. Find the probability that it is actually a six. Write any three benefits of speaking the truth.

Sol. Let E_1 : man is speaking truth, E_2 : man is speaking lie, A : actually six appears.

So, $P(E_1) = \frac{3}{4}$, $P(A|E_1) = \frac{1}{6}$, $P(E_2) = \frac{1}{4}$, $P(A|E_2) = \frac{5}{6}$.

By Bayes' Theorem, $P(E_1 \mid A) = \frac{P(E_1)P(A \mid E_1)}{P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)} = \frac{\frac{3}{4} \times \frac{1}{6}}{\frac{3}{4} \times \frac{1}{6} + \frac{1}{4} \times \frac{5}{6}} = \frac{3}{8}$

Values : (i) Everyone likes a person speaking the truth. (iii) It is good life skill.

Q14. Two third of the students in a class are sincere about their study and rest are careless. Probability of passing in examination are 0.7 and 0.2 for sincere and careless students 9 respectively, A Student is chosen and is found to be passed what is the probability that he/she was sincere. Explain the importance of sincerity for a student.

Sol. Let E_1 : sincere students, E_2 : careless students, A : passing the exam.

So, $P(E_1) = \frac{2}{3}$, $P(A|E_1) = \frac{7}{10}$, and $P(E_2) = \frac{1}{3}$, $P(A|E_2) = \frac{2}{10}$.

By Bayes' Theorem, $P(E_1 \mid A) = \frac{P(E_1)P(A \mid E_1)}{P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)} = \frac{\frac{2}{3} \times \frac{7}{10}}{\frac{2}{3} \times \frac{7}{10} + \frac{1}{3} \times \frac{2}{10}} = \frac{14}{16}$.

Values : A sincere student is true to his responsibilities. As the future of any society lies on the students, so a sincere student carries this duty to make a better world around.

Q15. A manufacturer has three machine operators A (skilled), B (semi-skilled), and C (non-skilled). The first operator A produces 1% defective items where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of time B in the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced what is the probability that it was produced by B? What is the value of skill?

Sol. Let E_1 : item is produced by A, E_2 : item is produced by B, E_3 : item is produced by C and, A : defective item is produced.

$\therefore$ $P(E_1) = 50\%$, $P(A|E_1) = 1\%$, $P(E_2) = 30\%$, $P(A|E_2) = 5\%$, $P(E_3) = 20\%$, $P(A|E_3) = 7\%$,.

By Bayes' Theorem, $P(E_2 \mid A) = \frac{P(E_2)P(A \mid E_2)}{P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2) + P(E_3)P(A \mid E_3)}$

$$\Rightarrow \quad = \frac{\frac{3}{10} \times \frac{5}{100}}{\frac{5}{10} \times \frac{1}{100} + \frac{3}{10} \times \frac{5}{100} + \frac{2}{10} \times \frac{7}{100}} = \frac{15}{34}.$$

Values : Skill plays an important part in quality product. Skilled person can complete a work in better way than a non-skilled person.

Q16. In a group of students, 200 attend coaching classes, 400 students attend school regularly and 600 students study themselves with help of peers. The probability that a student will succeed in life who attend coaching classes, attend school regularly and study themselves with help of peers are 0.1, 0.2 and 0.5 respectively. One student is selected who succeeded in life, what is the probability that he study himself with help of peers. What type of study can be considered for the success in life and why?

Sol. Let E_1 : students attending coaching classes, E_2 : students attending school regularly, E_3 : students doing self study, and A : the students who succeeded in life.

$\therefore P(E_1) = \frac{200}{1200} = \frac{2}{12}$, $P(A|E_1) = \frac{1}{10}$, $P(E_2) = \frac{400}{1200} = \frac{4}{12}$, $P(A|E_2) = \frac{2}{10}$, $P(E_3) = \frac{600}{1200} = \frac{6}{12}$, $P(A|E_3) = \frac{5}{10}$.

By Bayes' Theorem, $P(E_3 \mid A) = \frac{P(E_3)P(A \mid E_3)}{P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2) + P(E_3)P(A \mid E_3)}$

$$\Rightarrow \quad = \frac{\frac{6}{12}\times\frac{5}{10}}{\frac{2}{12}\times\frac{1}{10}+\frac{4}{12}\times\frac{2}{10}+\frac{6}{12}\times\frac{5}{10}} = \frac{30}{40}$$

Values : Self studies with the help of peers is best as it helps the students in getting in depth knowledge of each concept. The students should be regular in school and if they feel the need the they could join different classes on their wish.

Q17. A shopkeeper sells three types of flower seeds A_1, A_2 and A_3. They are sold as a mixture where the proportions are 4:4:2 respectively. The germination rates of three types of seeds are 45%, 60% and 35%. Calculate the probability
(a) of a randomly chosen seed to germinate.
(b) that it is of type A_2, given that a randomly chosen seed doesn't germinate.

Sol. Let E_1 : randomly selected seed is of type A_1, E_2 : randomly selected seed is of type A_2, and E_3 : randomly selected seed is of type A_3.
$\therefore$ $P(E_1) = 4/10$, $P(E_2) = 4/10$, $P(E_3) = 2/10$.
(i) Let A : selected seed germinates.
$\therefore P(A|E_1) = 45/100$, $P(A|E_2) = 60/100$, $P(A|E_3) = 35/100$
So, $P(A) = P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2) + P(E_3)\,P(A|E_3) = 49/100$ or 0.49
(ii) Let A : selected seed does not germinates.
$\therefore P(A|E_1) = 55/100$, $P(A|E_2) = 40/100$, $P(A|E_3) = 65/100$

$$\text{So, } P(E_2|A) = \frac{P(E_2)\,P(A\,|\,E_2)}{P(E_1)\,P(A\,|\,E_1) + P(E_2)\,P(A\,|\,E_2) + P(E_3)\,P(A\,|\,E_3)} = \frac{16}{51}.$$

Q18. Assume that the chances of a patient having heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time, a patient can chose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course a course of meditation and yoga. Interpret the result and state which of the above stated methods is more beneficial for the patient.

Sol. Let A be the event that the patient follows a course of meditation and yoga, B be the event that he takes a certain drug.
Then, $P(A) = 1/2$, $P(B) = 1/2$.
Let E be the event that the patient suffers a heart-attack.

$$\text{Also } P(E|A) = \frac{70}{100}.\frac{40}{100},\ P(E|B) = \frac{75}{100}.\frac{40}{100}.$$

By Bayes' Theorem, we get :

$$\text{Required probability, } P(A\,|\,E) = \frac{P(E\,|\,A)P(A)}{P(E\,|\,A)P(A) + P(E\,|\,B)P(B)}$$

$$= \frac{\left(\frac{70}{100}.\frac{40}{100}\right)\times\frac{1}{2}}{\left(\frac{70}{100}.\frac{40}{100}\right)\times\frac{1}{2} + \left(\frac{75}{100}.\frac{40}{100}\right)\times\frac{1}{2}}$$

$$\therefore \quad P(A\,|\,E) = \frac{14}{29}.$$

Interpretation of result: It is evident that if a patient follows a course of meditation and yoga, then he is less likely to get heart-attack [since $P(B|E) = 15/29$] So, clearly a course of meditation and yoga is more beneficial as compared to the intake of drugs.

Q19. A company has two plants of scooter manufacturing. Plant I manufacture 70% Scooter and plant II manufactures 30%. At plant I, 80% of the scooter's are maintaining pollution norms and in plant II, 90% of the scooter maintaining pollution norms. A Scooter is chosen at

random and is found to be fit on pollution norms. What is the probability that it has come from plant II. What is importance of pollution norms for a vehicle?

Sol. Let E_1 : scooter manufactured in plant I, E_2 : scooter manufactured in plant II, A : the scooter fits on the pollution norms.

$$\therefore\ P(E_1) = \frac{70}{100},\ P(A|E_1) = \frac{80}{100},\ P(E_2) = \frac{30}{100},\ P(A|E_2) = \frac{90}{100}.$$

$$\text{By Bayes' Theorem, } P(E_2 \mid A) = \frac{P(E_2)P(A \mid E_2)}{P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)} = \frac{\frac{3}{10}\times\frac{9}{10}}{\frac{7}{10}\times\frac{8}{10} + \frac{3}{10}\times\frac{9}{10}} = \frac{27}{83}$$

Values : Pollution norms have a check on the pollution level in the vehicles. Pollution free environment minimize the health problems in the human being.

Q20. In a group of 400 people, 160 are smokers and non-vegetarian, 100 are smokers and vegetarian and the remaining are non-smokers and vegetarian. The probabilities of getting a special chest diseases are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the disease. What is the probability that the selected person is a smoker and non-vegetarian? What value is reflected in this question?

Sol. Let E be the event of persons suffering from special chest disease.
Let E_1 be the event of those people who are smokers and non-vegetarian.
Let E_2 be the event of those people who are smokers and vegetarian.
Let E_3 be the event of those people who are non-smokers and vegetarian.

$$\text{We have } P(E_1) = \frac{160}{400}, P(E_2) = \frac{100}{400}, P(E_3) = \frac{140}{400}$$

$$\text{And } P(E \mid E_1) = \frac{35}{100}, P(E \mid E_2) = \frac{20}{100}, P(E \mid E_3) = \frac{10}{100}.$$

By using Bayes' theorem, we have :

$$P(E_1 \mid E) = \frac{P(E \mid E_1)P(E_1)}{P(E \mid E_1)P(E_1) + P(E \mid E_2)P(E_2) + P(E \mid E_3)P(E_3)}$$

$$\Rightarrow P(E_1 \mid E) = \frac{\frac{35}{100}\times\frac{160}{400}}{\frac{35}{100}\times\frac{160}{400} + \frac{20}{100}\times\frac{100}{400} + \frac{10}{100}\times\frac{140}{400}} = \frac{35\times16}{35\times16 + 20\times10 + 10\times14}$$

$$\therefore\ P(E_1 \mid E) = \frac{28}{45}.$$

Values: One should avoid smoking as it can be the reason of several dangerous diseases.