

OPG'S

Practice Problem Series

Based on Derivatives (Advanced Level)

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❖ Differentiate the following functions with respect to x [Q01 – Q47] :

- Q01. $3^{2\log_3 x}$ Q02. $\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$, $x \in (0, \pi) - \frac{\pi}{2}$ Q03. 3^{x+3}
- Q04. $\log\left(\frac{1}{\sqrt{x}}\right)$ Q05. $\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)^2$ Q06. $x^a - a^x$
- Q07. $\frac{2-3\sin x}{\cos x}$ Q08. $e^{2x}(x^2 + \log 2x)$ Q09. $\frac{x^3 \sin x}{\cos x}$
- Q10. $x \sin x \log x$ Q11. $x^n \log x$ Q12. $\log_{x^2}(x)$
- Q13. $\frac{x + \cos x}{x - \sin x}$ Q14. $\frac{1 + \tan x^\circ}{1 - \tan x^\circ}$ Q15. $\frac{\sec x + \tan x}{\sec x - \tan x}$
- Q16. $\frac{\sin x - x \cos x}{x \sin x + \cos x}$ Q17. $\frac{1}{px^2 + qx + r}$ Q18. $\frac{x \tan x}{(\sec x + \tan x)}$
- Q19. $\frac{x^2 - x + 1}{x^2 + x + 1}$ Q20. $\frac{\sqrt{a} - \sqrt{x}}{\sqrt{a} + \sqrt{x}}$ Q21. $\frac{10^x}{\tan x}$
- Q22. $\frac{1 + \log x}{1 - \log x}$ Q23. $\frac{1 + 5^x}{1 - 5^x}$ Q24. $\sin(e^{x^2})$
- Q25. $\log(\sec x + \tan x)$ Q26. $e^{x \sin x} + \sin x^e$ Q27. $e^x - \cos \sqrt{e^{2x}} + e^{3 \log x}$
- Q28. $\log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ Q29. $\log \tan \frac{x}{2}$ Q30. $\log \sqrt{\sin\left(\frac{x^2}{3} - 1\right)}$
- Q31. $\log(x + \sqrt{x^2 + a^2})$ Q32. $\log(x + \sqrt{x^2 - a^2})$ Q33. $(x + \sqrt{x^2 + a^2})^n$
- Q34. $\frac{\sqrt{a^2 + x^2} + \sqrt{a^2 - x^2}}{\sqrt{a^2 + x^2} - \sqrt{a^2 - x^2}}$ Q35. $\sqrt{\frac{1-x}{1+x}}$ Q36. $\log\left(\frac{\sin x}{1 + \cos x}\right)$
- Q37. $\tan e^{\sin x}$ Q38. $\frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$ Q39. $\sin^2 \log(3x - 2)$
- Q40. $(\log \cos x)^2$ Q41. $\cos \log \sin x$ Q42. $\log x^x$
- Q43. $x + 3^{\sqrt{\cot \log x}}$ Q44. $\sin \sqrt{\tan x + \cot x}$ Q45. $\cos\left(\frac{1-x^2}{1+x^2}\right)$
- Q46. $\log_7 x$ Q47. $\log_x x$
- Q48. If $y = \sqrt{x^2 + a^2}$, then show that $y \frac{dy}{dx} - x = 0$.
- Q49. If $y = e^x + e^{-x}$, then show that $\frac{dy}{dx} = \sqrt{y^2 - 4}$.

Q50. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then prove that $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$.

Q51. Find the rate at which the function $f(x) = x^3 - 3x^2 + 5x - 7$ changes with respect to x .

Q52. If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$, show that $\frac{dy}{dx} = y$.

Q53. If $y = \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}$ then, show that $2xy \frac{dy}{dx} = \frac{x}{a} - \frac{a}{x}$.

Q54. If $y = \frac{x}{x+2}$, prove that $x \frac{dy}{dx} = y(1-y)$.

Q55. Using induction, prove that $\frac{d}{dx}(x^n) = nx^{n-1}$ for all $n \in \mathbb{N}$.

Note that the derivative of logarithmic function (i.e., $\log x$) is not in the syllabus of class XI. However if you memorize $\frac{d}{dx}(\log x) = \frac{1}{x}$ and consider the rules like chain rule, product rule etc. as you've studied, you could be able to solve the questions involving logarithmic functions. [By the way, it shall be coming up in class XII.]

❖ ANSWERS ❖

- Q01. Q01. $2x$ Q02. $\sec^2 x$, if $0 < x < \frac{\pi}{2}$ and $-\sec^2 x$, if $\frac{\pi}{2} < x < \pi$
- Q03. $3^{x+3} \log_e 3$ Q04. $-\frac{1}{2x}$ Q05. $\cos x$
- Q06. $ax^{a-1} - a^x \log_e a$ Q07. $\sec x(2 \tan x - 3 \sec x)$ Q08. $e^{2x} [2x^2 + 2 \log 2x + 2x + \frac{1}{x}]$
- Q09. $x^3 \sec^2 x + 3x^2 \tan x$ Q10. $\sin x + x \cos x \log x + \sin x \log x$ Q11. $x^{n-1} [1 + n \log x]$
- Q12. 0 Q13. $\frac{1 - \cos x - \sin x + x \cos x - x \sin x}{(x - \sin x)^2}$ Q14. $\frac{\pi}{180} \sec^2 \left(\frac{\pi}{4} + x^0 \right)$
- Q15. $\frac{2 \cos x}{(1 - \sin x)^2}$ Q16. $\frac{x^2}{(x \sin x + \cos x)^2}$ Q17. $-\frac{2px + q}{(px^2 + qx + r)^2}$
- Q18. $\frac{(\sec x + \tan x)(x \sec^2 x + \tan x) - x \sec x \tan x (\tan x + \sec x)}{(\sec x + \tan x)^2}$ Q19. $\frac{2(x^2 - 1)}{(x^2 + x + 1)^2}$
- Q20. $-\sqrt{\frac{a}{x}} \cdot \frac{1}{[\sqrt{a} + \sqrt{x}]^2}$ Q21. $10^x (\cot x \cdot \log_e 10 - \operatorname{cosec}^2 x)$ Q22. $\frac{2}{x} \cdot \frac{1}{(1 - \log x)^2}$
- Q23. $\frac{5^x \log_e 25}{(1 - 5^x)^2}$ Q24. $2xe^{x^2} \cos(e^{x^2})$ Q25. $\sec x$
- Q26. $e^{x \sin x} (x \cos x + \sin x) + ex^{e-1} \cos x^e$ Q27. $e^{e^x + x} + \sqrt{e^{2x}} \sin \sqrt{e^{2x}} + 3x^2$
- Q28. $\sec x$ Q29. $\operatorname{cosec} x$ Q30. $\frac{x}{3} \cot \left(\frac{x^2 - 1}{3} \right)$
- Q31. $\frac{1}{\sqrt{x^2 + a^2}}$ Q32. $\frac{1}{\sqrt{x^2 - a^2}}$ Q33. $\frac{n(x + \sqrt{x^2 + a^2})^n}{\sqrt{x^2 + a^2}}$
- Q34. $\frac{-4a^2 x}{\sqrt{a^4 - x^4}}$ Q35. $-\frac{1}{(1+x)\sqrt{1-x^2}}$ Q36. $\cot x + \frac{\sin x}{1 + \cos x}$
- Q37. $e^{\sin x} \sec^2 e^{\sin x} \cos x$ Q38. $-\frac{8e^{4x}}{(e^{4x} - 1)^2}$ Q39. $\frac{3 \sin 2 \log(3x - 2)}{3x - 2}$
- Q40. $-2 \tan x \log \cos x$ Q41. $-\cot x \sin \log \sin x$ Q42. $\log(e x)$
- Q43. $1 - \frac{3^{\sqrt{\cot \log x}} (\log 3) \cdot [\operatorname{cosec}^2(\log x)]}{2x \sqrt{\cot \log x}}$ Q44. $\frac{\cos \sqrt{\tan x + \cot x}}{2\sqrt{\tan x + \cot x}} [\sec^2 x - \operatorname{cosec}^2 x]$
- Q45. $\frac{4x}{(1+x^2)^2} \sin \left(\frac{1-x^2}{1+x^2} \right)$ Q46. $\frac{\log_7 e}{x}$ Q47. 0 Q51. $3x^2 - 6x + 5.$

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