Complete Study Guide & Notes On
GENERAL USEFUL FORMULAE

A Formulae Guide By OP Gupta (Indira Award Winner)

If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is.

IMPORTANT TERMS, DEFINITIONS & RESULTS

01. Indices (Laws Of Exponents):

a) $a^m.a^n = a^{m+n}$ **b)** $\frac{a^m}{a^n} = a^{m-n}$ **c)** $\left(a^m\right)^n = a^{mn}$ **d)** $a^{-n} = \frac{1}{a^n}$

e) $a^0 = 1$ **f)** $a^m.b^m = \left(ab\right)^m$ **g)** $\left(\frac{a}{b}\right)^m = \left(\frac{b}{a}\right)^{-m}$.

02. Componendo & Dividendo:

If $\frac{a}{b} = \frac{c}{d}$ then we have, $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ and $\frac{a-b}{a+b} = \frac{c-d}{c+d}$.

03. Solving of a Quadratic Equation:
Consider a quadratic equation of the form, $ax^2+bx+c=0$ then, its roots are given by

$$x = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and, } x = \frac{-b-\sqrt{b^2-4ac}}{2a} \quad where\ D = b^2-4ac \ .$$

04. Logarithmic Relations:

a) $log_a p = \frac{log_b p}{log_b a}$ **b)** $log_a p = \frac{1}{log_p a}$

c) $log_a\left(m^n\right) = nlog_a\left(m\right)$ **d)** $log_a\left(m.n\right) = log_a\left(m\right)+log_a\left(n\right)$

e) $log_a\left(\frac{m}{n}\right) = log_a\left(m\right)-log_a\left(n\right)$ **f)** $log_a a = 1$

g) $log_a a^p = plog_a a = p$ **h)** $a^{log_a f(x)} = f(x)$.

05. Exponential Series:

a) $a^x = 1+x.\left(log_e a\right)+\frac{x^2}{2!}.\left(log_e a\right)^2+...+\frac{x^n}{n!}.\left(log_e a\right)^n+...\infty$

b) $e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+...+\frac{x^n}{n!}+...\infty$ for all x.

06. Logarithmic Series:

a) $log_e\left(1+x\right) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+...\infty,\ -1<x\le 1$

b) $log_e\left(1-x\right) = -x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}-...\infty,\ -1\le x<1$.

07. Binomial Expansions:

a) $\left(a+b\right)^n = {}^nC_0 a^{n-0}b^0+{}^nC_1 a^{n-1}b^1+{}^nC_2 a^{n-2}b^2+...+{}^nC_r a^{n-r}b^r+...+{}^nC_n a^{n-n}b^n$ if $n\in Z^+$.

b) $\left(1+x\right)^n = 1+{}^nC_1 x+{}^nC_2 x^2+...+{}^nC_r x^r+...+{}^nC_n x^n$ if n is a *positive integer*.

c) $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + ...\infty$ such that $-1 < x < 1$ and $n \in Z^-$ or $n \in Q$.

d) $\frac{x^n - a^n}{x-a} = x^{n-1} + a.x^{n-2} + a^2.x^{n-3} + ... + a^{n-1}$.

08. Trigonometric Series:

a) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...\infty$

b) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ...\infty$

c) $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + ...\infty$.

09. Sum of Special Sequences:

a) Sum of first *n* natural numbers: $1+2+3+...+n = \sum_{r=1}^{n} r = \frac{n(n+1)}{2}$

b) Sum of squares of first *n* natural numbers: $1^2+2^2+3^2+...+n^2 = \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$

c) Sum of cubes of first *n* natural numbers: $1^3+2^3+3^3+...+n^3 = \sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2$

d) Sum of any constant k to *n* times: $k+k+k+...+k(\text{to } n \text{ times}) = \sum_{r=1}^{n} k = nk$.

NUMBER SYSTEM

01. Natural numbers: The numbers used in ordinary counting i.e. $1, 2, 3, ...,$ are called natural numbers (and *positive integers* as well). The set of natural nos. is denoted by N. Also if we include 0 to the set of natural numbers, we get set of the *whole numbers* which is denoted by the symbol W.

Therefore $\text{N}=\{1, 2, 3, ...\}$ and, $\text{W}=\{0, 1, 2, 3, ...\}$.

02. Integers: The numbers $...-3, -2, -1, 0, 1, 2, 3, ...$ are called integers. The set of integers is denoted by I *or* Z. Though now we use Z instead of I to symbolize the set of integers.

Therefore, I *or* $\text{Z}=\{...-3, -2, -1, 0, 1, 2, 3, ...\}$.

Clearly $\text{N} \subset \text{Z}$.

- Also from the above discussion, it is evident that integers are of three types viz.:

***a)** Positive integers i.e.* $\text{Z}^+=\{1, 2, 3, ...\}$

***b)** Negative integers i.e.* $\text{Z}^-=\{-1, -2, -3, ...\}$

***c)** Zero integer i.e. non-positive and non-negative integer.*

03. Rational numbers: A number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a rational no. The set of rational nos. is denoted by Q.

Therefore $\text{Q}=\left\{\frac{p}{q}; p, q \in Z \text{ and } q \neq 0\right\}$

Clearly $\text{N} \subset \text{Z} \subset \text{Q}$.

- Zero being an integer, is also a rational number.

04. Irrational numbers: An irrational number has a non-terminating and non-repeating decimal representation *i.e.* it can't be expressed in the form of $\frac{p}{q}$. The set of irrational nos. is denoted by T.

Few examples of irrational numbers are $\sqrt{2}, 5\sqrt{7}, 8+\sqrt{3}, \sqrt[3]{5}, e, \pi, \ldots$ *etc.*

- Note that π is irrational while $\frac{22}{7}$ is rational.

05. Real numbers: The set of all numbers either rational or irrational, is called real number. The set of real nos. is denoted by R .

Clearly $\mathrm{N} \subset \mathrm{Z} \subset \mathrm{Q} \subset \mathrm{R}$.

Algebraic Identities:

a) $(a \pm b)^2 = a^2 \pm 2ab + b^2$

b) $a^2 - b^2 = (a+b)(a-b)$

c) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

d) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

e) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

f) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

g) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

h) $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$.

➲ Concept of Infinity:

We consider the existence of **two symbols** $-\infty$ and ∞ **outside** the set of real numbers R and would call them ***minus infinity*** and ***plus infinity*** respectively with the fact $-\infty < x < \infty$ for every $x \in \mathrm{R}$.
Thus $-\infty$ and ∞ are not real numbers but just the symbols (like we use *x, y* etc.).

When we write $x = \infty$, we mean that:

a) x is larger than any real number however large.

b) x is not a fixed number.

Also $c^{\infty} = \begin{cases} \infty, \text{ if } c > 1 \\ 0, \text{ if } 0 \le c < 1 \\ 1, \text{ if } c = 1 \end{cases}$.

Any queries and/or suggestion(s), please write to me at
theopgupta@gmail.com
Please mention your details : Name, Student/Teacher/Tutor, School/Institution, Place, Contact No. (if you wish)