RELATIONS, FUNCTIONS & ALGEBRA

Q01. Prove that $f : \text{R} \rightarrow \text{R}$ is a bijection given by $f(\text{x}) = \text{x}^2 + 3$. Find $f^{-1}(\text{x})$. Does the truthfulness and honesty may have any relation?

Q02. Set $A = \{a_1, a_2, a_3, a_4, a_5\}$ and $B = \{b_1, b_2, b_3, b_4\}$ where a_i's and b_i's are school going students. Define a relation from a set A to set B by xRy iff y is a true friend of x.
If $R = \{(a_1, b_1), (a_2, b_1), (a_3, b_3), (a_4, b_2), (a_5, b_2)\}$. Is R a bijective function? Do you think that true friendship is important in life? How?

Q03. If h denotes the number of honest people and p denotes the number of punctual people and a relation between honest people and punctual people is given as h = p + 5. If P denotes the number of people who progress in life and a relation between number of people who progress and honest people is given as P = (h/8) + 5. Find the relation between number of people who progress in life and punctual people. How does the punctuality plays an important role in the progress of life?

Q04. Let A be the set of all students of class XII in a school and R be the relation having the same sex (*i.e.*, male or female) on set A, then prove that R is an equivalence relation. Do you think, co-education may be helpful in child development and why?

Q05. Everyone wants to be a perfect ideal human being. Let us assume that dishonesty is one of the factors that affects our perfectness and perfectness has an inverse square relationship with dishonesty. For any value x of level of dishonesty, we have a unique value y of perfection.
(i) Write down the equation that relates *y* with *x*.
(ii) Does this relationship from $x \in (0, \infty)$ to $y \in (0, \infty)$, form a function?
(iii) For what level of dishonesty one can achieve $\left(\dfrac{1}{4}\right)^{\text{th}}$ level of perfection?
(iv) What will be the change in level of perfection when the level of dishonesty changes from 4 to 2?

Q06. Three shopkeepers A, B, C are using polythene, handmade bags (prepared by prisoners), and newspaper's envelope as carry bags. It is found that the shopkeepers A, B, C are using (20,30,40), (30,40,20,), (40,20,30) polythene, handmade bags and newspapers envelopes respectively. The shopkeepers A, B, C spent ₹250, ₹270 & ₹200 on these carry bags respectively. Find the cost of each carry bags using matrices. Keeping in mind the social & environmental conditions, which shopkeeper is better? Why?

Q07. In a Legislative assembly election, a political party hired a public relation firm to promote its candidate in three ways: telephone, house calls and letters. The numbers of contacts of each type in three cities A, B & C are (500, 1000, 5000), (3000, 1000, 10000) and (2000, 1500, 4000), respectively. The party paid ₹3700, ₹7200, and ₹4300 in cities A, B & C respectively. Find the costs per contact using matrix method. Keeping in mind the economic condition of the country, which way of promotion is better in your view?

Q08. A trust fund has ₹30,000 is to be invested in two different types of bonds. The first bond pays 5% interest per annum which will be given to orphanage and second bond pays 7% interest per annum which will be given to an N.G.O. cancer aid society. Using matrix multiplication, determine how to divide ₹30,000 among two types of Bonds if the trust fund obtains an annual total interest of ₹1800. What are the values reflected in this question?

Q09. Using matrix method, solve the following system of equations:
x + 2y + z = 7, x – y + z = 4, x + 3y +2z = 10.
Suppose x represents the number of persons who take food at home, y represents the number of persons who take junk food in market and z represent the number of persons who take food at hotel. Which way of taking food you prefer and why?

Q10. A school has to reward the students participating in co-curricular activities (Category I), with 100% attendance (Category II) and brave students (Category III) in a function. The sum of

the numbers of all the three category students is 6. If we multiply the number of students of category III by 2 and add to the number of students of category I to the result, we get 7. By adding II and III category students to three times the I category students, we get 12. Form the matrix equation and, hence solve it.

Q11. For keeping fit, X people believe in morning walk, Y people believe in yoga and Z people join gym. Total no. of people are 70. Further 20%, 30% and 40% people are suffering from any diseases who believe in morning walk, yoga and gym respectively. Total no. of such people is 21. If morning walk costs ₹0, yoga costs ₹500/month and gym costs ₹400/month and total expenditure is ₹23000.
(i) Formulate a matrix problem.
(ii) Calculate the no. of each type of people.
(iii) Why exercise is important for health?

Q12. An amount of ₹600 crores is spent by the government in three schemes. Scheme A is for saving girl child from the cruel parents who don't want girl child and get the abortion before her birth. Scheme B is for saving of newlywed girls from death due to dowry. Scheme C is planning for good health for senior citizen. Now twice the amount spent on Scheme C together with amount spent on Scheme A is ₹700 crores. And three times the amount spent on Scheme A together with amount spent on Scheme B and Scheme C is ₹1200 crores. Find the amount spent on each schemes using matrices. What is the importance of saving girl child from the cruel parents who don't want girl child and get the abortion before her birth?

Q13. There are three families. First family consists of 2 male members, 4 female members and 3 children. Second family consists of 3 male members, 3 female members and 2 children. Third family consists of 2 male members, 2 female members and 5 children. Male member earns ₹500 per day and spends ₹300 per day. Female member earns ₹400 per day and spends ₹250 per day, child member spends ₹40 per day. Find the money each family saves per day using matrices? What is the necessity of saving in the family?

Q14. Two schools A and B decided to award prizes to their students for three values honesty (x), punctuality (y) and obedience (z). School A decided to award a total of ₹11000 for the three values to 5, 4 and 3 students respectively while school B decided to award ₹10700 for the three values to 4, 3 and 5 students respectively. If all the three prizes together amount to ₹2700, then:
    i. Represent the above situation by a matrix equation and form linear equations using matrix multiplication.
    ii. Is it possible to solve the system of equations so obtained using matrices?
    iii. Which value you prefer to be rewarded most and why?

Q15. Let $A = \begin{bmatrix} 8 & 16 \\ 32 & 48 \end{bmatrix}$, where first row represents the number of table fans and second row represents the number of ceiling fans which two manufacturing units x and y makes in one day. Compute 7A and, state what does it represents?

Q16. Three friends A, B and C visited a Super Market for purchasing fresh fruits. A purchased 1kg apples, 3kg grapes and 4kg oranges and paid ₹800. B purchased 2kg apples, 1kg grapes and 2kg oranges and paid ₹500. While C paid ₹700 for 5kg apples, 1kg grapes and 1kg oranges. Find the cost of each fruit per kg by using matrix method. Why are the fruits good for health?

Q17. Mr. Priyanshu has invested a part of his income in 10% (bond A) and another part of his income in 15% (bond B). His interest during a certain period is ₹4000. Had he invested 20% more in bond A and 10% more in bond B, his interest would have been increased by ₹500 for the same period. Then: (i) Represent the above situation by a matrix equation and form linear equations using matrix multiplication. (ii) Is it possible to solve the system of equations so obtained by matrices? If yes, solve it too.

Q18. In XII class examination, 25 students from school A and 35 students from school B appeared. Only 20 students from each school could get through the examination. Out of them, 15 students from school A and 10 students from school B secured full marks. Write down this information in matrix from.

Q19. A trust fund has ₹30,000 that is to be invested in two different types of bonds. The first bond pays 5% p.a. interest which will be given to orphanage and second bond pays 7% interest p.a. which will be used for the financial benefits of the trust. Using matrix multiplication, determine how to divide ₹30,000 among two types of bonds if the trust fund obtains an ₹annual total interest of ₹1800.
(i) What are the values reflected in the question?
(ii) Why is it required to help orphan children?

Q20. In a survey of 20 richest person of three cities A, B and C it is found that in city A, 5 believe in honesty, 10 in hardwork, 5 in unfair means while in city B, 5 believe in honesty, 8 in hardwork, 7 in unfair means and in city C, 6 believe in honesty, 8 in hardwork, 6 in unfair means. If the per day income of these richest persons of cities A, B and C are ₹32.5K, ₹30.5K and ₹31K respectively, then find the per day income of each type of person by matrix method.
(i) Which type of persons have more per day income?
(ii) According to you, which type of person is better for country?

Q21. To promote "Save Environment" awareness, a university gives scholarships for those students who take any of the below subjects as an additional subject in first year, second year, third year of graduation. From the table given below, form a set of simultaneous equation and check the consistency. Which subject has to be promoted the most and why? What are the values to be promoted?

Sr. No.	Subject	No. of students in 1st year	No. of students in 2nd year	No. of students in 3rd year
1.	Industrial Waste	1	3	6
2.	Organic Waste	1	1	7
3.	e-Waste	1	1	8
	Amount Received	₹5000	₹7000	₹35800

Q22. In a legislative assembly election, a political group hired a public relations firm to promote its candidate in 3 ways namely Posters, Pamphlets, Public Addressing system. The cost per contract is :

$$A = \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} \text{Posters} \\ \text{Pamphlets} \\ \text{Public Addressing System} \end{matrix}$$

Number of contacts of each type made by a city is given as : $B = [100 \quad 500 \quad 5000]$

Find the amount spent by the city on these ways. Is it a good practice to campaign in such a manner? Suggest an alternative to the above for campaigning.

Q23. There are 2 families A & B. There are 4 men, 6 women and 2 children in family A, 2 men, 2 women and 4 children in family B. The recommended daily allowance for calories is 2400 for men, 1900 for women, 1800 for children and 45gms of proteins for men, 55gms for women and 33gms for children. Represent the above information using matrices. Using matrix multiplication, calculate the total requirement of calories and proteins for each of the 2 families. What awareness can you create among people about the balanced diet?

Q24. Rekha donates 3 pens, 2 bags and 1 instrument box worth Rs.41, Diksha donates 2 pens, 1 bag and 2 instrument boxes worth Rs.29, while Ankita donates 2 pens, 2 bags and instrument boxes to a rural school. Translate the problem into a system of linear equations. Solve & find the cost of each item. By the act of these three people what values do you learn?

Q25. There are 3 families. Family A consists of 2 men, 3 women and 1 child. Family B has 2 men 1 woman and 3 children. Family C has 4 men, 2 women and 6 Children. Daily expenses of men, women and children are 200,155 and 200 respectively. Only men and women earn and children do not. Using matrix multiplication, calculate the daily expenses of each family. What impact does having more children in the family create on the society?

Q26. The cost of 4 chocolates, 3 samosas and 2 apples is Rs.60 and that of 2 chocolates, 4 samosas, and 6 apples is Rs.90. The cost of 6 chocolates, 2 samosas, 3 apples is Rs.70. Find the cost of each item by matrix method. What do you think is the healthiest diet? Suggest an item that could replace chocolates and samosas to make the diet healthier?

Q27. Last year, 1 packet of tea and 3 packets of sugar together cost Rs.96.This year, the rate of tea increased by 15% and that of sugar by 10%. So, the same amounts of tea and sugar now cost Rs.108.60. Find the rates of sugar and tea per packet last year and this year using matrix method. What do you think is the impact of inflation on family expenses?

Q28. A part of the monthly expenses of a family is constant while the remaining varies with the price of rice, fuel etc., When the price of rice is Rs.25/Kg the monthly expenses of the family is Rs.1000. when it is Rs.24/Kg the monthly expenses is Rs.980. Find the total monthly expenses of the family when the cost of rice is Rs 35/Kg. Is this family below poverty line? Give some suggestions to improve their standard of living.

Q29. Let R be a relation defined as R = {(x, y) : x and y study in the same class}. Show that R is an equivalence relation. If x is a brilliant student and y is a slow learner and x helps y in his studies, what quality does x possess?

Q30. Show that the function $f : R_+ \to R_+$ defined by $f(x) = 3x + 4$ is an invertible function. If x represents the number of systematic hours of study that a student puts in and f(x) represents the marks scored by him, from the graph of the above function, which value will be rewarded?

Q31. Let L be the set of all the lines in a plane and R be the relation in L defined by R = {(L$_1$, L$_2$) : L$_1$ || L$_2$}. Show that R is an equivalence relation. Let L$_1$ represents the ideologies of Gandhi ji and L$_2$ represents the ideologies of Neta ji Subhash Chandra Bose. Even though their ideaologies ran on parallel tracks, both had the common goal to achieve independence for India. Which common value did they both exhibit?

Q33. Let X be a non-empty set. Let * be a binary operation on the power set P(X) defined by A*B = A$\cup$B. Prove that * is commutative and associative. What is the identity element for the operation *? If X is a set of people in a locality, A is a set of children and B is a set of citizens aged above 75 years in the same locality, is * a binary operation for these sets as defined above? What qualities would you suggest that elements of A should have towards elements of B?

Q34. Consider a relation R in the set A of people in a colony defined as aRb iff a and b are members of joint family. Is R is an equivalence relation? Staying with grand parents in a joint family imbibes the moral values in us. Can you elicit 2 such values?

CONTINUITY, DIFFERENTIAL CALCULUS & ITS APPLICATIONS

Q01. A car driver is driving a car on the dangerous path given by

$$f(x) = \begin{cases} \dfrac{1-x^m}{1-x}, & \text{if } x \neq 1 \\ m-1, & \text{if } x = 1 \end{cases} , \ m \in N.$$

Find the dangerous point (point of discontinuity) on the path. Whether the driver should pass that point or not? Justify your answer.

Q02. A car parking company has 500 subscribers and collects fixed charges of ₹300 per subscriber per month. The company proposes to increase the monthly subscription and it is believed that for every increase of ₹1, one subscriber will discontinue the service. What increase will bring maximum income for the company? What values are driven by this problem?

Q03. Check whether the function $f(x) = x^{100} + \sin x - 1$ is strictly increasing or strictly decreasing or none of both on (–1, 1). Should the nature of a man be like this function? Justify your answer.

Q04. If $y = x^4 - \dfrac{x^3}{3}$, when x denotes the number of hours worked and y denotes the amount (in ₹) earned. Then find the value of (in interval) for which the income remains on the growth? Explain the importance of earning in life?

Q05. If performance of the students 'y' depends on the number of hours 'x' of hard work done per day is given by the relation: $y = 4x - \dfrac{x^2}{2}$. Find the number of hours, the students work to have the best performance. 'Hours of hard work are necessary for success.' Justify.

Q06. A farmer wants to construct a circular well and a square garden in his field. He wants to keep sum of their perimeters fixed. Then prove that the sum of their areas is least when the side of square garden is double the radius of the circular well. Do you think good planning can save energy, time and money?

Q07. Profit function of a company is given as $p(x) = \dfrac{24x}{5} - \dfrac{x^2}{100} - 500$ where x is the number of units produced. What is the maximum profit of the company? Company feels its social responsibility and decided to contribute 10% of his profit for the orphanage. What is the amount contributed by the company for the charity? Justify that every company should do it.

Q08. A window is in the form of rectangle surmounted by a semi-circular opening. Total perimeter of the window is 10m. What will be dimensions of the whole opening to admit maximum light and air?
(i) How does having large windows help us in saving electricity & conserving environment?
(ii) Why optimum use of energy is required in the Indian context?

Q09. In a kite festival, a kite is at a height of 120m and 130m string is out. If the kite is moving horizontally at the rate of 5.2m/sec, find the rate at which the string is being pulled out at that instant. How does a festival enhance national integration?

Q10. An expensive square piece of golden color board of side 24cm is to be made into a box without top by cutting a square from each corner and folding the flaps to form a box. What should be the side of the square piece to be cut from each corner of the board to hold maximum volume and minimize the wastage? What is the importance of minimizing the wastage in utilizing the resources? How can a student utilise the resources?

Q11. A student is given card board of area 27 square centimeters. He wishes to form a box with square base to have maximum capacity and no wastage of the board. What are the dimensions of the box so formed? Do you agree that students don't utilize the resources properly? Justify.

Q12. The contentment obtained after eating x units of new dish at a trial function is given by the function $C(x) = x^3 + 7x^2 + 8x + 6$. If the marginal contentment is defined as the rate of change of $C(x)$ with respect to the number of units consumed at an instant, then find the marginal contentment when four units of dish are consumed.

Q13. The income I of Dr.Rastogi is given by $I(x) = ₹ (x^3 - 3x^2 + 5x)$. Can an insurance agent ensure him for the growth of his income?

Q14. The amount of pollution content added in air in a city due to *x*-diesel vehicles is given by $P(x) = 0.005x^3 + 0.02x^2 + 30x$. Find the marginal increase in pollution content when 3 diesel vehicles are added and write which value is indicated in the above question.

Q15. In a competition, a brave child tries to inflate a huge spherical balloon bearing slogans against child labour at the rate of 900cubic centimeter of gas per second. Find the rate at which the radius of the balloon is increasing when its radius is 15cm?
(i) Which values have been reflected in the question?
(ii) Why child labour is not good for society?

Q16. The confidence gained by playing x games of tennis at a trial function is given by $C(x) = 11 + 15x + 6x^2 - x^3$. Find the marginal confidence gained after playing 5 games.

Q17. The radius of a spherical diamond is measured as 7cm with an error of 0.04cm. Find the approximate error in calculating its volume. If the cost of 1cm^3 diamond is ₹1000, what is the loss to the buyer of the diamond? What lesson do you get from this observation?

Q18. The path of a moving bike is given by $f(x) = \begin{cases} 2x - 1, \text{if } x < 0 \\ 2x + 1, \text{if } x \geq 0 \end{cases}$. Find the dangerous point on the path. Whether the rider should pass that point or not? Justify your answers.

Q19. The edge of a cubical gold is measured as 8 cm with an error of 0.03cm. Find the approximate error in its volume. What is the loss to the buyer of the gold if the cost of 1 cubic cm of gold is Rs.3000. What lesson do you get?

Q20. The bottom of a rectangular fish tank is 30 cm by 15 cm. Water is pumped into the tank either by man-made pump at the rate of 50 cubic cm/minute or by motor pump at the rate of 75 cubic cm/minute. Find the rate at which the level of the water in the tank is rising when water is pumped by motor pump. Which pump would you prefer and why?

Q21. Find the volume of the largest cylinder that can be scooped out from a given wooden solid cone. Give a valued method of using the left out wood and justify?

INTEGRAL CALCULUS, AREAS & DIFFERENTIAL EQUATIONS

Q01. Evaluate: $\int \dfrac{x^3 - x + 1}{x^2 + 1} dx$. Discuss the importance of integration (unity) in life.

Q02. A manufacturer's marginal revenue function is MR $= 275 - x - 0.3x^2$. Find the increase in the manufacturer's total revenue if production is increased from 10 units to 20 units.

Q03. The marginal cost of a manufacturer is given by MC $= \dfrac{500x}{\sqrt{x^2 + 40}}$, where x is the number of units of a product in thousands. If x increases from 3000 to 9000 units, find the total increase in cost.

Q04. The marginal cost is defined as the rate of change of total cost with respect to the number of units of the product. The marginal cost of producing x units of a product is given by Marginal Cost $= 2x\sqrt{x + 5}$. If the cost of producing 4 units of the product is ₹314.40, find the cost function.

Q05. An electric manufacturing company makes small house-hold switches. The company estimates the marginal revenue function for these switches to be MR $= \dfrac{x^2}{(x + 2)^2} e^x$, where x represents the number of units (in thousand) and MR is the rate of change of revenue R w.r.t. x. What is the total revenue function? Use integrals.

Q06. A farmer has a piece of land. He wishes to divide equally in his two sons to maintain peace and harmony in the family. If his land is denoted by area bounded by curve $y^2 = 4x$ and $x = 4$ and to divide the area equally, he draws a line $x = a$. What is the value of 'a'? What is the importance of equality among the people?

Q07. A circular Olympic gold medal has a radius of 2cm and taking the centre at the origin, find its area by the method of integration. What is the importance of Olympic Games for a sportsman and why?

Q08. A poor deceased farmer had agriculture land bounded by the curve $y = \cos x$, between $x = 0$ and the line $x = 2\pi$. He had two sons. Now they want to distribute this land in two parts as decided by their deceased father, such that both of them have equal share of land. Find the area of each part. Do you think that the decision taken by the farmer before his death was good? Justify your answer.

Q09. If a triangular field is bounded by the lines $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$. Using integration, compute the area of the field.
(i) If in each square unit area, 4 trees may be planted. Find the number of trees can be planted in the field. (ii) Why plantation of trees is necessary?

Q10. Solve the differential equation: $(x + 2y^2)\dfrac{dy}{dx} = y$; given that when $x = 2$, $y = 1$.

Let x denotes the % of people who are Polite and y denotes the % of people who are Intelligent. Find x when $y = 2$%. A polite child is always liked by all in society. Do you agree? Justify.

Q11. If $\dfrac{dy}{dx} + \dfrac{y}{x} = 0$, where x denotes the percentage population living in a city & y denotes the area for living a healthy life of population. Find the particular solution when x = 100, y = 1. Is higher density of population harmful? Justify yours answer.

Q12. The velocity v and mass m of a rocket at time t are given by the equation: $m\dfrac{dv}{dt} + V\dfrac{dm}{dt} = 0$, where V is the constant velocity of emission. If the rocket starts from rest when t = 0 with mass m_0, then prove that: $v = V\,log\left(\dfrac{m_0}{m}\right)$.

Should we encourage the rocket technology? Comment.

Q13. The male-female ratio of a village increases continuously at the rate proportional to the rato at any time. If the ratio of male-female of the village was 1000:980 in 1999 and 1000:950 in 2009, what will be the ratio in 2019?
(i) Why is the gender equality valueable for the society?
(ii) What should society do to reduce the male-female ratio to 1?

Q14. A parking lot in an IT company as an area bounded by the curve $y = 4 - x^2$ and the lines y = 0 and y = 3. The line y = 3 divides the area into two parts out of which the greater area is allotted for car owners who practice car pooling. Find this area using integration. Write any two benefits of carpooling.

Q15. Evaluate using limit as a sum : $\displaystyle\int_1^3 (2x^2 + 5x + 1)dx$. Mention two suitable situations in your everyday life (be it food, travel, matters, media, etc.) where limiting yourself will help you evolve as a better human being.

Q16. Using integration find the area of the bounded region $y = 1 + |\,x + 1\,|$; $x = \pm 3$ and $y = 0$. The area obtained is divided into two regions and the students are asked to put a posters on "Child Labour" and "Education for all" in those two. Which theme according to you deserves a bigger space? Justify.

Q17. Find the area of the region enclosed by the curve $y = x^2$ and the lines x = 0, y = 1 and y = 4. A farmer plans to construct an electrical fence around this bounded region to protect his crop from animals. But his son rejects this idea and wants a wooden fence to be erected. Who would you favour? Mention two values demonstrated by the son.

3 DIMENSIONAL GEOMETRY

Q01. Considering the earth as a plane having equation 5x + 9y – 10z + 138 = 0, a monument is standing vertically such that its peak is at the point (1, 2,–3). Find the height of the monument. How can we save our monument?

Q02. Let the point P(5, 9, 3) lies on the top of Qutub Minar, Delhi. Find the image of the point on the line:
$\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$. Do you think that the conservation of monuments is important? Why?

Q03. Two bikers are running at the speed more than allowed speed on the road along the lines $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = 4\hat{i} - \hat{k} + \mu(2\hat{i} + 3\hat{k})$. Using Shortest distance formula check whether they meet to an accident or not?
'While driving, the driver should maintain the speed limit as allowed.' Comment.

Q04. Two airships are moving in space along the following lines
$\dfrac{x-3}{1} = \dfrac{5-y}{2} = \dfrac{z-7}{1}$ and $\dfrac{x+1}{7} = \dfrac{y+1}{-6} = \dfrac{z+1}{1}$.
An astronaut wants to move from one ship to another ship when two airships are closest. What is the least distance between the ships that he has to travel?

Q05. A bird is located at the point A(3, 2, 8) in space. It wants to move to the plane whose equation is given by $3x + 2y + 6z + 16 = 0$ in the shortest time. Find the distance she covered.

Q06. From the point A(2, 3, 8) in space, a shooter aims to hit the target at P(6, 5, 11). If the line of fire is $\dfrac{x-2}{4} = \dfrac{y-3}{2} = \dfrac{z-8}{3}$, what do you think about the success of the shooter?

Q07. A bird at A(7, 14, 5) in space wants to reach a point P on the plane $2x + 4y - z = 2$ when AP is least. Find the position of P and also the distance AP travelled by the bird.

Q08. Show that a powerful bomb shot along the line of fire $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$ will never hit a helicopter flying in the plane $2x + 4y - 4z + 11 = 0$.

Q09. A gunner who is hiding himself from the enemy is at the point G(2, 1, 3) and observes an enemy bomber flying along the plane $3x + 6y + 2z + 10 = 0$. What is the least distance of G from the plane?

LINEAR PROGRAMMING PROBLEM

Q01. A dietician wishes to mix two types of food in such a way that the vitamin content of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2units/kg of vitamin A and 1unit/kg of vitamin C, while food II contains 1unit/kg of vitamin A and 2units/kg of vitamin C. It cost ₹5.00 per kg to purchase food I and ₹7.00 per kg to purchase food II. Determine the minimum cost of the mixture. Formulate the linear programming problem and, hence solve it.
Why a person should take balanced food?

Q02. A farmer has a supply of chemical fertilizers of type 'A' which contains 10% nitrogen and 6% phosphoric acid and type 'B' contains 5% of nitrogen and 10% of phosphoric acid. After soil testing, it is found that at least 7kg of nitrogen and same quantity of phosphoric acid is required for a good crop. The fertilizers of type A and type B cost ₹5 and ₹8 per kilograms respectively. Using L.P.P., find how many kgs. of each type of fertilizers should be bought to meet the requirement and cost be minimum. Solve the problem graphically. What are the side-effects of using excessive fertilizers?

Q03. If a class XII student aged 17 years, rides his motor cycle at 40km/hr, the petrol cost is ₹2 per km. If he rides at a speed of 70km/hr, the petrol cost increases to ₹7per km. He has ₹100 to spend on petrol and wishes to cover the maximum distance within one hour.
(i) Express this as an L .P.P and solve graphically.
(ii) What is benefit of driving at an economical speed?
(iii) Should a child below 18years be allowed to drive a motorcycle? Give reasons.

Q04. Avinash has been given two lists of problems from his mathematics teacher with the instructions to submit not more than 100 of them correctly solved for marks. The problems in the first list are worth 10 marks each and those in the second list are worth 5 marks each. He knows from past experience that he requires on an average of 4 minutes to solve a problem of 10 marks and 2 minutes to solve a problem of 5 marks. He has other subjects to worry about; he cannot devote more than 4 hours to his mathematics assignment. With reference to manage his time in best possible way, how many problems from each list shall he do to maximize his marks? What is the importance of time management for students?

Q05. An NGO is helping the poor people of earthquake hit village by providing medicines. In order to do this, they set up a plant to prepare two medicines A and B. There is sufficient raw material available to make 20000 bottles of medicine A and 40000 bottles of medicine B but there are 45000 bottles into which either of the medicines can be put. Further it takes 3 hours to prepare enough material to fill 1000 bottles of medicine A and takes 1 hour to prepare enough material to fill 1000 bottles of medicine B. There are 66 hours available for the operation. If the bottle of medicine A is used for 8 patients and bottle of medicine B is used for 7 patients. How the NGO should plan its production to cover maximum patients? How can you help others in case of natural disasters?

Q06. A dealer in rural area wishes to purchase a number of sewing machines. He has only ₹5760.00 to invest and has space for at most 20 items. An electronic sewing machine costs him ₹360.00 and a manually operated sewing machine ₹240.00. He can sell an Electronic Sewing Machine at a profit of ₹22.00 and a manually operated sewing machine at a profit of ₹18.00. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit? Make it as a linear programming problem and then, solve it graphically. Keeping the rural background in mind justify the 'values' to be promoted for the selection of the manually operated machine.

Q07. A manufacturing company makes two type of teaching aids A and B of mathematics of class XII. Each type of A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 hours, respectively. The profit on type A and B is ₹80 and ₹120 per piece, respectively. How many pieces of each type should be manufactured per week by the company to maximize its profit? What is the maximum profit per week? Is teaching aid necessary for teaching-learning process? Justify your answer.

Q08. A village has 500 hectares of land to grow two types of plants X and Y. The contribution of total amount of oxygen produced by plant X and Y are 60% and 40% per hectare respectively. To control weeds, a liquid herbicide has to be used for the plants X and Y at the rate of 20 litres and 10 litres per hectare, respectively. Further no more than 8000 litres of herbicides should be used in order to protect aquatic animals in a pond which collects drainage from this land. How much land should allocated to each crop so as to maximize the total production of oxygen?
(i) How do you think excess use of herbicides affect our environment?
(ii) What are the general implications of this question towards planting trees around us?

Q09. A cooperative society of farmers has 50 hectares of land to grow two crops A and B. The profit from crops A and B per hectares are estimated as ₹10500 and ₹9000 respectively. To control weeds, a liquid herbicide has to be used for crops A and B at the rate of 20 litres and 10 litres per hectare, respectively. Further not more than 800 litres of herbicide should be used in order to protect fish and wildlife using a pond which collects drainage from this land. Keeping in mind that the protection of fish and other wildlife is more important than earning profit, how much land should be allocated to each crop so as to maximize the total profit? Form an LPP from the above and solve it graphically. Do you agree with the message that the protection of wildlife is utmost necessary to preserve the balance in environment?

Q10. A firm manufactures jute bags and cloth bags. The total number of items it can manufacture is at most 24. A Jute bag requires 1 hour to be made while a cloth bag requires only half an hour. The maximum number of hours available per day is 6 hours. If the profit on a jute bag is Rs.30 and on a cloth bag is Rs.20, how many bags of each type must be made for maximum profit? Solve it graphically. The manufacturer wants to replace cloth bags by plastic bags to increase his profit margin. Is it a good idea? If not, give reasons.

Q11. A manufacturer makes cycles and scooters. Processing of these products is done on two machines A and B. The cycle parts need 2 hours on machine A and 6 hours on machine B. Parts of a scooter needs 4 hours on machine A and 2 hours on machine B. Machine A is available for 16 hours per day and Machine B is available for 30 hours per day. Profit gained by the manufacturers from a cycle and a scooter is Rs.1000 and Rs.3000 respectively. Find with the help of a graph what should be the daily production of each of the two products to maximize the profit? Which of the above modes of transport is a better option and why?

Q12. There are two types of fertilizers that a farmer uses in his farm namely A and B. A consists of 10% of nitrogen and 6% phosphoric acid while B contains 5% nitrogen and 10% phosphoric acid. The farmer needs at least 14 kg of nitrogen and 14 kg of phosphoric acid. If A costs Rs.8/kg and B costs Rs.6/kg, determine how much of each type of fertilizer should be used by the farmer so that the required nutrient levels are met at minimum cost. What is the minimum cost? The farmer is considering the option of using pesticides to increase his yield. What is your opinion? Can he convert it into an organic farm?

Q13. A farmer decides to plant upto 10 hectares with cabbages and

potatoes. He decides to grow at least 2 but not more than 8 hectares of cabbages and at least 1 but not more than 6 hectares of potatoes. He can make a profit of Rs.1500 per hectare on cabbages and Rs.2000 per hectare on potatoes. How should he plan his farming so as to maximize his profit. Keeping in mind the nutritional value of both vegetables, do you think the farmer's production will help the community? Mention one value point displayed by the farmer.

Q14. A manufacturer produces two types of disposable plates, one using plastic and the other using bamboo. Three machines are required to produce the plates and the time in minutes is given below :

Type Of Plate	Machine		
	I	II	III
Plastic	12	18	6
Bamboo	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each plate of plastic is 75 paise and profit on bamboo plate is 50 paise, how many plates of each type should the factory produce to maximize the profit. Which type of plates do you think should be promoted more and mention one value point as the reason.

Q15. A school administrator decides to buy colour boxes and books as prizes for children. A colour box costs Rs.5 and a book costs Rs.10. He wants to buy at least 4 of each of them. How many of each should he buy so that the expenditure does not exceed Rs.100 and at the same time can give maximum number of prizes? Which of the two, do you think will benefit the children more and mention one value point as the reason.

PROBABILITY

Q01. Probability of winning when batting coach A and bowling coach B working independently are ½ and ⅓ respectively. If both try for the win independently, find the probability that there is a win. Will the independently working may be effective? Justify your answer.

Q02. A person has undertaken a construction job. The probabilities are 0.65 that there will be strike, 0.80 that the construction job will be completed on time if there is no strike and 0.32 that the construction job will be completed on time if there is strike. Determine the probability that the construction job will be completed on time. What values are driven by this question?

Q03. A clever student used a biased coin so that the head is 3 times as likely to occur as tail. If the coin is tossed twice find the probability distribution and mean of numbers of tails.
Is this a good tendency? Justify your answer.

Q04. A man is known to speak truth 5 out of 6 times. He draws a ball from the bag containing 4 white and 6 black balls and reports that it is white. Find the probability that it is actually white? Do you think that speaking truth is always good?

Q05. A drunkard man takes a step forward with probability 0.6 and takes a step backward with probability 0.4. He takes 9 steps in all. Find the probability that he is just one step away from the initial point. Do you think drinking habit can ruin one's family life?

Q06. If group A contains the students who try to solve the problem by knowledge, Group B contains the students who guess to solve the problem whereas Group C contains the students who give answer by cheating. Given that $n(A) = 20$, $n(B) = 15$, $n(C) = 10$. Two students are selected at random. Find the probability that they are from Group C. Do you think that cheating habit spoils the career?

Q07. In a school, 30% of the student has 100% attendance. Previous year result report tells that 70% of all students having 100% attendance attain A grade and 10% of remaining students attain A grade in their annual examination. At the end of the year, one student is chosen at random and he has an A grade. What is the probability that the student has 100% attendance? Also state the factors which affect the result of a student in the examination.

Q08. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is six. Find the probability that it is actually a six. Write any three benefits of speaking the truth.

Q09. There are 20 People in a group. Out of them 7 people are non-vegetarian, 2 people are selected randomly. Write the probability distribution of non-vegetarian people. Explain whether you would like to be vegetarian or non-vegetarian and why? Also keeping life of animals in mind, how would you promote a person to be vegetarian?

Q10. Two third of the students in a class are sincere about their study and rest are careless. Probabilities of passing in examination are 0.7 and 0.2 for sincere and careless students respectively. A student is chosen and is found to be passed. What is the probability that he/she was sincere? Explain the importance of sincerity for a student.

Q11. A company has two plants of scooter manufacturing. Plant I manufacture 70% scooter and plant II manufactures 30%. At plant I, 80% of the scooters are maintaining pollution norms and in plant II 90% of the scooters are maintaining pollution norms. A Scooter is chosen at random and is found to be fit on pollution norms. What is the probability that it has come from plant II. What is importance of pollution norms for a vehicle?

Q12. A chairman is biased so that he selects his relatives for a job 3 times as likely as others. If there are 3 posts for a job, find the probability distribution for selection of persons other than his relatives.
If the chairman is biased then which value of life will be demolished?

Q13. A manufacturer has three machine operators A (skilled), B (semi-skilled) and C (non-skilled).The first operator A produces 1% defective items whereas the other two operators B and C produces 5% and 7 % defective items respectively. A is on the job for 50% of time, B in the job for 30% of the time and C is on the job for 20 % of the time. A defective item is produced, what is the probability that it was produced by B? What is the value of skill in industries?

Q14. In a group of 100 families, 30 families like male child, 25 families like female child and 45 families feel both children are equal. If two families are selected at random out of 100 families, find the probability distribution of the number of families who feel both children are equal. What is the importance in the society to develop the feeling that both the children are equal?

Q15. In a group of 200 people, 50% believe that anger and violence will ruin the country, 30% do not believe that anger and violence will ruin the country and 20% are not sure about anything. If 3 people are selected at random, find the probability that 2 people believe and 1 does not believe that anger and violence will ruin the country. How do you consider that anger and violence will ruin the country?

Q16. In a group of students, 200 attend coaching classes, 400 students attend school regularly and 600 students study themselves with help of peers (self study). The probability that a student will succeed in life who attend coaching classes, attend school regularly and study themselves with help of peers (self study) are 0.1, 0.2 and 0.5 respectively. One student is selected who succeeded in life, what is the probability that he studied himself with the help of peers (self study)? What type of study can be considered for the success in life and why?

Q17. There is a group of 50 people who are patriotic out of which 20 believe in non-violence. Two persons are selected at random out of them, write the probability distribution for the selected persons who are non-violent. Also find the mean of the distribution. Explain the importance of Non-violence in patriotism.

Q18. In answering a question on a MCQ test with 4 choices per question, a student knows the answer, guesses or copies the answer. Let 1/2 be the probability that he knows the answer, 1/4 be the probability that he guesses and 1/4 that he copies it. Assuming that a student, who copies the answer, will be correct with the probability 3/4, what is the probability that the student knows the answer, given that he answered it correctly?
Arjun does not know the answer to one of the questions in the test. The evaluation process has negative marking. Which value would Arjun violate if he resorts to unfair means? How would an act like the above hamper his character development in the coming years?

Q19. An insurance company insured 2000 cyclists, 4000 scooter drivers and 6000 motorbike drivers. The probability of an accident involving a cyclist, scooter driver and a motorbike driver are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an

accident. What is the probability that he is a scooter driver? Which mode of transport would you suggest to a student and why?

Q20. In a backward state, there are 729 families having six children each. If probability of survival of a girl child is 1/3 and that of boy child is 2/3, find the number of families having 2 girls and 4 boys. Do you believe that a female child is neglected in backward areas? What steps will you take to restore the respect of a female child in society?

Q21. There are 40 hardworking scholars in a class. Out of which 10 are sports-persons. Three scholars are selected at random out of them. Write the probability distribution for selected persons who are sports-persons. Find the mean of distribution. Explain the importance of sports in education.

Q22. A survey revealed that 70% men and 30% women eat pan-masala. 10% of these men and 20% of these women eat brand X pan-masala. What is the probability that a person seen eating brand X will be a man? Why would you discourage intake of pan-masala?

Q23. Of the students in a school, it is known that 30% has 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who has 100% attendance attain grade A and 10% irregular student attain grade A in their annual examinations. At the end of the year, one student is chosen at random from the school and it is found that he has grade A. What is the probability that the student has 100% attendance?
(i) Write any two values reflected in the question.
(ii) Is regularity required only in school? Justify your answer.

Q24. In shop A, 30 tin pure ghee and 40 tin adulterated ghee are kept for sale while in shop B, 50 tin pure ghee and 60 tin adulterated ghee are kept. One tin of ghee is purchased from one of the shops randomly and it is found to be adulterated. Find the probability that it is purchased from shop B.
(i) How is the adulteration dangerous for human?
(ii) What can you do against adulteration?

Q25. In a self-assessment survey, 60% persons claimed that they never indulged in corruption, 40% claimed that they always speak truth and 20% say that they neither indulged in corruption nor tell lies. A person is selected at random out of this group.
(i) What is the probability that the person is either corrupt or tells lie?
(ii) If the person never indulged in corruption, find the probability that she/he tell the truth.
(iii) If the person always speaks truth, find the probability that she/he claims to have never indulged in the corruption.
(iv) What values have been discussed in this question?
(v) Why is it must for all to practice values in our life?

Q26. A speaks truth in 60% of the cases, while B in 90% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact?
In the cases of contradiction do you think, the statement of B will carry more weight as he speaks truth in more number of cases than A?

Q27. Assume that the chances of a patient having a heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time, a patient can chose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course a course of meditation and yoga. Interpret the result and state which of the above stated methods is more beneficial for the patient.

Q28. P speaks truth in 70% of the cases and Q in 80% of the cases. In what percent of cases are they likely to agree in stating the same fact?
Do you think, when they agree, means both are speaking truth?

Q29. A speaks truth in 75% of the cases, while B in 90% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact?
Do you think that statement of B is true?

Q30. In a group consisting of equal number of men and women, 10% men and 45% women are unemployed. What is the probability that a person selected at random is employed?
Suggest two steps to enhance employability of youth.

Q31. A group contains 10 men and 4 women. A three member committee is formed from the group, containing at least 1 woman. Find the probability that the committee so formed has more women than men.

Write any 2 benefits of including more women in Panchayat committees.

Q32. Three rifle men take one shot each at the same target. The probabilities of the first, second and third rifle men hitting the target are 0.4, 0.5, 0.8 respectively. Find the probability that exactly 2 of them hit the target.

The National Cadet Corps (NCC) provides opportunities for training in gun firing. Write 2 benefits of students joining NCC.

Q33. A lot of 100 bulbs from a manufacturing unit is known to contain 10 defective and 90 non-defective bulbs. If a sample of 8 bulbs is selected at random, what is the probability that (a) the sample has 3 defective and 5 non-defective bulbs.   (b) The sample has at least 1 defective bulb. Write two advantages of using Compact fluorescent lamp (CFL) bulbs over incandescent bulbs.

Q34. A shopkeeper has 5 customers who rent his cycles. He has 3 cycles and the probability that a customer will a hire a cycle for a day is 3/4. If he charges Rs.2 as rent for a cycle, find the probability that he earns exactly Rs.6 per day. Give 2 advantages of using bicycles.

❖ HINTS / ANSWERS ❖

RELATIONS, FUNCTIONS & ALGEBRA

Q01. $f^{-1}(x) = (x - 3)^{1/2}$. Truthfulness and honesty among people may have the bijective (one-one onto) relation as people who are honest, are usually truthful as well and vice versa.

Q02. Neither one-one nor onto hence not bijective. Yes, true friendship makes life easier.

Q03. P = (p + 45)/8. Punctuality develops discipline in life and hence progressive in life.

Q04. The relation R is reflexive, symmetric and transitive. Co-education is very helpful because it leads to the balanced development of the children and in future they become good citizens.

Q05. (i) $y = \dfrac{1}{x^2}, x \neq 0$ (ii) Yes (iii) For $y = \dfrac{1}{4}$, we have $\dfrac{1}{4} = \dfrac{1}{x^2} \Rightarrow x = 2$ (iv) $\Delta y = \dfrac{1}{4} - \dfrac{1}{16} = \dfrac{3}{16}$

Q06. Polythene = ₹1, Handmade bag = ₹5, Newspaper's envelope = ₹2. Shopkeeper A is better for environmental conditions. As he is using least no. of polythene. Shopkeeper B is also better for social conditions as he is using handmade bags (prepared by prisoners). Shopkeeper C is better too as the newspaper's envelope used by him give employment to some people.

Q07. Cost per Contact: Telephone = ₹0.40, House calls = ₹1.00, Letters = ₹0.50. Telephone is better as it is cheap.   **Q08.** ₹15000 each type of bond. **Values reflected in the question:** (i) Charity (ii) Helping orphans or poor people (iii) Awareness about diseases.

Q09. x = 3, y = 1, z = 2. Food taken at home is always the best way.

Q10. x + y + z = 6, x + 2z = 7, 3x + y + z = 12 where x, y, z represent the number of students in categories I, II, III respectively. Also x = 3, y = 1, z = 2. Participating in co-curricular activities is very important. It is very essential for all round development.

Q11. (i) x + y + z = 70, 2x + 3y + 4z = 210, 5y + 4z = 230 (ii) x = 20, y = 30, z = 20
(iii) Exercise keeps fit and healthy to a person.

Q12. ₹300crores, ₹200crores and ₹100crores. (i) In our country, male population is more than female population. (ii) It is essential for a human being to save the life of all.

Q13. ₹880, ₹970, ₹500. Saving is necessary for each family as in case of emergency our saving in good time helps us to survive in bad time.

Q14. (i) $5x + 4y + 3z = 11000$, $4x + 3y + 5z = 10700$, $x + y + z = 2700$ (ii) -3 (iii) We prefer to reward most the value of **honesty** as it is of highest value in the character development.

Q15. $7A = \begin{bmatrix} 56 & 112 \\ 224 & 336 \end{bmatrix}$. It represents the number of table fans and ceiling fans that the manufacturing units x and y produce in 7 days.

Q16. Let cost of each fruit be x, y, z respectively. Then solve the equations so formed by using matrix method. So, x=100, y=100, z=100. Hence the cost of each fruit is ₹100 per kg. **Importance of fruits:** Fruits contain nutrients and vitamins which help our body in its proper growth and maintenance.

Q17. Let x and y be the initial investments by Mr. Priyanshu in bond A and bond B respectively.

(i) $\begin{bmatrix} 2 & 3 \\ 8 & 11 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 80000 \\ 300000 \end{bmatrix}$ (ii) x = ₹10000, y = ₹20000.

Q18. The given information is expressed in matrix:

	School	
	A	B
Appeared	25	35
Got through exam	20	20
Secured full marks	15	10

Q19. Amount invested in both the bonds = ₹15000. (i) **Values reflected:** Helping poor and needy children. Also since, the interest rate in bond II is more than the bond I, the trust decides to invest the funds equally. This shows that the motive of the trust is not to only earn the interest but also to help the needy orphan children.   (ii) The children living in orphanage are also talented and carry potential. If they are given the proper education with good opportunities, they can also come up

with the flying colors and can contribute to the development of society. So, the orphan children should be helped out.

Q20. Per day income of person who believe in honesty = ₹1500, Per day income of person who believe in hardwork = ₹2000 and, Per day income of person who believe in unfair means = ₹1000. (i) The persons who believe in hardwork ahs more per day income. (ii) A person who believe in hardwork and honesty, is better for country.

Q21. Values : 1. Sense of belonging 2. Pollution control 3. About harmful radiation 4. Simple living without electronic gadgets

Q22. Alternatives : 1. Advertise using mass media 2. Through sms

Q23. Values : 1. Vegetarianism will help us to control many ailments 2. Should not skip break fast 3. Aerated drinks should be totally avoided 4. Over eating/ eating while watching T.V should be avoided

Q24. Values : 1. Sharing 2. Bring happiness to everybody 3. Bridge the gap between the rich and poor 4. Following the motto "giving more than what we take from the society"

Q25. Values : 1. Can't give better standard of living 2. Cannot educate them 3. Good food, clothes and shelter cannot be given by the government 4. Poverty increases

Q26. Values : 1. fruits 2. sprouts 3. salads 4. fresh juices

Q27. Values : 1. People in the family has to be educated 2. They should be made skilled laborers 3. Government should take suitable steps to reduce the inflation 4. The benefits of the government schemes should reach the poor people correctly

Q28. Values : 1. Government should take suitable steps to reduce the inflation 2. The benefits of the government schemes should reach the poor people correctly 3. Price of essential commodities should be reduced. 4. The corruption menace has to be curbed.

Q29. Values : 1. Concern about fellow students 2. Helping others 3. Sharing of knowledge 4. Empathy

Q30. Values : 1. Systematic study 2. Quality of hard work 3. Study habit 4. Time management

Q31. Values : 1. Patriotisim 2. Sacrifice 3. Pride in our country 4. Leadership

Q33. Values : 1. Respect for elders 2. Concern for the aged 3. Lending a helping hand

Q34. Values : 1. Love and concern for grandparents 2. Respect for old people 3. Tolerance

CONTINUITY, DIFFERENTIAL CALCULUS & ITS APPLICATIONS

Q01. Point of discontinuity: x = 1. No, because life is precious so vehicles should be driven carefully.

Q02. Increase of ₹100 monthly subscription for Max.Income of the company. (i) The sharing (2-3 persons on the same route) will be promoted. (ii) Decrease in pollution. (iii) Decrease vehicle density on road (less traffic jams). (iv) Saving of energy (petrol/ diesel etc.).

Q03. As $f'(x) > 0$ for $0 < x < 1$ but, $f'(x)$ can be positive as well as negative when $-1 < x < 0$. So, $f'(x)$ can be positive as well as negative in $-1 < x < 1$. Hence, $f(x)$ is neither strictly increasing nor strictly decreasing. Yes, because strictness in not always good in life.

Q04. $x \in \left(\dfrac{1}{4}, \infty \right)$. To support the family, regular increasing income is must.

Q05. Four hours per day. By hard work, we can create skill in using the things learnt by us. So that, we don't make mistakes in the competition when the things are asked.

Q06. Yes, every work done in a planned way proves to be more fruitful. If a student makes a planning for his studies he can do wonders.

Q07. Maximum profit = ₹76 when x = 240. Yes it is good for society.

Q08. Length of rectangular aprt of window = $\dfrac{20}{4+\pi}$ m, and its width = $\dfrac{10}{4+\pi}$ m. (i) Large windows allow more light during daytime and hence will reduce the use of electricity. Saving energy (here, in the form of electricity) helps in conserving the environment as electricity is produced by using natural resources (coal, diesel etc.) which should be preserved for the sake of future generations. (ii) India has less sources of energy production as compared to its vast population. So, it is required to optimize the energy usage in India.

Q09. 4.8m/sec. In the festival-time, many people participate with full happiness and share their lives and enjoy it.

Q10. 4centimeters. As our country is still developing and most of the Indian people are from the middle class, so we should utilize our resources in proper way. Students should buy only those books which they feel really important. Instead of buying books for only one or two chapters, they should borrow it from the library.

Q11. Length of square base is 3cm and height of the box is 1.5cm. Yes, I agree that students don't utilize the resources properly. They get various notes photocopies and waste one side of the paper. Whereas other side of paper can be utilized for making notes/comments.

Q12. Find MC $(x) = \dfrac{d}{dx}C(x) = 3x^2 + 14x + 8 \Rightarrow MC(4) = 112$.

Q13. Show that $\dfrac{d}{dx}I(x) > 0$ for all x.

Q14. $P'(3) = 30.255$. **Value:** We should cut down the use of vehicles running on diesel as they cause damage to the environment by polluting air.

Q15. $\dfrac{1}{\pi}$ cm /s. (i) **Values reflected:** Bravery, sympathy for child labour, right of a child and raising voice against child labour. **Q16.** 0

Q17. Approximate error in volume = 24.64cm^3. Loss to the buyer = (Error in volume).(Cost) = ₹24,640. **Lesson:** A small error of 0.04cm can result in huge loss of ₹24,640. So it is needed to be careful while taking measurements.

INTEGRAL CALCULUS, AREAS & DIFFERENTIAL EQUATIONS

Q01. $\dfrac{x^2}{2} - \log|x^2 + 1| + \tan^{-1}x + k$. **Importance of integration (unity):** (i) United we stand, divided we fall. (ii) Union is strength.

Q02. Find R(x) = $\displaystyle\int_{10}^{20} MR(x)\,dx \Rightarrow 1900$ **Q03.** Use $\displaystyle\int_{3}^{9} MC(x)\,dx \Rightarrow `2000$

Q04. Use MC = $\dfrac{d}{dx}C(x) \Rightarrow dC(x) = MC\,dx \Rightarrow \int dC(x) = \int MC\,dx$

$\Rightarrow C(x) = \int MC\,dx \Rightarrow C(x) = \dfrac{4}{15}(x+5)^{3/2}(3x - 10) + k$. Use x = 4, C(x)= 314.40 to get k = 300. Hence write C(x). **Q05.** $R = \left(\dfrac{x-2}{x+2}\right)e^x + k$

Q06. Value of a = $(16)^{1/3}$. Equality helps to maintain peace and harmony in all aspect of society.

Q07. 4π cm^2 **Q08.** First part of the land from x = 0 to x = π having area of 2 Sq. units and Second part from x = π to x = 2π having area of 2 Sq. units.

Q09. Area of the field = 6 sq.units (i) 24 trees (ii) Plants provide us oxygen and play major role in rain, so plantation is essential for all human beings. **Q10.** x = $2y^2$, 8. Yes, polite child has a peaceful mind and peaceful mind grasps the ideas easily and understand the complicated concept with ease.

Q11. xy = 100. Yes, as the population increases area for living decreases, that is very harmful for us.

Q12. Use $m\dfrac{dv}{dt} + V\dfrac{dm}{dt} = 0 \Rightarrow \int dv + V\int \dfrac{dm}{m} = 0 \Rightarrow v + V\log m = C$. It is given that when v = 0, m = m_0. So, $V\log m_0 = C$. Hence $v = V\log\left(\dfrac{m_0}{m}\right)$. We should not encourage the rocket technology as they can be used to carry nuclear weapons to destroy countries.

Q13. Given $\dfrac{dr}{dt} \propto r \Rightarrow \dfrac{dr}{dt} = kr$, where k is proportionality constant and r is male-female ratio at any time. So, $\int \dfrac{dr}{r} = \int k\,dt \Rightarrow r = Ce^{kt}$, where C is the integral constant. Now in 1999, $t = 0$, and $r = \dfrac{1000}{980}$ so, $\dfrac{1000}{980} = Ce^0 \Rightarrow C = \dfrac{50}{49}$. That is, $r = \dfrac{50}{49}e^{kt}$ …(i). Now, in 2009, $t = 10$ and

$r = \dfrac{1000}{950}$ so, $\dfrac{1000}{950} = \dfrac{50}{49} e^{10k} \Rightarrow e^{10k} = \dfrac{98}{95}$.   Substituting this in (i), we get:

$r = \dfrac{50}{49} \left(e^{10k} \right)^{t/10} \Rightarrow r = \dfrac{50}{49} \left(\dfrac{98}{95} \right)^{t/10}$ …(ii). Hence in the year 2019, $t = 20$, $r = \dfrac{50}{49} \left(\dfrac{98}{95} \right)^{20/10}$

i.e., $r = \dfrac{50}{49} \left(\dfrac{98}{95} \right) \left(\dfrac{98}{95} \right) \approx 1.085 \approx \dfrac{1085}{1000}$.

(i) Gender equality promotes economic growth, reduce fertility, child mortality and under nutrition.
(ii) a. Stop female-foeticide.   b. Empower women to realize their rights.   c. Provide special opportunities to women to come at par with men in all walks of life.
Q14. (a) Fuel saving (b) less pollution   (c) space management
Q17. Concern for animals, kind hearted, not being cruel, bold decision making

3 DIMENSIONAL GEOMETRY

Q01. $\dfrac{191}{\sqrt{206}}$ units. **To save monuments:** (i) We should not harm any monument. (ii) We should not write anything on it. (iii) We should respect our national heritage.
Q02. The point of image is (1, 1, 11). Conservation of monuments is very important because it is a part of our history and their contribution.   **Q03.** S.D. = 0, this means they meet to an accident. If a driver follow speed limit there will be minimum chance of accident.
Q04. Find S.D. $\Rightarrow 2\sqrt{29}$ units.   **Q05.** 11 units
Q06. Since both the points lie on the line of fire so, the shooter will be successful in his attempt.
Q07. Note that point P is foot of perpendicular $\Rightarrow$ P(1, 2, 8). Also, AP = $3\sqrt{21}$ units.
Q08. Show that the line is parallel to the given plane.   **Q09.** 4 units.

LINEAR PROGRAMMING PROBLEM

Q01. Minimum cost = ₹38.00 at x = 2, y = 4. Balanced diet keeps fit, healthy and disease free life for a person.
Q02. Type A fertilizers = 50 kg, Type B = 40 kg. Minimum cost = ₹570. **Side effects:** Excessive use of fertilizers can spoil the quality of crop also it may cause infertility of land.
Q03. Max. Z = x + y. Subject to constraints: x/40 + y/70 $\leq$ 1, 2x + 7y $\leq$ 100; x, y $\geq$ 0. Here x & y represents the distance travelled by the boy at speed of 40km/hr & 70km/h respectively. (i) x = 1560/41km, y = 140/41km. (ii) It saves petrol. It saves money. (iii) No, because according to the law driving license is issued when a person is above the 18 years of age.
Q04. 20 problems from first list and 80 problems from second list. Students who divide the time for each subject per day according to their need don't feel burden of any subject before the examination.
Q05. 10500 bottles of medicine A and 34500 bottles of medicine B and they can cover 325500 patients. We should not get panic and should not create panic in case of natural disaster. We must have the helpline numbers of government agencies and NGO working in case of natural disaster.
Q06. Max. Z = ₹360. No. of electronic machines = 8 and no. of manually operated machines = 12. Keeping the **'save environment'** factor in mind the manually operated machine should be promoted so that energy could be saved.
Q07. Maximum profit = ₹1680 at (12,6). Yes, teaching aid is necessary for teaching-learning process because: (a) it makes learning very easy. (b) it offers active learning.
Q08. Maximum production of Oxygen will be achieved when plant X and Y are planted in 300 hectares and 200 hectares of land respectively.
(i) Excess herbicide will get absorbed in the soil and may contaminate the water sources also.   (ii) Care should be taken while planting tress so that the variety of plants is such that they provide more oxygen for environment.
Q09. 30 hectares of land is to be allocated for crop A and 20 hectares for the crop B for maximum profit. Keeping the **'save environment'** factor in mind, we agree that the protection of wildlife is utmost necessary to maintain the balance in environment as their protection preserves the food-

chain. **Q10.** No, it's not a good idea. Cloth bags are biodegradable while plastic is not. Also cloth bags can be reused.

Q11. Using cycles is a better option since it is environment friendly and reduces pollution. It is also a good exercise option.

Q12. Pesticides may increase the yield but are harmful to mankind. Organic farming is a good option as it is free of use of any chemicals.

Q13. Potatoes are produced more and the farmer's production will help the community as the potatoes have high protein value than cabbages. So the farmer promotes health awareness.

Q14. Bamboo plates should be promoted more as they are bio-degradable and eco friendly.

Q15. Books can benefit more as it promotes reading habit of the children. Or Colour Boxes helps to improve their creativity.

PROBABILITY

Q01. 2/3. Chances of success increase when ideas flow independently. Hard work pays the fruits.

Q02. 0.488. Peace is better than strike. As the probability of completion of job on time if there is strike is less then ½.

Q03. Probability Distribution:

X	0	1	2
P(X)	9/16	6/16	1/16

Mean = 1/2.

i. No, it may be good once or twice but not forever.

ii. Honesty pays in a long run.

Q04. Required probability = 10/13. Speaking truth pays in the long run. Although sometimes lie told for a good cause is not bad.

Q05. Since the man is just one step away from the initial point, he is either a step forward or a step backward from the initial point at the end of nine steps. **Case I :** If he is one step forward, then he must have taken five steps in forward direction while 4 steps in backward direction. **Case II :** If he is one step backward, then he must have taken four steps in forward and five steps in backward direction. So required probability = $P(X = 5 \text{ or } 4) = P(5) + P(4) = 126 \times (0.6)^4 (0.4)^4 = 126 \times (0.24)^4$.

Yes, addiction of wine or smoking is definitely harmful for a person and its family.

Q06. (i) 1/22 (ii) Yes, because a cheater finds it hard to do any work independently. It may prove beneficial and tempting in the beginning but it is harmful in long run.

Q07. 3/4. **Factors:** (i) Regular study (ii) Hard work (iii) Good memory (iv) Well time management (v) Writing skills.

Q08. 3/8. **Benefits of speaking truth:** (i) It gives positive thinking & satisfaction.
(ii) Everyone loves it. (iii) It is good life skill.

Q09. Probability Distribution:

X	0	1	2
P(X)	156/380	182/380	42/380

I would like to be vegetarian as vegetarian food is much easier to digest than non-vegetarian. Also, for non-vegetarian food we have to kill animals which is not good thing because everybody has right to survive.

Q10. 7/8. A Student is future of a country. If a student is sincere then he/she can serve the country in a better way.

Q11. 27/83. Pollution free environment minimizes the health problems in the human being.

Q12. Probability Distribution:

X	0	1	2	3
P(X)	27/64	27/64	9/64	1/64

Values lost by chairman:
Honesty, integrity.

Q13. 15/34. Skilled person can complete a work in better way than other person.

Q14. Probability Distribution:

X	0	1	2
P(X)	$(11/20)^2$	$2(9/20)(11/20)$	$(9/20)^2$

To maintain the ratio of male and female equally. This is important to consider both children are equal.

Q15. 0.225. People in anger cannot use their presence of mind and become violent and destroy public property in riots which is indirectly their own property.

Q16. 0.75. Self studies with the help of peers is best as through it students can get the knowledge in depth of each concept. But students should be regular in school and if they feel need they could join different classes.

Q17. Probability Distribution:

X	0	1	2
P(X)	87/245	120/245	38/245

Mean = 196/245.

Importance of Non-violence in patriotism: In order to have a peaceful environment both the values *i.e.*, patriotism and non-violence are required. Only patriotism with the violence could be very dangerous.

Q18. 2/3. If Arjun copies the answer, he will be violating the value of honesty in his character. He should not guess the answer as well as that may fetch him negative marking for a wrong guess. He should accept the question the way it is and leave it unanswered as cheating may get him marks in this exam but this habit may not let him develop integrity of character in the long run.

Q19. 3/26. **Suggestion:** Cycle should be promoted as it is good for: i. Health, ii. No pollution, iii. Saves energy (i.e., no need of petrol).

Q20. Let N = 729 be the no. of families having six children. Let probability of survival of a girl child be p = 1/3 and, that of boy be q = 2/3. Let X be the no. of girls in the family. Then X = 0, 1, 2, 3, 4, 5, 6. So, probability of 2 girls and 4 boys in the family of six is = P(2) = $^6C_2 \left(\dfrac{1}{3}\right)^2 \left(\dfrac{2}{3}\right)^{6-2} = \dfrac{80}{243}$.

So, no. of families having 2 girls and 4 boys in that state $= N \times P(2) = 729 \times \dfrac{80}{243} = 240$. Yes, the female child is neglected in the backward state. **Steps to be taken to restore respect of female child:** Moral education to society to spread awareness among people and incentives for the female child (just like free education, various schemes should be organised like one run by Delhi Govt. i.e., Laadli-Yojna).

Q21. Probability Distribution:

X	0	1	2	3
P(X)	406/988	435/988	135/988	12/988

Mean = 741/988 = 3/4.

Importance of sports in Education: It takes care of the mental and physical fitness of the body. It is helpful for the growth of student in the field of studies as well.

Q22. $P(M|E) = \dfrac{P(M)P(E|M)}{P(M)P(E|M) + P(W)P(E|W)}$, where E: event of taking brand X.

$\Rightarrow \quad = \dfrac{(70/100).(10/100)}{(70/100).(10/100) + (30/100).(20/100)} = \dfrac{7}{13}$.

Reason: Intake of pan-masala could be highly injurious to health. It causes cancer. So we would discourage its intake.

Q23. 3/4; (i) **Values reflected:** Regularity and intelligence. (ii) Regularity is required at every stage in the life. Regularity increases our capabilities and makes us able to put the best of our potential. We are able to achieve certain targets with our continuous efforts.

Q24. 21/43. (i) Adulteration is dangerous as it is harmful for user's health. (ii) To prevent adulteration, we should spread awareness against it in the society.

Q25. (i) 4/5 (ii) 1/3 (iii) 1/2 (iv) **Values discussed:** Never indulgence in any type of corruption and, always speaking the truth. (v) Values contribute to intellectual development, use of abilities, achieve creativity, personal development and hence development of society. So we must practice values in our life.

Q26. 42%. Since no one trusts a liar, so the statement of B will carry more weight as he speaks truth in more number of cases than A.

Q27. 14/29. **Interpretation of result:** It is evident that if a patient follows a course of meditation and yoga, then he is less likely to get heart-attack. [Since P(B|E) = 15/29.] So, clearly a course of meditation and yoga is more beneficial as compared to the intake of drugs.

Q28. P and Q are likely to agree in 62% of the cases in stating the same fact.   No, when they agree, it doesn't mean that they are speaking the truth. In fact, it may be that they both of them are lying.

Q29. A and B are likely to contradict each other in 30% of the cases in stating the same fact.
Though B speaks truth in 90% of the cases but he also lies in 10% of the case. So, his statement is not always true.

Q30. 1. Education up to class 12th (minimum) 2. Acquiring soft skills 3. Acquiring linguistic skills 4. Cultivating working qualities like responsibility and team work 5. Enrolling in vocational courses

Q31. 1. Family needs will be addressed faster 2. Gender bias and social evils targeted at women will wane 3. Needs of women will come to the fore and improve self confidence amongst them

Q32. 1. Personal discipline 2. Acquiring survival skills 3. Team spirit and learning to work as a team 4. Patriotism 5. Care for others and social service

Q33. 1. Uses energy more efficiently (saves energy) 2. Lower price 3. Brighter 4. Releases lesser amount of heat.

Q34. 1. Eco-friendly (no pollution) 2. Health benefits of individual 3. Lower price 4. Relieves parking menace 5. Saves energy

☞ For any clarification(s), please contact on any of the followings :

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