Test Booklet Code

E

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Answers & Solutions

Time : 3 hrs.

JEE (MAIN)-2014

M.M. : 360

MATHEMATICS

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Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted 4 **(four)** marks for each correct response.
4. *Candidates will be awarded marks as stated above in Instructions No. 3 for correct response of each question. ¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
5. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.
6. Use ***Blue/Black Ball Point Pen only*** for writing particulars/marking responses on ***Side-1*** and ***Side-2*** of the Answer Sheet. ***Use of pencil is strictly prohibited.***
7. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination hall/room.
8. The **CODE** for this Booklet is **E.** Make sure that the **CODE** printed on Side-2 of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.

PART–C : MATHEMATICS

61. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to

(1) X (2) Y

(3) N (4) $Y - X$

Answer (2)

Sol. $X = \{(1+3)^n - 3n - 1, n \in N\}$

$= 3^2({}^nC_2 + {}^nC_3.3 + ... + 3^{n-2}), n \in N\}$

= {Divisible by 9}

$Y = \{9(n-1), n \in N\}$

= (All multiples of 9}

So, $X \subseteq Y$

i.e., $\boxed{X \cup Y = Y}$

62. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$

(1) Is strictly greater than $\frac{5}{2}$

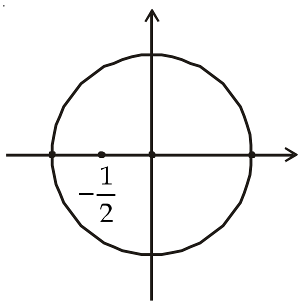
(2) Is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

(3) Is equal to $\frac{5}{2}$

(4) Lies in the interval (1, 2)

Answer (4)

Sol.



$\left|z + \frac{1}{2}\right|$

So, $\left||z| - \frac{1}{2}\right| \leq \left|z + \frac{1}{2}\right|$

$\Rightarrow \left|z + \frac{1}{2}\right| \geq \left|2 - \frac{1}{2}\right|$

$\Rightarrow \left|z_{min.} = \frac{3}{2}\right|$

63. If $a \in R$ and the equation

$-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$

(where $[x]$ denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval

(1) (–2, –1)

(2) $(-\infty, -2) \cup (2, \infty)$

(3) $(-1, 0) \cup (0, 1)$

(4) (1, 2)

Answer (3)

Sol. $-3(x - [x])^2 + 2[x - [x]) + a^2 = 0$

$3\{x\}^2 - 2\{x\} - a^2 = 0$

$a \neq 0,\ 3\left(\{x\}^2 - \frac{2}{3}\{x\}\right) = a^2$

$a^2 = 3\left(\{x\} - \frac{1}{3}\right)^2 - \frac{1}{3}$

$0 \leq \{x\} < 1$ and $-\frac{1}{3} \leq \{x\} - \frac{1}{3} < \frac{2}{3}$

$0 \leq 3\left(\{x\} - \frac{1}{3}\right)^2 < \frac{4}{3}$

$-\frac{1}{3} \leq 3\left(\{x\} - \frac{1}{3}\right)^2 - \frac{1}{3} < 1$

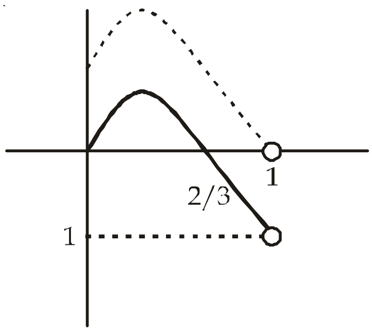
For non-integral solution

$0 < a^2 < 1$ and $a \in (-1, 0) \cup (0, 1)$

Alternative

$-3\{x\}^2 + 2\{x\} + a^2 = 0$

Now, $-3\{x\}^2 + 2\{x\}$



to have no integral roots $0 < a^2 < 1$

$\therefore\ a \in (-1, 0) \cup (0, 1)$

64. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in A.P. and $\frac{1}{\alpha}+\frac{1}{\beta}=4$, then the value of $|\alpha - \beta|$ is

(1) $\frac{\sqrt{34}}{9}$ (2) $\frac{2\sqrt{13}}{9}$

(3) $\frac{\sqrt{61}}{9}$ (4) $\frac{2\sqrt{17}}{9}$

Answer (2)

Sol.

$\because$ p, q, r are in AP

$2q = p + r$ ...(i)

Also $\frac{1}{\alpha}+\frac{1}{\beta}=4$

$\Rightarrow \frac{\alpha+\beta}{\alpha\beta}=4$

$=\frac{-\frac{q}{p}}{\frac{r}{p}}=4 \Rightarrow q=-4r$ ...(ii)

From (i)

$2(-4r) = p + r$

$p = -9r$

$q = -4r$

$r = r$

Now $|\alpha-\beta|=\sqrt{(\alpha+\beta)^2-4\alpha\beta}$

$=\sqrt{\left(\frac{-q}{p}\right)^2-\frac{4r}{p}}$

$=\frac{\sqrt{q^2-4pr}}{|p|}$

$=\frac{\sqrt{16r^2+36r^2}}{|-9r|}$

$=\frac{2\sqrt{13}}{9}$

65. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$= K(1-\alpha)^2 (1-\beta)^2 (\alpha-\beta)^2$, then K is equal to

(1) 1 (2) –1

(3) $\alpha\beta$ (4) $\frac{1}{\alpha\beta}$

Answer (1)

Sol. $$\begin{vmatrix} 1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$=\begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & 1 \\ \alpha^2 & \beta^2 & 1 \end{vmatrix} \times \begin{vmatrix} 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \\ 1 & 1 & 1 \end{vmatrix}$$

$= [(1-\alpha)(1-\beta)(1-\beta)]^2$

So, $\boxed{k=1}$

66. If A is an 3×3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals

(1) B^{-1} (2) $(B^{-1})'$

(3) $I + B$ (4) I

Answer (4)

Sol. $BB' = (A^{-1}.A')(A(A^{-1})')$

$= A^{-1}.A.A'.(A^{-1})^1$ {as $AA' = A'A$}

$= I(A^{-1}A)'$

$= I.I = I^2 = I$

67. If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to

(1) $\left(14, \frac{272}{3}\right)$ (2) $\left(16, \frac{272}{3}\right)$

(3) $\left(16, \frac{251}{3}\right)$ (4) $\left(14, \frac{251}{3}\right)$

Answer (2)

Sol. $(1 + ax + bx^2)(1 - 2x)^{18}$

$(1 + ax + bx^2)[{}^{18}C_0 - {}^{18}C_1(2x) + {}^{18}C_2(2x)^2 - {}^{18}C_3(2x)^3 + {}^{18}C_4(2x)^4 - ....]$

Coeff. of $x^3 = -{}^{18}C_3.8 + a \times 4.{}^{18}C_2 - 2b \times 18 = 0$

$= -\frac{18\times17\times16}{6}.8+\frac{4a+18\times17}{2}-36b=0$

$= -51 \times 16 \times 8 + a \times 36 \times 17 - 36b = 0$

$= -34 \times 16 + 51a - 3b = 0$

$= 51a - 3b = 34 \times 16 = 544$

$= 51a - 3b = 544$ ... (i)

Only option number (2) satisfies the equation number (i).

68. If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9 = k(10)^9$, then k is equal to

(1) 100 (2) 110

(3) $\frac{121}{10}$ (4) $\frac{441}{100}$

Answer (1)

Sol. $10^9 + 2\cdot(11)(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9 = k(10)^9$

$x = 10^9 + 2\cdot(11)(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9$

$\frac{11}{10}x = 11\cdot10^8 + 2\cdot(11)^2\cdot(10)^7 + \ldots + 9(11)^9 + 11^{10}$

$x\left(1-\frac{11}{10}\right) = 10^9 + 11(10)^8 + 11^2\times(10)^7 + \ldots + 11^9 - 11^{10}$

$\Rightarrow -\frac{x}{10} = 10^9\left(\frac{\left(\frac{11}{10}\right)^{10}-1}{\frac{11}{10}-1}\right) - 11^{10}$

$\Rightarrow -\frac{x}{10} = (11^{10} - 10^{10}) - 11^{10} = -10^{10}$

$\Rightarrow x = 10^{11} = k\cdot10^9$

$\Rightarrow k = 100$

69. Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is

(1) $2-\sqrt{3}$ (2) $2+\sqrt{3}$

(3) $\sqrt{2}+\sqrt{3}$ (4) $3+\sqrt{2}$

Answer (2)

Sol. $a, ar, ar^2 \rightarrow$ G.P.

$a, 2ar, ar^2 \rightarrow$ A.P.

$2 \times 2ar = a + ar^2$

$4r = 1 + r^2$

$\Rightarrow r^2 - 4r + 1 = 0$

$r = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$

$\boxed{r = 2+\sqrt{3}}$

$r = 2-\sqrt{3}$ is rejected

$\because$ $(r > 1)$

G.P. is increasing.

70. $\lim_{x\to0}\frac{\sin(\pi\cos^2 x)}{x^2}$ is equal to

(1) $-\pi$ (2) π

(3) $\frac{\pi}{2}$ (4) 1

Answer (2)

Sol. $\lim_{x\to0}\frac{\sin(\pi\cos^2 x)}{x^2}$

$= \lim_{x\to0}\frac{\sin(\pi(1-\sin^2 x))}{x^2}$

$= \lim_{x\to0} \sin\frac{(\pi-\pi\sin^2 x)}{x^2}$

$= \lim_{x\to0} \sin\frac{(\pi\sin^2 x)}{x^2}$ $[\because \sin(\pi-\theta) = \sin\theta]$

$= \lim_{x\to0} \sin\frac{(\pi\sin^2 x)}{(\pi\sin^2 x)} \times \frac{\pi\sin^2 x}{x^2}$

$= \lim_{x\to0} 1\times\pi\left(\frac{\sin x}{x}\right)^2 = \pi$

71. If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$, then $g'(x)$ is equal to

(1) $\frac{1}{1+\{g(x)\}^5}$ (2) $1 + \{g(x)\}^5$

(3) $1 + x^5$ (4) $5x^4$

Answer (2)

Sol. $f'(x) = \frac{1}{1+x^5} = f(g(x)) = x \rightarrow f'(g(x))\, g'(x) = 1$

$g'(x) = \frac{1}{f'(g(x))} = 1 + (g(x))^5$

72. If f and g are differentiable functions in [0, 1] satisfying $f(0) = 2 = g(1)$, $g(0) = 0$ and $f(1) = 6$, then for some $c \in]0, 1[$

(1) $f'(c) = g'(c)$ (2) $f'(c) = 2g'(c)$

(3) $2f'(c) = g'(c)$ (4) $2f'(c) = 3g'(c)$

Answer (2)

Sol. Using, mean value theorem

$f'(c) = \frac{f(1)-f(0)}{1-0} = 4$

$g'(c) = \frac{g(1)-g(0)}{1-0} = 2$

so, $\boxed{f'(c) = 2g'(c)}$

73. If $x = -1$ and $x = 2$ are extreme points of $f(x)=\alpha\log|x|+\beta x^2+x$ then

(1) $\alpha=2,\ \beta=-\frac{1}{2}$ (2) $\alpha=2,\ \beta=\frac{1}{2}$

(3) $\alpha=-6,\ \beta=\frac{1}{2}$ (4) $\alpha=-6,\ \beta=-\frac{1}{2}$

Answer (1)

Sol. $f(x)=\alpha\log|x|+\beta x^2+x$

$f'(x)=\frac{\alpha}{x}+2\beta x+1=0$ at $x = -1, 2$

$-\alpha-2\beta+1=0 \Rightarrow \alpha+2\beta=1$ ...(i)

$\frac{\alpha}{2}+4\beta+1=0 \Rightarrow \alpha+8\beta=-2$ ...(ii)

$6\beta=-3 \Rightarrow \beta=-\frac{1}{2}$

$\therefore\ \alpha=2$

74. The integral $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to

(1) $(x+1)e^{x+\frac{1}{x}}+c$ (2) $-x\,e^{x+\frac{1}{x}}+c$

(3) $(x-1)e^{x+\frac{1}{x}}+c$ (4) $x\,e^{x+\frac{1}{x}}+c$

Answer (4)

Sol. $I = \int\left\{e^{\left(x+\frac{1}{x}\right)}+x\left(1-\frac{1}{x^2}\right)e^{x+\frac{1}{x}}\right\}dx$

$= x.e^{x+\frac{1}{x}}+c$

As $\int(xf'(x)+f(x))dx=xf(x)+c$

75. The integral

$\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$ equals

(1) $4\sqrt{3}-4$ (2) $4\sqrt{3}-4-\frac{\pi}{3}$

(3) $\pi - 4$ (4) $\frac{2\pi}{3}-4-4\sqrt{3}$

Answer (2)

Sol. $\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$

$=\int_0^{\pi}\left|2\sin\frac{x}{2}-1\right|dx$ $\left[\begin{array}{l}\sin\frac{x}{2}=\frac{1}{2}\\ \Rightarrow\frac{x}{2}=\frac{\pi}{6}\to x=\frac{\pi}{3}\\ \frac{x}{2}=\frac{5\pi}{6}\to x=\frac{5\pi}{3}\end{array}\right]$

$=\int_0^{\pi/3}\left(1-2\sin\frac{x}{2}\right)dx+\int_{\pi/3}^{\pi}\left(2\sin\frac{x}{2}-1\right)dx$

$=\left[x+4\cos\frac{x}{2}\right]_0^{\pi/3}+\left[-4\cos\frac{x}{2}-x\right]_{\pi/3}^{\pi}$

$=\frac{\pi}{3}+4\frac{\sqrt{3}}{2}-4+\left(0-\pi+4\frac{\sqrt{3}}{2}+\frac{\pi}{3}\right)$

$=4\sqrt{3}-4-\frac{\pi}{3}$

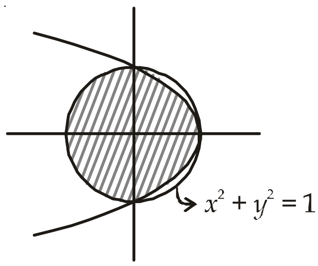
76. The area of the region described by $A = \{(x, y) : x^2 + y^2 \le 1 \text{ and } y^2 \le 1 - x\}$ is

(1) $\frac{\pi}{2}-\frac{2}{3}$ (2) $\frac{\pi}{2}+\frac{2}{3}$

(3) $\frac{\pi}{2}+\frac{4}{3}$ (4) $\frac{\pi}{2}-\frac{4}{3}$

Answer (3)

Sol.



Shaded area

$=\frac{\pi(1)^2}{2}+2\int_0^1\sqrt{(1-x)}\,dx$

$=\frac{\pi}{2}+\frac{2(1-x)^{3/2}}{3/2}(-1)\Big|_0^1$

$=\frac{\pi}{2}+\frac{4}{3}(0-(-1))$

$=\frac{\pi}{2}+\frac{4}{3}$

77. Let the population of rabbits surviving at a time t be governed by the differential equation

$\frac{dp(t)}{dt}=\frac{1}{2}p(t)-200$. If $p(0) = 100$, then $p(t)$ equals

(1) $600 - 500\, e^{t/2}$

(2) $400 - 300\, e^{-t/2}$

(3) $400 - 300\, e^{t/2}$

(4) $300 - 200\, e^{-t/2}$

Answer (3)

Sol. $\frac{dp(t)}{dt}=\frac{1}{2}p(t)-200$

$$\int\frac{d(p(t))}{\left(\frac{1}{2}p(t)-200\right)}=\int dt$$

$$2\log\left(\frac{p(t)}{2}-200\right)=t+c$$

$$\frac{p(t)}{2}-200=e^{\frac{t}{2}}k$$

Using given condition $p(t) = 400 - 300\, e^{t/2}$

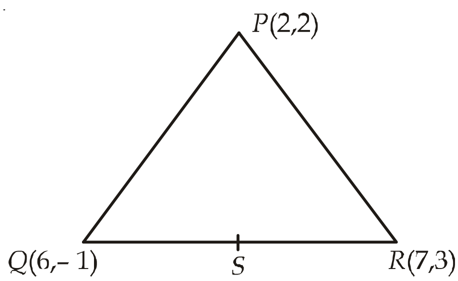
78. Let PS be the median of the triangle with vertices $P(2, 2)$, $Q(6, -1)$ and $R(7, 3)$. The equation of the line passing through $(1, -1)$ and parallel to PS is

(1) $4x + 7y + 3 = 0$ (2) $2x - 9y - 11 = 0$

(3) $4x - 7y - 11 = 0$ (4) $2x + 9y + 7 = 0$

Answer (4)

Sol.



S is mid-point of QR

So $S=\left(\frac{7+6}{2},\frac{3-1}{2}\right)$

$=\left(\frac{13}{2},1\right)$

Slope of PS = $\frac{2-1}{2-\frac{13}{2}}=-\frac{2}{9}$

Equation of line $\Rightarrow y-(-1)=-\frac{2}{9}(x-1)$

$9y + 9 = -2x + 2 \Rightarrow 2x + 9y + 7 = 0$

79. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then

(1) $3bc - 2ad = 0$

(2) $3bc + 2ad = 0$

(3) $2bc - 3ad = 0$

(4) $2bc + 3ad = 0$

Answer (1)

Sol. Let $(\alpha, -\alpha)$ be the point of intersection

$\therefore\ 4a\alpha - 2a\alpha + c = 0 \Rightarrow \alpha=-\frac{c}{2a}$

and $5b\alpha - 2b\alpha + d = 0 \Rightarrow \alpha=-\frac{d}{3b}$

$\Rightarrow 3bc = 2ad$

$\Rightarrow 3bc - 2ad = 0$

Alternative method :

The point of intersection will be

$$\frac{x}{2ad-2bc}=\frac{-y}{4ad-5bc}=\frac{1}{8ab-10ab}$$

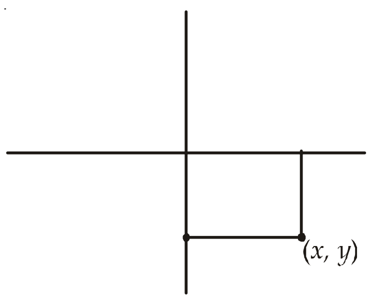
$\Rightarrow x=\frac{2(ad-bc)}{-2ab}$

$\Rightarrow y=\frac{5bc-4ad}{-2ab}$

$\because$ Point of intersection is in fourth quadrant so x is positive and y is negative.

Also distance from axes is same

So $x = -y$ ($\because$ distance from x-axis is $-y$ as y is negative)



$$\frac{2(ad-bc)}{-2ab}=\frac{-(5bc-4ad)}{-2ab}$$

$2ad - 2bc = -5bc + 4ad$

$\Rightarrow 3bc - 2ad = 0$ ...(i)

80. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is

(1) $(x^2 + y^2)^2 = 6x^2 + 2y^2$

(2) $(x^2 + y^2)^2 = 6x^2 - 2y^2$

(3) $(x^2 - y^2)^2 = 6x^2 + 2y^2$

(4) $(x^2 - y^2)^2 = 6x^2 - 2y^2$

Answer (1)

Sol. Here ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $a^2 = 6$, $b^2 = 2$

Now, equation of any variable tangent is

$$y = mx \pm \sqrt{a^2m^2 + b^2} \quad \text{...(i)}$$

where m is slope of the tangent

So, equation of perpendicular line drawn from centre to tangent is

$$y = \frac{-x}{m} \quad \text{...(ii)}$$

Eliminating m, we get

$$(x^2 + y^2)^2 = a^2x^2 + b^2y^2$$

$$\Rightarrow \boxed{(x^2 + y^2)^2 = 6x^2 + 2y^2}$$

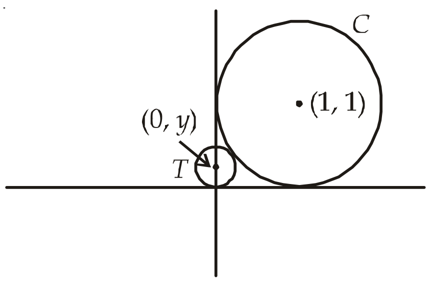
81. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) $\frac{\sqrt{3}}{\sqrt{2}}$ (4) $\frac{\sqrt{3}}{2}$

Answer (2)

Sol.



$C \equiv (x-1)^2 + (y-1)^2 = 1$

Radius of $T = |y|$

T touches C externally

$(0 - 1)^2 + (y - 1)^2 = (1 + |y|)^2$

$\Rightarrow 1 + y^2 + 1 - 2y = 1 + y^2 + 2|y|$

If $y > 0$,

$y^2 + 2 - 2y = y^2 + 1 + 2y$

$\Rightarrow 4y = 1$

$\Rightarrow y = \frac{1}{4}$

If $y < 0$,

$y^2 + 2 - 2y = y^2 + 1 - 2y$

$\Rightarrow 1 = 2$ (Not possible)

$\therefore y = \frac{1}{4}$

82. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is

(1) $\frac{1}{8}$ (2) $\frac{2}{3}$

(3) $\frac{1}{2}$ (4) $\frac{3}{2}$

Answer (3)

Sol. $y^2 = 4x$ ...(1)

$x^2 = -32y$ ...(2)

m be slope of common tangent

Equation of tangent (1)

$$y = mx + \frac{1}{m} \quad \text{...(i)}$$

Equation of tangent (2)

$$y = mx + 8m^2 \quad \text{...(ii)}$$

(i) and (ii) are identical

$$\frac{1}{m} = 8m^2$$

$$\Rightarrow m^3 = \frac{1}{8}$$

$$\boxed{m = \frac{1}{2}}$$

Alternative method :

Let tangent to $y^2 = 4x$ be

$$y = mx + \frac{1}{m}$$

as this is also tangent to $x^2 = -32y$

Solving $x^2 + 32mx + \frac{32}{m} = 0$

Since roots are equal

$\therefore D = 0$

$$\Rightarrow (32)^2 - 4 \times \frac{32}{m} = 0$$

$$\Rightarrow m^3 = \frac{4}{32}$$

$$\Rightarrow m = \frac{1}{2}$$

83. The image of the line

$\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line

(1) $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$

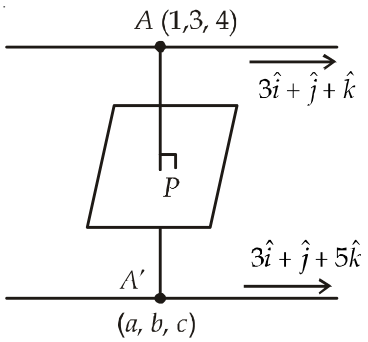
(2) $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$

(3) $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

(4) $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z+2}{5}$

Answer (3)

Sol.



$\frac{a-1}{2}=\frac{b-3}{-1}=\frac{c-4}{1}=\lambda$

$\Rightarrow$ $a = 2\lambda + 1$

$b = 3 - \lambda$

$c = 4 + \lambda$

$P \equiv \left(\lambda+1,\ 3-\frac{\lambda}{2},\ 4+\frac{\lambda}{2}\right)$

$2(\lambda+1)-\left(3-\frac{\lambda}{2}\right)+\left(4+\frac{\lambda}{2}\right)+3=0$

$2\lambda+2-3+\frac{\lambda}{2}+4+\frac{\lambda}{2}+3=0$

$3\lambda + 6 = 0 \Rightarrow \lambda = -2$

$a = -3,\ b = 5,\ c = 2$

So the equation of the required line is

$\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

84. The angle between the lines whose direction consines satisfy the equations $l + m + n = 0$ and $l^2 = m^2 + n^2$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

Answer (3)

Sol. $l + m + n = 0$

$l^2 = m^2 + n^2$

Now, $(-m - n)^2 = m^2 + n^2$

$\Rightarrow mn = 0$

$m = 0$ or $n = 0$

If $m = 0$

then $l = -n$

$l^2 + m^2 + n^2 = 1$

Gives

$\Rightarrow n=\pm\frac{1}{\sqrt{2}}$

i.e. (l_1, m_1, n_1)

$=\left(-\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$

If $n = 0$

then $l = -m$

$l^2 + m^2 + n^2 = 1$

$\Rightarrow 2m^2 = 1$

$\Rightarrow m^2=\frac{1}{2}$

$\Rightarrow m=\pm\frac{1}{\sqrt{2}}$

Let $m=\frac{1}{\sqrt{2}}$

$l=-\frac{1}{\sqrt{2}}$

$n = 0$

(l_2, m_2, n_2)

$=\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right)$

$\therefore \cos\theta=\frac{1}{2}$

$\theta=\frac{\pi}{3}$

85. If $\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right]=\lambda[\vec{a}\ \vec{b}\ \vec{c}]^2$ then λ is equal to

(1) 0 (2) 1

(3) 2 (4) 3

Answer (2)

Sol. L.H.S.

$=(\vec{a}\times\vec{b})\cdot[(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})]$

$=(\vec{a}\times\vec{b})\cdot[(\vec{b}\times\vec{c}\cdot\vec{a})\vec{c}-(\vec{b}\times\vec{c}\cdot\vec{c})\vec{a}]$

$=(\vec{a}\times\vec{b})\cdot[[\vec{b}\ \vec{c}\ \vec{a}]\vec{c}]$ $[\because \vec{b}\times\vec{c}.\vec{c}=0]$

$=[\vec{a}\ \vec{b}\ \vec{c}]\cdot(\vec{a}\times\vec{b}\cdot\vec{c})=[\vec{a}\ \vec{b}\ \vec{c}]^2$

$[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]=[\vec{a}\ \vec{b}\ \vec{c}]^2$

So $\lambda = 1$

86. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then the events A and B are

(1) Independent but not equally likely
(2) Independent and equally likely
(3) Mutually exclusive and independent
(4) Equally likely but not independent

Answer (1)

Sol. $P(\overline{A \cup B}) = \frac{1}{6} \Rightarrow P(A \cup B) = 1 - \frac{1}{6} = \frac{5}{6}$

$P(\overline{A}) = \frac{1}{4} \Rightarrow P(A) = 1 - \frac{1}{4} = \frac{3}{4}$

$\because \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{5}{6} = \frac{3}{4} + P(B) - \frac{1}{4}$

$P(B) = \frac{1}{3}$

$\because \quad P(A) \neq P(B)$ so they are not equally likely.

Also $P(A) \times P(B) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$

$= P(A \cap B)$

$\because \quad P(A \cap B) = P(A) \cdot P(B)$ so A & B are independent.

87. The variance of first 50 even natural numbers is

(1) 437 (2) $\frac{437}{4}$

(3) $\frac{833}{4}$ (4) 833

Answer (4)

Sol. Variance $= \frac{\Sigma x_i^2}{N} - (\overline{x})^2$

$\Rightarrow \sigma^2 = \frac{2^2 + 4^2 + ... + 100^2}{50} - \left(\frac{2 + 4 + ... + 100}{50}\right)^2$

$= \frac{4(1^2 + 2^2 + 3^2 + ... + 50^2)}{50} - (51)^2$

$= 4\left(\frac{50 \times 51 \times 101}{50 \times 6}\right) - (51)^2$

$= 3434 - 2601$

$\Rightarrow \sigma^2 = 833$

88. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in R$ and $k \ge 1$. Then $f_4(x) - f_6(x)$ equals

(1) $\frac{1}{4}$ (2) $\frac{1}{12}$

(3) $\frac{1}{6}$ (4) $\frac{1}{3}$

Answer (2)

Sol. $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$

$f_4(x) - f_6(x) = \frac{1}{4}(\sin^4 x + \cos^4 x) - \frac{1}{6}(\sin^6 x + \cos^6 x)$

$= \frac{1}{4}[1 - 2\sin^2 x \cos^2 x] - \frac{1}{6}[1 - 3\sin^2 x \cos^2 x]$

$= \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

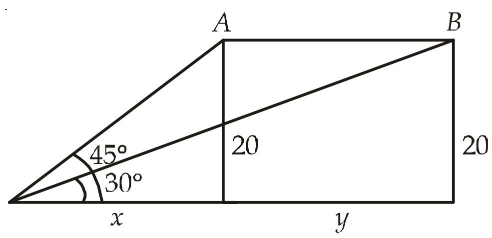
89. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45°. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. Then the speed (in m/s) of the bird is

(1) $20\sqrt{2}$ (2) $20\left(\sqrt{3} - 1\right)$

(3) $40\left(\sqrt{2} - 1\right)$ (4) $40\left(\sqrt{3} - \sqrt{2}\right)$

Answer (2)

Sol.



$t = 1$ s

From figure $\tan 45° = \frac{20}{x}$

and $\tan 30° = \frac{20}{x + y}$

so, $y = 20(\sqrt{3} - 1)$

i.e., speed $= 20(\sqrt{3} - 1)$ m/s.

90. The statement $\sim(p \leftrightarrow \sim q)$ is

(1) A tautology
(2) A fallacy
(3) Equivalent to $p \leftrightarrow q$
(4) Equivalent to $\sim p \leftrightarrow q$

Answer (3)

Sol. $\sim(p \leftrightarrow \sim q)$

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$
F	F	T	F	T
F	T	F	T	F
T	F	T	T	F
T	T	F	F	T

Clearly equivalent to $p \leftrightarrow q$