ASSERTION REASON Type Questions

By O.P. GUPTA
Indira Award Winner
M.+919650350480

In the following questions, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Unit 1 (Relations & Functions)

Relations & Functions, Inverse Trig. Functions

Q01. **Assertion (A) :** The relation $R = \{(a,b) : a \le b^2\}$ on the set R of real nos. is not reflexive.
Reason (R) : A relation on a set A is reflexive if $(a,a) \in R \ \forall \ a \in A$.

Q02. **Assertion (A) :** Let a relation R on the set $\mathbb{R}$ of real numbers be defined as $(a,b) \in R \Leftrightarrow 1+ab>0 \ \forall a,b \in \mathbb{R}$, is transitive relation.
Reason (R) : A relation on a set A is transitive if (a,b) and $(b,c) \in R$ implies $(a,c) \in R$ for all $a,b,c \in A$.

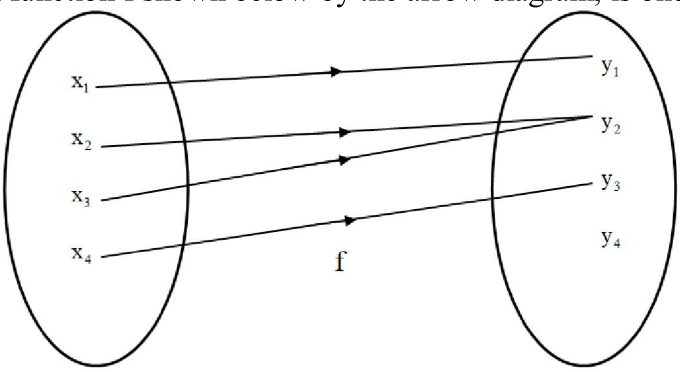
Q03. **Assertion (A) :** If R be the relation defined on Q (set of rational numbers) as $aRb \Leftrightarrow |a-b| \le \frac{1}{2}$, then is not a symmetric relation.
Reason (R) : A relation on a set A is symmetric if $(a,b) \in R$ implies $(b,a) \in R$ for all $a,b \in A$.

Q04. **Assertion (A) :** A reflexive relation may or may not be an identity relation.
Reason (R) : A relation R on A is identity relation iff $R = \{(a,b) : a \in A, b \in A \text{ and } a = b\}$.

Q05. **Assertion (A) :** Principal value of $\cos^{-1}\frac{1}{2} = \frac{\pi}{3}$.
Reason (R) : Range of $\cos^{-1} x$ is $x \in [0, \pi]$.

Q06. **Assertion (A) :** For $f(x) = \sin^{-1} x,\ D_f = x \in [-1, 1]$.
Reason (R) : $\sin^{-1} : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Q07. **Assertion (A) :** A function f shown below by the arrow diagram, is one-one.



Reason (R) : A function $f: A \to B$ is one-one if $f(\alpha) = f(\beta)$ implies $\alpha = \beta$ for all $\alpha, \beta \in A$.

Q08. **Assertion (A) :** $\sec^{-1}(-2) = \frac{2\pi}{3}$.

Reason (R) : $\sec^{-1}: A \to B$, where $A = R - (-1, 1)$ and $B = \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$.

Unit 2 (Algebra)
Matrices, Determinants

Q01. Let A and B be two symmetric matrices of order 3.
Assertion (A) : A(BA) and (AB)A are symmetric matrices.
Reason (R) : AB is symmetric matrix if matrix multiplication of A with B is commutative.

Q02. Let A be a square matrix of order 2.
Assertion (A) : $adj.(adj.A) = A$.
Reason (R) : $|adj.A| = |A|$.

Q03. **Assertion (A) :** If A is a skew-symmetric matrix of order 3 then, $|A| = 0$.
Reason (R) : If A is square matrix of order 2, then $|A| = |A^T| = |-A^T|$.

Q04. **Assertion (A) :** The inverse of $A = \begin{pmatrix} 5 & 1 \\ 2 & 2 \end{pmatrix}$ does not exist.
Reason (R) : Matrix A is non-singular.

Q05. **Assertion (A) :** If $A = \begin{pmatrix} 1 & 2 \\ 4 & 9 \end{pmatrix}$ then, $A^{-1} = \begin{pmatrix} 9 & -2 \\ -4 & 1 \end{pmatrix}$.
Reason (R) : For $A = \begin{pmatrix} a & c \\ d & b \end{pmatrix}$, $adj.A = \begin{pmatrix} -a & d \\ c & -b \end{pmatrix}$.

Unit 3 (Calculus)
Continuity & Differentiability, Applications Of Derivatives, Integrals, Application of Integrals, Differential Equations

Q01. **Assertion (A) :** $f(x) = \log|x|$ is always continuous for all real values of x.
Reason (R) : A function is always continuous at all the points of its domain.

Q02. **Assertion (A) :** $f(x) = |x-3|$ is always differentiable for all $x \in R - 3$.
Reason (R) : A continuous function in R (real nos.), is always differentiable in R (real nos.).

Q03. **Assertion (A) :** $f(x) = \begin{cases} 3 - x^2, \text{ if } x > 1 \\ x^2 + 1, \text{ if } x \le 1 \end{cases}$ is differentiable at $x = 1$.
Reason (R) : $f(x)$ is said to be differentiable at $x = c$, if $Rf'(c) = Lf'(c)$.

Q04. **Assertion (A) :** Function $f(x) = |x|$ is continuous everywhere in R (real nos.) but not differentiable at $x = 0$.
Reason (R) : For the given function $f(x) = |x|$, we have $\lim_{x \to 0^+} f(x) \ne \lim_{x \to 0^-} f(x)$.

Q05. **Assertion (A) :** $f(x)=e^x$ is an increasing function, where $x \in R$.

Reason (R) : If $f'(x) \le 0$ then, $f(x)$ is an increasing function.

Q06. **Assertion (A) :** For $f(x)=\tan^{-1}\frac{2x}{1-x^2}$, $f'(x)=\frac{2}{1+x^2}$.

Reason (R) : Inverse of trigonometric functions exists only in their restricted domains.

Q07. **Assertion (A) :** An angle θ, $0<\theta<\frac{\pi}{2}$ which increases twice as fast as its sine, is $\frac{\pi}{6}$.

Reason (R) : Rate of change of $3x^2$ with respect to x is, $6x$.

Q08. **Assertion (A) :** $\int_{-1/2}^{1/2} \log\left(\frac{1+x}{1-x}\right)dx = 0$.

Reason (R) : $\int_{-a}^{a} f(x)\,dx = 0$, if $f(-x)=-f(x)$.

Q09. **Assertion (A) :** $\frac{dy}{dx}+y\cot x = x$ is a linear differential equation in y.

Reason (R) : $y^2=x^2+c$ represents the general solution for differential equation $\frac{dy}{dx}=\frac{x}{y}$.

Q10. **Assertion (A) :** $\int_0^{\pi}\log(1+\cos x)\,dx=-\pi\log 2$, as $\int_0^{\pi/2}\log\cos x\,dx=-\frac{\pi}{2}\log 2$.

Reason (R) : $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$, if $f(x)$ is an even function.

Unit 4 (Vectors & 3 D Geometry)

Vector Algebra, Three Dimensional Geometry

Q01. **Assertion (A) :** $\vec{a}.\vec{b}=0$ implies, $\vec{a}\perp\vec{b}$, if $\vec{a}$ and $\vec{b}$ are non-zero vectors.

Reason (R) : Value of $\lambda=2$, if $\vec{a}\,||\,\vec{b}$, where $\vec{a}=2\hat{i}+4\hat{j}+3\lambda\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$.

Q02. **Assertion (A) :** For P(1, –2, 0) and Q(3, 5, 4), $\overrightarrow{PQ}=2\hat{i}+7\hat{j}+4\hat{k}$.

Reason (R) : $\overrightarrow{AB}=(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k}$, where $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$.

Q03. **Assertion (A) :** If the adjacent sides of a parallelogram are represented by $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+3\hat{j}-4\hat{k}$, then one of its diagonal is $3\hat{i}+2\hat{j}-3\hat{k}$.

Reason (R) : For a parallelogram whose adjacent sides are given by vectors $\vec{a}$ and $\vec{b}$, its diagonals are $(\vec{a}+\vec{b})$ and $\pm(\vec{a}-\vec{b})$.

Q04. **Assertion (A) :** If $\vec{a}=\hat{i}+2\hat{j}-2\hat{k}$ then, $\hat{a}=\frac{\hat{i}+2\hat{j}-2\hat{k}}{3}$, where $\hat{a}$ is a unit vector along $\vec{a}$.

Reason (R) : If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ then, a unit vector along $\vec{a}$ is given by $\frac{a_1\hat{i}+a_2\hat{j}+a_3\hat{k}}{a_1^2+a_2^2+a_2^2}$.

Q05. **Assertion (A) :** If a line passes through a point whose position vector is $\vec{a}=\hat{i}-4\hat{j}-2\hat{k}$ and is parallel to the vector $\vec{b}=2\hat{i}+2\hat{j}+3\hat{k}$ then, its equation is $\vec{r}=\hat{i}-4\hat{j}-2\hat{k}+\lambda(2\hat{i}+2\hat{j}+3\hat{k})$.

Reason (R) : If a line passes through a point whose position vector is $\vec{a}$ and is parallel to the vector $\vec{b}$ then, its equation is $\vec{r}=\vec{a}+\lambda\vec{b}$.

Q06. **Assertion (A) :** $\frac{x}{1}=\frac{y-3}{-2}=\frac{z}{2}$ and $\frac{x-4}{2}=\frac{y+7}{2}=\frac{z-1}{1}$ are perpendicular lines.

Reason (R) : The direction ratios of parallel lines are proportional.

Q07. **Assertion (A) :** The shortest distance between the lines $\vec{r}=8\hat{i}-9\hat{j}+10\hat{k}+\lambda(3\hat{i}-16\hat{j}+7\hat{k})$ and $\vec{r}=15\hat{i}+29\hat{j}+5\hat{k}+\mu(3\hat{i}+8\hat{j}-5\hat{k})$ is given by 14 units.

Reason (R) : The shortest distance between the parallel lines $\vec{r}=\vec{a}_1+\lambda\vec{b}$ and $\vec{r}=\vec{a}_2+\mu\vec{b}$, is given by $S.D.=\frac{\left|\vec{b}\times(\vec{a}_2-\vec{a}_1)\right|}{\left|\vec{b}\right|}$.

Unit 5 (Linear Programming)
Linear Programming Problems

Q01. **Assertion (A) :** In a particular LPP whose objective function is given as $Z=x+y$, the corner points of the feasible region are found to be (25, 0), (0, 40) and (0, 0) and so, $Z_{max}=40$.

Reason (R) : The maximum or minimum values of objective function occur at the corner point of the feasible region.

Q02. **Assertion (A) :** In a particular linear programming problem whose objective function is given as $Z=x+y$, the corner points of the feasible region are (25, 0), (0, 20), (20, 10) and (0, 0). Then, we have $Z_{max}=30$.

Reason (R) : The maximum or minimum values of objective function occur at a point, which lies inside the feasible region

Unit 6 (Probability)
Probability

Q01. **Assertion (A) :** Let A and B are two independent events. If $P(A)=0.2,\ P(B)=0.1$, then $P(A\cap B)=0.02$.

Reason (R) : For independent events A and B, we always have $P(A\cap B)=P(A)\times P(B)$.

Q02. **Assertion (A) :** If $P(A)=0.4,\ P(B)=p,\ P(A\cup B)=0.6$ and the events A and B are given to be independent events, then $(3p)=1$.

Reason (R) : For independent events A and B, we always have $P(A\cap B)\neq P(A)\times P(B)$.

Q03. **Assertion (A) :** For $P(E_1)=\frac{4}{10},\ P(E_2)=\frac{4}{10},\ P(E_3)=\frac{2}{10},\ P(E|E_1)=\frac{45}{100},\ P(E|E_2)=\frac{60}{100},$ $P(E|E_3)=\frac{35}{100}$, we have $P(E)=0.49$ (using total probability).

Reason (R) : $P(E)=P(E_1)\times P(E|E_1)+P(E_2)\times P(E|E_2)+P(E_3)\times P(E|E_3)$.

Q04. **Assertion (A) :** The random variable X has a probability distribution P(X), which is of the form

$$P(X)=\begin{cases}k, \text{ if } x=0\\ 2k, \text{ if } x=1\\ 3k, \text{ if } x=2\\ 0, \text{ otherwise}\end{cases}$$. Then $k=1$.

Reason (R) : For a probability distribution of random variable X, $\sum P(X)=1$.

Q05. **Assertion (A) :** Three cards are drawn successively with replacement from a well shuffled pack of 52 cards. Then the mean of the number of red cards, is $\frac{3}{2}$.

Reason (R) : For the independent events E and F, we may or may not have $P(E\cap F)=P(E)\times P(F)$.