

# Questions

THE PINNACLE  
For CRT - 01  
BY O.P. GUPTA

Max. Marks : 40

Time : 60 Minutes

Topics : Sets Theory

INDIRA AWARD WINNER

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Q01. Write the followings in the set-builder form :

(a)  $\left\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \dots\right\}$  (b)  $\{2, 5, 10, 17, \dots\}$  (c)  $\{\pm 3\}$  (d)  $\{8, 27, 125, 343, \dots, 24389\}$ .

Q02. Write the followings in the roster form (tabular form) :

(a)  $\{x : x \text{ is a positive integer and } x^2 < 50\}$  (b)  $\{x : x \in \mathbb{N} \text{ and } x^2 + x - 2 = 0\}$ .

Q03. (a) Let  $A = \{x : x \in \mathbb{Z} \text{ and } x^2 \leq 4\}$ ,  $B = \{x : x \in \mathbb{R} \text{ and } x^2 - 3x + 2 = 0\}$ . State whether or not,  $A = B$ .

(b) If  $\mathbb{Z}$  is the set of integers and  $\mathbb{N}$  is the set of natural numbers, then what is  $\mathbb{Z} - \mathbb{N}$ ?

(c) Write the interval  $[-2, 6)$  in set-builder form.

(d) If  $A \subset B$ , then find the value of  $A \cap B$  and  $A \cup B$ .

Q04. If  $A$  and  $B$  are two sets and  $U$  is the universal set such that  $n(U) = 700$ ,  $n(A) = 200$ ,  $n(B) = 300$  and  $n(A \cap B) = 100$ . Find the value of (i)  $n(A \cup B)$  (ii)  $n(A' \cap B')$ .

Q05. (a) Let  $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ ,  $A = \{2, 4\}$  and  $B = \{2, 3, 7\}$ . Verify any one of the De Morgan's laws.

(b) If  $A = \{4, 5, 7, 8\}$ ,  $B = \{3, 5, 9, 10\}$ , then find  $A - (B \cap A)$  and  $(A - B) \cup (B - A)$ .

Q06. (i) If  $A = \{1, 2, 4, 5\}$ ,  $B = \{x : x \in \mathbb{N}, x \leq 5\}$ , find  $A \cap B$  and  $A \cup B$ . Show it in Venn diagram.

(ii) Prove that  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ .

Q07. Write down all the possible subsets of  $A = \{\phi, \{2, 3\}, \{\}\}$ . How many proper subsets does it have? State if it is possible to write its power set? If yes, write it.

Q08. Check which of the following pair of sets are disjoint? Give reason(s).

(a)  $\{a, e, i, o, u\}$  and  $\{c, d, e, f\}$  (b)  $\{x : x \text{ is an odd integer}\}$  and  $\{x : x \text{ is an even integer}\}$ .

Q09. A school awarded 38 medals to the students of Category I, 15 to the students of Category II and 20 to the students of Category III. If these medals went to a total of 58 students and only three students got medals in all the three categories, how many received medals in exactly two of the three categories?

Q10. Let  $T = \left\{x \mid \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x}\right\}$ . Is  $T$  an empty set? Justify your answer.

OR Assume that  $P(A) = P(B)$ . Show that  $A = B$ .

[4×10 = 40]

# Hints & Answers Of CRT-01

- Q01.** (a)  $\left\{x : x = \frac{1}{n^2}, n \in \mathbb{N}\right\}$  (b)  $\{x \mid x = n^2 + 1, n \in \mathbb{N}\}$   
(c)  $\{y \mid y^2 - 9 = 0\}$  (d)  $\{x : x = m^3, m \text{ is a positive prime no. s.t. } m \leq 29\}$ .
- Q02.** (a)  $\{1, 2, \dots, 7\}$  (b)  $\{1\}$ .
- Q03.** (a)  $A \neq B$  as  $A = \{-2, -1, 0, 1, 2\}$ ,  $B = \{1, 2\}$  which implies  $n(A) \neq n(B)$   
(b)  $\mathbb{Z}^- \cup \{0\}$  i.e.,  $\{\dots, -3, -2, -1, 0\}$  that is, set of non-positive integers.  
(c)  $\{x : x \in \mathbb{R}, -2 \leq x < 6\}$   
(d)  $A \cap B = A$ ,  $A \cup B = B$ .
- Q04.** Since  $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 200 + 300 - 100 = 400$   
So  $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B) = 700 - 400 = 300$ .
- Q05.** (a) De Morgan's laws are (i)  $(A \cup B)' = A' \cap B'$ , (ii)  $(A \cap B)' = A' \cup B'$ . Now verify.  
(b)  $A - (B \cap A) = \{4, 7, 8\}$  and  $(A - B) \cup (B - A) = \{3, 4, 7, 8, 9, 10\}$ .
- Q06.** (i) Here  $A = \{1, 2, 4, 5\}$ ,  $B = \{1, 2, 3, 4, 5\}$   
 $\therefore A \cap B = \{1, 2, 4, 5\} = A$ ,  $A \cup B = \{1, 2, 3, 4, 5\} = B$ . Draw the Venn diagram yourself.  
(ii) See Mathematicia Vol. 1.
- Q07.** Note that  $A = \{\phi, \{2, 3\}, \{\phi\}\} = \{\phi, \{2, 3\}\}$   
Subsets of  $A = \phi, \{\phi\}, \{\{2, 3\}\}, \{\phi, \{2, 3\}\}$ ; No. of proper subsets  $= 2^2 - 1 = 3$ ;  
And, Power set of  $A$ ,  $P(A) = \{\phi, \{\phi\}, \{\{2, 3\}\}, \{\phi, \{2, 3\}\}\}$ .
- Q08.** (a)  $\because \{a, e, i, o, u\} \cap \{c, d, e, f\} = \{e\} \neq \phi$  so, these sets aren't disjoint.  
(b)  $\because \{x : x \text{ is an odd integer}\} \cap \{x : x \text{ is an even integer}\} = \phi$  so, these are disjoint sets.
- Q09.** 9 medals.
- Q10.** As  $T = \{10\}$  so,  $T$  is not an empty set.  
**OR** See Mathematicia Vol. 1.

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Please find below my contact info when you decide to offer your valuable suggestions. I am looking forward for a response.

Moreover, I would wish **if you inform your friends/students** about my efforts for Maths so that they may also be benefited.

**Let's learn Maths with smile :-)**

☞ For any clarification(s), please contact :

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