Questions For CRT - 15

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Max. Marks : **30**
Time : **60 Minutes**
Topics : **Differential Equations & Linear Programming**

Advanced MATH Classes, 1st Floor (Above Burger Shop), Opp. HP Petrol Pump, Thana Road, Najafgarh.

Q01. **(a)** Find the integrating factor of $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1,\ x\neq 0$.

(b) Write the product of the degree and order of $\frac{d}{dt}\left\{1+\left(\frac{dx}{dt}\right)^2\right\}=\frac{d}{dt}\left\{\frac{d^2x}{dt^2}\right\}$. [$2\times 2=4$

Q02. Solve : $y-x\frac{dy}{dx}=a\left(y^2+\frac{dy}{dx}\right)$.

Q03. If a class XII student aged 17 years, rides his motor cycle at 40km/hr, the petrol cost is ₹2 per km. If he rides at a speed of 70km/hr, the petrol cost increases to ₹7per km. He has ₹100 to spend on petrol and wishes to cover the maximum distance within one hour. Express this as an L. P. P.

[$4\times 2=8$

Q04. Show that $x\frac{dy}{dx}\sin\left(\frac{y}{x}\right)+x-y\sin\left(\frac{y}{x}\right)=0$ is homogeneous and hence solve it.

OR Solve : $y\sin x\frac{dy}{dx}=\cos x\left(\sin x-\frac{y^2}{2}\right),\ y\left(\frac{\pi}{2}\right)=1$..

Q05. An airline agrees to charter planes for a group. The group needs at least 160 first class seats and at least 300 tourist class seats. The airline must use at least two of its Model-314 planes which have 20 first class and 30 tourist class seats. The airline will also use some of its Model-535 planes which have 30 first class and 60 tourist class seats. Each flight of a Model-314 plane costs the company ₹1.00 Lakhs, and each flight of Model-535 plane costs ₹1.50 Lakhs. Formulate an LPP for the airline to minimize the flight cost.

Q06. Minimise and maximize $z=5x+2y$ subject to the following constraints :

$$x-2y\leq 2,$$
$$3x+2y\leq 12,$$
$$-3x+2y\leq 3,$$
$$x\geq 0,\ y\geq 0.$$

[$6\times 3=18$

 # Test held on : 16 July, 2017

Solutions Of CRT-15

Q01. **(a)** $e^{2\sqrt{x}}$.

(b) $\frac{d}{dt}\left\{1+\left(\frac{dx}{dt}\right)^2\right\}=\frac{d}{dt}\left\{\frac{d^2x}{dt^2}\right\}$ $\Rightarrow 2\frac{dx}{dt}\times\frac{d}{dt}\left(\frac{dx}{dt}\right)=\frac{d^3x}{dt^3}$ $\Rightarrow 2\left(\frac{dx}{dt}\right)\times\left(\frac{d^2x}{dt^2}\right)=\frac{d^3x}{dt^3}$

$\therefore$ Order $=3$, Degree $=1$.

Q02. See **Mathematicia** by **O.P. Gupta** (General Differential Eqs.).

Q03. Let x and y (both in km) be the distances covered by the student at the speed of 40 kmph and 70 kmph respectively.

To maximize : Z = (x + y) km

Subject to constraints : $x, y \geq 0$, $2x+7y\leq 100$, $\frac{x}{40}+\frac{y}{70}\leq 1$.

Q04. $x\frac{dy}{dx}\sin\left(\frac{y}{x}\right)+x-y\sin\left(\frac{y}{x}\right)=0$ $\Rightarrow \frac{dy}{dx}=\frac{y}{x}-\operatorname{cosec}\left(\frac{y}{x}\right)$

Consider $f(x, y)=\frac{y}{x}-\operatorname{cosec}\left(\frac{y}{x}\right)$.

Now prove yourself that f (x, y) is homogeneous.

Then to solve, put $y = vx$ $\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore v+x\frac{dv}{dx}=\frac{vx}{x}-\operatorname{cosec}\left(\frac{vx}{x}\right)$ $\Rightarrow x\frac{dv}{dx}=-\operatorname{cosec} v$ $\Rightarrow -\int \sin v\, dv=\int\frac{dx}{x}$

$\Rightarrow \cos v=\log|x|+c$ $\therefore \cos\left(\frac{y}{x}\right)=\log|x|+c$.

OR See **Mathematicia** by **O.P. Gupta** (Linear Differential Eqs.).

Q05. See **Mathematicia** by **O.P. Gupta**.

Q06. See **Mathematicia** by **O.P. Gupta**.

❖ Dear Student/Teacher,

I would urge you for a little favour. Please notify me about any error (s) which you notice in this (or other Maths) work. It would be beneficial for all the future learners of Maths like us. Any constructive criticism will be well acknowledged.

Please find below my contact info. when you decide to offer me your valuable suggestions.

I'm looking forward for a response.

Apart from this, I would wish **if you inform your friend/students** about my efforts for Maths so that they may also be benefitted.

Let's *learn* Maths with smile :-)

☞ For any clarification(s), please contact :

O.P. Gupta, *Math Mentor*

[Maths (Hons.), E & C Engg., Indira Award Winner]

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