

Questions

For CRT - 13

BY O.P. GUPTA

Max. Marks : 40

INDIRA AWARD WINNER

Time : 60 Minutes

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Topics : Definite Integrals & Its Application

■ Advanced MATH Classes, 1st Floor (Above Burger Shop), Opp. HP Petrol Pump, Thana Road, Najafgarh.

- Q01. (a) Write the value of $\int_{-\pi}^{\pi} x^3 \tan^2 x \log \cos x \, dx$. (b) Evaluate : $\int_0^{\pi/2} |\cos 2x| \, dx$.
(c) Find the value of $\int_0^{\pi/2} [x] \, dx$ where $[x]$ is greatest integer function. $[2 \times 3 = 6]$
- Q02. Prove that : $\int_0^{\pi/2} \frac{\sin^2 \theta}{1 + \sin \theta \cos \theta} \, d\theta = \frac{\pi}{3\sqrt{3}}$.
- Q03. Evaluate : $\int_0^{\infty} \frac{\log x}{1+x^2} \, dx$.
- Q04. Prove that : $\int_a^b f(x) \, dx = \int_a^b f(a+b-x) \, dx$. Hence, evaluate : $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+3^x} \, dx$.
- Q05. Using integrals, find the area of the region bounded by the curves $y^2 = x$ and $x^2 = y$. $[4 \times 4 = 16]$
- Q06. Evaluate : $\int_0^{\pi} x \log \sin x \, dx$. OR Evaluate : $\int_{-\pi/4}^{\pi/4} \log(\sin x + \cos x) \, dx$.
- Q07. Find : $\int_0^{\pi/2} \frac{\sin 2x}{\sin^4 x + \cos^4 x} \, dx$. OR Find : $\int_0^{\pi} e^{2x} \cdot \sin\left(\frac{\pi}{4} + x\right) \, dx$.
- Q08. Find the value of $\int_0^{\pi/2} \log(\cot \theta + \tan \theta) \, d\theta$. $[6 \times 3 = 18]$

INDIRA Award Winner O.P. Gupta is author of several popular books on Mathematics for Classes 12th & 11th. These can be bought at webstore www.iMathematicia.com.

Solutions Of CRT-13

Q01. (a) As $f(x)$ is an odd function so, the integral value is 0.

(b) 1.

(c) $\pi/2 - 1$.

Q03. Refer to the Mathematicia by O.P. Gupta. Ans. 0

Q04. For the proof of property, refer to the Mathematicia by O.P. Gupta. Ans. 1

Q05. Refer to the Mathematicia by O.P. Gupta. Ans. $1/3$ Sq. units

Q06. Refer to the Mathematicia by O.P. Gupta.

OR Let $I = \int_{-\pi/4}^{\pi/4} \log(\sin x + \cos x) dx \dots (i) \Rightarrow I = \int_{-\pi/4}^{\pi/4} \log \left(\sin \left(-\frac{\pi}{4} + \frac{\pi}{4} - x \right) + \cos \left(-\frac{\pi}{4} + \frac{\pi}{4} - x \right) \right) dx$

$\Rightarrow I = \int_{-\pi/4}^{\pi/4} \log(-\sin x + \cos x) dx \dots (ii)$. Adding (i) & (ii), $2I = \int_{-\pi/4}^{\pi/4} \log \cos 2x dx$. Show that $\log \cos 2x$ is

an even function and, use the concerned property to get : $I = \int_0^{\pi/4} \log \cos 2x dx$. Now proceed.

Q07. $\pi/2$ OR See **Worked Out Examples** in Chapter 06 of O.P. Gupta's Mathematicia

Q08. Refer to the Mathematicia by O.P. Gupta.

❖ Dear Student/Teacher,

I would urge you for a little favour. Please notify me about any error (s) which you notice in this (or other Maths) work. It would be beneficial for all the future learners of Maths like us. Any constructive criticism will be well acknowledged.

Please find below my contact info. when you decide to offer me your valuable suggestions.

I'm looking forward for a response.

Apart from this, I would wish **if you inform your friend/students** about my efforts for Maths so that they may also be benefitted.

Let's learn Maths with smile :-)

☞ For any clarification(s), please contact :

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