Questions For CRT - 03

THE PINNACLE

BY O.P. GUPTA

Max. Marks : **40**
Time : **60 Minutes**
Topics : **Relations & Functions**

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Q01. **(a)** If $\{(x, 2), (4, y)\}$ represents an identity function, then write the value of ordered pair $(2x, y)$.
(b) If $\{(7, -1), (-5, a)\}$ represents a constant function, then what is the value of 'a'?
(c) Given $f(x) = (-1)^x$ is a function from N to Z. Then obtain the range of f.
(d) If $f = \{(6, 3), (8, 9), (5, 9), (-1, 6)\}$, then what are the pre-images of 9?

Q02. Write the domain of : **(a)** $f(x) = \dfrac{|x|}{1+|x|}$ **(b)** $y = \sqrt{[x]-x}$.

(c) $f = \{(4,-2),(9,-1),(6,1),(0,4)\}$ **(d)** $y = \dfrac{1}{\log[x]}$.

Q03. Redefine $f(x) = |x-2| + |x+2|, \ -3 \le x \le 3$.

Q04. Let $A = \{x : x$ is the name of month in a leap year$\}$, $B = \{28, 29, 30, 31\}$. Let $R : A \to B$ is defined by $R = \{(a, b) :$ 'a' month has 'b' number of days$\}$. Write the roster form of this relation. Hence write its domain.

Q05. If $f(x) = \dfrac{2^x + 2^{-x}}{2}$, then prove that $f(x+y) \times f(x-y) = \dfrac{1}{2}[f(2x) + f(2y)]$.

Q06. What is the domain of $f(x) = \dfrac{1}{\sqrt{[x]^2 - 2[x] - 8}}$?

Q07. Draw the graph of $f(x) = \text{sgn}.(x-2)$. Also mention its domain and range. $[4 \times 7 = 28$

Q08. Let $f = \left\{\left(x, \dfrac{x^2}{1+x^2}\right) : x \in R\right\}$ be a function from R into R. Determine the range of f.

Q09. If $f(x) = x + \dfrac{1}{x}$, then prove that $[f(x)]^3 = f(x^3) + 3f(1/x)$.

OR Sketch the graph for greatest integer function. Hence mention its domain and range.

$[6 \times 2 = 12$

 # Test held on : 29 July, 2017

Hints & Answers Of CRT-03

Q01. **(a)** (4, 4) **(b)** –1 **(c)** {–1, 1} **(d)** 8, 5.

Q02. **(a)** $x \in R$ **(b)** $x \in Z$ **(c)** $\{0,4,6,9\}$ **(d)** $x \in [2,\infty)$.

Q03. See the **NCERT Exemplar Problem Solutions** by **O.P. Gupta**.

Q04. R = {(January, 31), (February, 29), (March, 31), (April, 30), (May, 31), (June, 30), (July, 31), (August, 31), (September, 30), (October, 31), (November, 30), (December, 31)}.
Domain of R = {January, February, March, …, November, December}.

Q05. Here $f(x+y)=\dfrac{2^{x+y}+2^{-x-y}}{2}=\dfrac{1}{2\times 2^{x+y}}\left[2^{2(x+y)}+1\right]$ and $f(x-y)=\dfrac{2^{x-y}+2^{-x+y}}{2}=\dfrac{1}{2\times 2^{x-y}}\left[2^{2(x-y)}+1\right]$.

LHS : $f(x+y)\times f(x-y)=\dfrac{1}{2\times 2^{x+y}}\left[2^{2(x+y)}+1\right]\times\dfrac{1}{2\times 2^{x-y}}\left[2^{2(x-y)}+1\right]$

$\Rightarrow \quad =\dfrac{1}{4\times 2^{2x}}\left[2^{2x+2y}+1\right]\left[2^{2x-2y}+1\right]=\dfrac{1}{4}\left[2^{2y}+2^{-2x}\right]\left[2^{2x-2y}+1\right]$

$\Rightarrow \quad =\dfrac{1}{4}\left[2^{2x}+2^{2y}+2^{-2y}+2^{-2x}\right]=\dfrac{1}{4}\left[2^{2x}+2^{-2x}+2^{2y}+2^{-2y}\right]$

$\Rightarrow \quad =\dfrac{1}{2}\left[\dfrac{2^{2x}+2^{-2x}+2^{2y}+2^{-2y}}{2}\right]=\dfrac{1}{2}\left[\dfrac{2^{2x}+2^{-2x}}{2}+\dfrac{2^{2y}+2^{-2y}}{2}\right]$

$\Rightarrow \quad =\dfrac{1}{2}\left[f(2x)+f(2y)\right]=\text{RHS}$.

Q06. See the Examples in **Mathematicia by O.P. Gupta**.

Q07. See the Examples in **Mathematicia by O.P. Gupta**. Ans. Domain $D_f : x \in R$, $R_f : \{\pm 1, 0\}$.

Q08. See the Solutions in **Mathematicia by O.P. Gupta**.

Q09. See the Solutions in **Mathematicia by O.P. Gupta**.
OR See the graph in **Mathematicia by O.P. Gupta**. Domain = R and range = Z.

$[6\times 2=12$

❖ Dear Student/Teacher,

I would urge you for a little favour. Please notify me about any error (s) which you notice in this (or other Maths) work. It would be beneficial for all the future learners of Maths like us. Any constructive criticism will be well acknowledged.

Please find below my contact info when you decide to offer your valuable suggestions. I am looking forward for a response.

Moreover, I would wish **if you inform your friends/students** about my efforts for Maths so that they may also be benefited.

Let's *learn* Maths with smile :-)

☞ For any clarification(s), please contact :

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