

Questions

For CRT - 08

BY O.P. GUPTA

Max. Marks : 40

Time : 60 Minutes

INDIRA AWARD WINNER

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Topics : Application Of Derivatives (Increasing & Decreasing Functions, Maxima & Minima)

■ Advanced MATH Classes, 1st Floor (Above Master Of Burgers), Opp. HP Petrol Pump, Thana Road, Najafgarh.

- Q01.** (a) Show that the exponential function is strictly increasing on R.
(b) Find the values of a and b for which the function $f(x) = \sin x - ax + b$ increases on R.
(c) Let f have second derivative at c such that $f'(c) = 0$ and $f''(c) < 0$, then c is a point of _____.
(d) Find the maximum and/or minimum value of $|\sin 5x + 4|$, $x \in R$. [1×4 = 4]
- Q02.** Show that the local maximum value of $x + \frac{1}{x}$ is less than local minimum value.
- Q03.** Determine the least value of the function $f(x) = px + \frac{q}{x}$ ($p > 0$, $q > 0$, $x > 0$). [2×2 = 4]
- Q04.** For $f(x) = \log(1+x) - \frac{2x}{2+x}$, find the intervals in which it is increasing and/or decreasing.
- Q05.** Find the absolute maximum and minimum values of $f(x) = 4x - \frac{1}{2}x^2$, $-2 \leq x \leq \frac{9}{2}$.
- Q06.** Find local maximum and/or local minimum values for $f(x) = (x-1)^3(x+1)^2$ using 1st derivative test.
- Q07.** Determine the interval(s) in which x^x is increasing or decreasing.
- Q08.** Show that the function $f(x) = \tan^{-1}(\sin x + \cos x)$, $x > 0$ is always a strictly increasing function in the interval $\left(0, \frac{\pi}{4}\right)$. [4×5 = 20]
- Q09.** Show that the surface area of a closed cuboid with square base and given volume is minimum when it is a cube.
- Q10.** Show that the height of the right circular cone of maximum volume that can be inscribed in a sphere of radius 12cm is 16cm.
- OR** Show that $f(x) = 2x + \cot^{-1}x + \log(\sqrt{1+x^2} - x)$ is increasing in R. [6×2 = 12]

INDIRA Award Winner O.P. Gupta is author of several popular books on Mathematics for Classes 12th & 11th. These can be bought at webstore www.iMathematicia.com.

Solutions Of CRT-08

- Q01.** (a) Let $f(x) = e^x \quad \therefore f'(x) = e^x > 0 \forall x \in \mathbb{R}$. So, exponential function is always increasing on \mathbb{R} .
(b) $f(x) = \sin x - ax + b \Rightarrow f'(x) = \cos x - a$. For increasing function, $f'(x) = \cos x - a > 0 \Rightarrow \cos x > a$
Since $\cos x \in [-1, 1]$ so, $a < -1 \Rightarrow a \in (-\infty, -1)$. Also, b can take any real values.
(c) Local maxima
(d) Maximum and minimum values are respectively 5 and 3.
- Q02.** Show that max. value which occurs at $x = -1$ is -2 and min. value which occurs at $x = 1$ is 2.
- Q03.** $2\sqrt{pq}$
- Q04.** See Q13 (Mathematicia by O.P. Gupta)
- Q05.** Absolute Max. = 8, Absolute Min. = -10
- Q06.** Local maximum value at $x = -1$ is $f(-1) = 0$. Local minimum value at $x = -\frac{1}{5}$ is $f\left(-\frac{1}{5}\right) = -\frac{136}{125}$.
Also $x = 1$ is the point of inflection.
- Q07.** Increasing : (e^{-1}, ∞) ; decreasing : $(0, e^{-1})$

❖ Dear Student/Teacher,

I would urge you for a little favour. Please notify me about any error (s) which you notice in this (or other Maths) work. It would be beneficial for all the future learners of Maths like us. Any constructive criticism will be well acknowledged.

Please find below my contact info. when you decide to offer me your valuable suggestions.

I'm looking forward for a response.

Apart from this, I would wish **if you inform your friend/students** about my efforts for Maths so that they may also be benefitted.

Let's learn Maths with smile :-)

☞ For any clarification(s), please contact :

O.P. Gupta, Math Mentor

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