Questions For CRT - 07

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Max. Marks : **40**
Time : **60 Minutes**
Topics : **Derivatives**

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Q01. **(a)** If $y=\cot\left(\frac{\pi}{2}-\tan^{-1}x\right)$ then, find $\frac{dy}{dx}$.

(b) If $f(x)=|\sin x|$, then find $f'\left(\frac{\pi}{4}\right)$. $[1\times 2=2$

Q02. **(a)** If $f(1)=4$, $f'(1)=2$, then find the value of derivative of $\log f(e^x)$ w. r. t. x at $x=0$.

(b) If $f(x)=\sqrt{\frac{\sec x-1}{\sec x+1}}$, find $f'(x)$. Also find $f'\left(\frac{\pi}{2}\right)$.

Q03. **(a)** If $f(x)=\cos^{-1}\left(\frac{1-16^x}{1+16^x}\right)$, then find $f'(x)$. **(b)** Find $\frac{dy}{dx}$, if $y=\cos^{-1}\left(\frac{6x+2\sqrt{1-4x^2}}{\sqrt{13}}\right)$.

Q04. **(a)** Find $\frac{d}{dx}\{|x-1|+|x-5|\}$, $1<x<5$. **(b)** Differentiate $\sin(x^x)$ w. r. t. x.

Q05. Given that $y=e^{ax}\sin bx$ then, prove that $y_2-2ay_1+(a^2+b^2)y=0$.

Q06. If $y=\sqrt{\frac{1-x}{1+x}}$ then, prove that $(1-x^2)\frac{dy}{dx}+y=0$.

Q07. Differentiate $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, when $x\neq 0$.

Q08. If $y^3(x-y)^{11}=x^{14}$ then, prove that $\frac{dy}{dx}=\frac{y}{x}$. Hence, find $\frac{d^2y}{dx^2}$ as well.

Q09. If $x=a\cos\theta+b\sin\theta$ and $y=a\sin\theta-b\cos\theta$, show that $y^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=0$. $[4\times 8=32$

Q10. Let $f(x)=\sin^{-1}\left(\frac{1}{\sqrt{x+1}}\right)+\begin{vmatrix} x & 1 & \cos x \\ x & 2x & 2\sin x \\ x & x & \sin x \end{vmatrix}$. Determine $\frac{d}{dx}[f(x)]$.

OR Differentiate $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2}x}{1-x^2}\right)-\frac{1}{2\sqrt{2}}\log\left(\frac{1-\sqrt{2}x+x^2}{1+\sqrt{2}x+x^2}\right)$ w. r. t. x.

$[6\times 1=6$

Test held on : 21 May, 2017

Solutions Of CRT-07

Q01. **(a)** $y = \cot\cot^{-1} x = x \quad \Rightarrow \dfrac{dy}{dx} = 1$.

(b) $\sin x > 0$ in $x \in \left(0, \dfrac{\pi}{2}\right)$ $\therefore f(x) = \sin x \Rightarrow f'(x) = \cos x \quad \therefore f'\left(\dfrac{\pi}{4}\right) = \cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$.

Q02. **(a)** Let $y = \log f(e^x) \quad \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{f(e^x)} \times f'(e^x) \times e^x \quad \Rightarrow \left.\dfrac{dy}{dx}\right]_{\text{at } x=0} = \dfrac{1}{2}$.

(b) $f(x) = \sqrt{\dfrac{\sec x - 1}{\sec x + 1}} = \sqrt{\dfrac{1-\cos x}{1+\cos x}} = \sqrt{\dfrac{2\sin^2(x/2)}{2\cos^2(x/2)}} = \tan\dfrac{x}{2} \quad \therefore f'(x) = \dfrac{1}{2}\sec^2\left(\dfrac{x}{2}\right),\ f'\left(\dfrac{\pi}{2}\right) = 1$.

Q03. **(a)** Use $f(x) = \cos^{-1}\left(\dfrac{1-16^x}{1+16^x}\right) = \cos^{-1}\left(\dfrac{1-4^{2x}}{1+4^{2x}}\right)$.

Then replace $4^x = \tan\theta$ so that $f(x) = 2\tan^{-1}(4^x)$. Finally, differentiate to get $f'(x) = \left(\dfrac{4\log 2}{1+16^x}\right)4^x$

(b) $\dfrac{2}{\sqrt{1-4x^2}}$ or $-\dfrac{2}{\sqrt{1-4x^2}}$

Q04. **(a)** $\dfrac{d}{dx}\{|x-1| + |x-5|\} = \dfrac{d}{dx}\{(x-1) - (x-5)\} = \dfrac{d}{dx}(4) = 0 \qquad [\because 1 < x < 5$.

(b) $x^x(1+\log x).\cos(x^x)$

Q05. Given $y = e^{ax}\sin bx \ldots$(i) [Differentiating w.r.t. x both the sides

$\Rightarrow y_1 = be^{ax}\cos bx + ae^{ax}\sin bx \Rightarrow y_1 = be^{ax}\cos bx + ay \ldots$(ii) [By (i)

$\Rightarrow y_2 = -b^2 e^{ax}\sin bx + abe^{ax}\cos bx + ay_1$

$\Rightarrow y_2 = -b^2 y + abe^{ax}\cos bx + abe^{ax}\cos bx + a^2 y$ [By (i) & (ii)

$\Rightarrow y_2 = -b^2 y + 2abe^{ax}\cos bx + a^2 y \quad \Rightarrow y_2 = -b^2 y + 2(ay_1 - a^2 y) + a^2 y$ [By (ii)

$\therefore \ y_2 - 2ay_1 + (a^2+b^2)y = 0$.

Q06. Take logarithm both sides and apply log properties.

Q07. 4 or –4

Q08. $y'' = 0$

Q09. See Mathematicia by O.P. Gupta

Q10. Obtain $f(x) = \cot^{-1}\sqrt{x} + x\sin x - x^2\cos x$ by substituting $x = \cot^2\theta$ (then simplifying) and expanding the determinant respectively. Therefore, $f'(x) = \sin x + x^2\sin x - x\cos x - \dfrac{1}{2\sqrt{x}(1+x)}$.

OR $\dfrac{2}{1+x^4}$.

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