

Questions

For CRT - 07

BY O.P. GUPTA

Max. Marks : 40

Time : 60 Minutes

Topics : Derivatives

INDIRA AWARD WINNER

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Q01. (a) If $y = \cot\left(\frac{\pi}{2} - \tan^{-1}x\right)$ then, find $\frac{dy}{dx}$.

(b) If $f(x) = |\sin x|$, then find $f'\left(\frac{\pi}{4}\right)$.

[1 × 2 = 2]

Q02. (a) If $f(1) = 4$, $f'(1) = 2$, then find the value of derivative of $\log f(e^x)$ w. r. t. x at $x = 0$.

(b) If $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$, find $f'(x)$. Also find $f'\left(\frac{\pi}{2}\right)$.

Q03. (a) If $f(x) = \cos^{-1}\left(\frac{1-16^x}{1+16^x}\right)$, then find $f'(x)$.

(b) Find $\frac{dy}{dx}$, if $y = \cos^{-1}\left(\frac{6x + 2\sqrt{1-4x^2}}{\sqrt{13}}\right)$.

Q04. (a) Find $\frac{d}{dx}\{|x-1| + |x-5|\}$, $1 < x < 5$.

(b) Differentiate $\sin(x^x)$ w. r. t. x .

Q05. Given that $y = e^{ax} \sin bx$ then, prove that $y_2 - 2ay_1 + (a^2 + b^2)y = 0$.

Q06. If $y = \sqrt{\frac{1-x}{1+x}}$ then, prove that $(1-x^2)\frac{dy}{dx} + y = 0$.

Q07. Differentiate $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, when $x \neq 0$.

Q08. If $y^3(x-y)^{11} = x^{14}$ then, prove that $\frac{dy}{dx} = \frac{y}{x}$. Hence, find $\frac{d^2y}{dx^2}$ as well.

Q09. If $x = a \cos \theta + b \sin \theta$ and $y = a \sin \theta - b \cos \theta$, show that $y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$.

[4 × 8 = 32]

Q10. Let $f(x) = \sin^{-1}\left(\frac{1}{\sqrt{x+1}}\right) + \begin{vmatrix} x & 1 & \cos x \\ x & 2x & 2 \sin x \\ x & x & \sin x \end{vmatrix}$. Determine $\frac{d}{dx}[f(x)]$.

OR Differentiate $\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\sqrt{2}x}{1-x^2}\right) - \frac{1}{2\sqrt{2}} \log\left(\frac{1-\sqrt{2}x+x^2}{1+\sqrt{2}x+x^2}\right)$ w. r. t. x .

[6 × 1 = 6]

INDIRA Award Winner O.P. Gupta is author of several popular books on Mathematics for Classes 12th & 11th. These can be bought at webstore www.iMathematicia.com.

Solutions Of CRT-07

Q01. (a) $y = \cot \cot^{-1} x = x \Rightarrow \frac{dy}{dx} = 1.$

(b) $\sin x > 0$ in $x \in \left(0, \frac{\pi}{2}\right) \therefore f(x) = \sin x \Rightarrow f'(x) = \cos x \therefore f'\left(\frac{\pi}{4}\right) = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}.$

Q02. (a) Let $y = \log f(e^x) \Rightarrow \frac{dy}{dx} = \frac{1}{f(e^x)} \times f'(e^x) \times e^x \Rightarrow \frac{dy}{dx} \Big|_{\text{at } x=0} = \frac{1}{2}.$

(b) $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}} = \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \sqrt{\frac{2 \sin^2(x/2)}{2 \cos^2(x/2)}} = \tan \frac{x}{2} \therefore f'(x) = \frac{1}{2} \sec^2\left(\frac{x}{2}\right), f'\left(\frac{\pi}{2}\right) = 1.$

Q03. (a) Use $f(x) = \cos^{-1}\left(\frac{1-16^x}{1+16^x}\right) = \cos^{-1}\left(\frac{1-4^{2x}}{1+4^{2x}}\right).$

Then replace $4^x = \tan \theta$ so that $f(x) = 2 \tan^{-1}(4^x)$. Finally, differentiate to get $f'(x) = \left(\frac{4 \log 2}{1+16^x}\right) 4^x$

(b) $\frac{2}{\sqrt{1-4x^2}}$ or $-\frac{2}{\sqrt{1-4x^2}}$

Q04. (a) $\frac{d}{dx} \{|x-1| + |x-5|\} = \frac{d}{dx} \{(x-1) - (x-5)\} = \frac{d}{dx} (4) = 0 \quad [\because 1 < x < 5.]$

(b) $x^x (1 + \log x) \cdot \cos(x^x)$

Q05. Given $y = e^{ax} \sin bx \dots$ (i) [Differentiating w.r.t. x both the sides]

$\Rightarrow y_1 = b e^{ax} \cos bx + a e^{ax} \sin bx \Rightarrow y_1 = b e^{ax} \cos bx + a y \dots$ (ii) [By (i)]

$\Rightarrow y_2 = -b^2 e^{ax} \sin bx + a b e^{ax} \cos bx + a y_1$

$\Rightarrow y_2 = -b^2 y + a b e^{ax} \cos bx + a b e^{ax} \cos bx + a^2 y$ [By (i) & (ii)]

$\Rightarrow y_2 = -b^2 y + 2 a b e^{ax} \cos bx + a^2 y \Rightarrow y_2 = -b^2 y + 2(a y_1 - a^2 y) + a^2 y$ [By (ii)]

$\therefore y_2 - 2 a y_1 + (a^2 + b^2) y = 0.$

Q06. Take logarithm both sides and apply log properties.

Q07. 4 or -4

Q08. $y'' = 0$

Q09. See Mathematicia by O.P. Gupta

Q10. Obtain $f(x) = \cot^{-1} \sqrt{x} + x \sin x - x^2 \cos x$ by substituting $x = \cot^2 \theta$ (then simplifying) and expanding the determinant respectively. Therefore, $f'(x) = \sin x + x^2 \sin x - x \cos x - \frac{1}{2\sqrt{x}(1+x)}.$

OR $\frac{2}{1+x^4}.$

❖ Dear Student/Teacher,

I would urge you for a little favour. Please notify me about any error (s) which you notice in this (or other Maths) work. It would be beneficial for all the future learners of Maths like us. Any constructive criticism will be well acknowledged.

Please find below my contact info when you decide to offer me your valuable suggestions. I am looking forward for a response.

Also I would wish **if you inform your friend/students** about my efforts for Maths so that they may also be benefitted.

Let's learn Maths with smile :-)

☞ For any clarification(s), please contact :

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