Questions For CRT - 06

Set B

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Max. Marks : **40**
Time : **60 Minutes**
Topics : **Continuity & Differentiability**

Advanced MATH Classes, 1st Floor (Above Master Of Burgers), Opp. HP Petrol Pump, Thana Road, Najafgarh.

Q01. **(a)** Give an example of a function which is continuous but fails to be differentiable at exactly two points.
(b) The composition of two continuous functions is a continuous function. State if this statement is True / False? Give an example.
(c) What are the point (s) of non-differentiability for $|2x + 3|$?
(d) At what points, the function $f(x) = \dfrac{1}{x - [x]}$ is discontinuous?

Q02. **(a)** Given $f(x) = \dfrac{1}{x-1}$. Find the points of the discontinuity of the composite function $y = fof(x)$.
(b) Write the interval of discontinuity for $f(x) = \dfrac{\log x}{\sqrt{1-9x^2}}$.

Q03. Discuss the continuity of $f(x) = \begin{cases} \dfrac{e^{1/x}-1}{e^{1/x}+1}, & \text{if } x \neq 0 \\ -1, & \text{if } x = 0 \end{cases}$ at $x = 0$.

OR Show that the function defined by $g(x) = x - [x]$ is discontinuous at all integral points.

Q04. **(a)** Discuss the continuity of greatest integer function, $f(x) = [x]$ at non-integral points.
(b) Prove that the greatest integer function defined by $f(x) = [x]$ is not differentiable at $x = 1$.

Q05. If $f(x) = \dfrac{\tan(\pi/4 - x)}{\cot 2x}$ for $x \neq \pi/4$, find the value which can be assigned to $f(x)$ at $x = \pi/4$ so that $f(x)$ becomes continuous at every point in $[0, \pi/2]$.

Q06. Prove that $f(x) = \begin{cases} \dfrac{|x| + 2x^2}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ remains discontinuous at $x = 0$, regardless the choice of k.

Q07. Discuss the continuity and differentiability of the function $f(x) = |x| + |x-1|$ in $(-1, 2)$. $[4 \times 7 = 28$

Q08. Determine the value of μ; if possible so that the following function $f(x)$ is continuous at $x = 0$:

$$f(x) = \begin{cases} \dfrac{\sqrt{x}}{\sqrt{25+\sqrt{x}} - 5}, & \text{when } x > 0 \\ \mu, & \text{when } x = 0 \\ \dfrac{1 - \cos 2\sqrt{5x}}{x^2}, & \text{when } x < 0 \end{cases}.$$

Q09. If $f(x) = \begin{cases} \dfrac{\sin(a+1)x + 2\sin x}{x}, & \text{if } x < 0 \\ 2, & \text{if } x = 0 \\ \dfrac{\sqrt{1+bx} - 1}{x}, & \text{if } x > 0 \end{cases}$ is continuous at $x = 0$, then find the values of a and b.

$[6 \times 2 = 12$

INDIRA Award Winner O.P. Gupta is author of several popular books on Mathematics for Classes 12th & 11th. These can be bought at webstore www.iMathematicia.com.

 # Test held on : 14 May, 2017

Solutions Of CRT-06 Set B

Q01. **(a)** $f(x) = |x| + |x-1|$.

(b) Let $f(x) = x^2$, $g(x) = \sin x$, $\therefore fog(x) = f[g(x)] = f(\sin x) = \sin^2 x$. Here f and g both are continuous functions and the composite function $fog(x)$ is also continuous.

(c) $x = -3/2$.

(d) As $x - [x] = 0 \ \forall \ x \in Z$ so, f is discontinuous at all $x \in Z$.

Q02. **(a)** Note that $f(x) = \dfrac{1}{x-1}$ is discontinuous at $x = 1$. For $x \neq 1$, $fof(x) = \dfrac{x-1}{2-x}$.

Clearly, y is discontinuous at $x = 2$. Hence the points of discontinuity are $x = 1$ and $x = 2$.

(b) See Mathematicia Q09 (VSA).

Q03. Discontinuous.

OR The given function g (x) is defined at all integral points.

Let n be an integer. Then g (n) = n – [n] = n – n = 0.

LHL (at x = n) : $\lim\limits_{x \to n^-} g(x) = \lim\limits_{x \to n^-} (x - [x]) = n - (n-1) = 1$.

RHL (at x = n) : $\lim\limits_{x \to n^+} g(x) = \lim\limits_{x \to n^+} (x - [x]) = n - (n) = 0$.

Since LHL (at x = n) ≠ RHL (at x = n).

Therefore g is not continuous at x = n i.e., g (x) is discontinuous at all integral points.

Q04. **(a)** Discontinuous at all integral points.

Q05. 1/2.

Q06. Show that $\lim\limits_{x \to 0^-} f(x) \neq \lim\limits_{x \to 0^+} f(x)$.

Q07. Non-differentiable at $x = 3$.

Q08. $\mu = 10$.

Q09. See Mathematicia Q67 (SA).

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Please find below my contact info when you decide to offer me your valuable suggestions. I am looking forward for a response.

Also I would wish **if you inform your friend/students** about my efforts for Maths so that they may also be benefitted.

Let's *learn* Maths with smile :-)

☞ For any clarification(s), please contact :

O.P. Gupta, *Math Mentor*

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