Questions

For CRT - 06

By O.P. Gupta
INDIRA AWARD WINNER
M.+91 9650350480

Max. Marks : **40**
Time : **60 Minutes**
Topics : **Continuity & Differentiability**

Set A

■ **Advanced MATH Classes**, 1st Floor (Above Master Of Burgers), Opp. HP Petrol Pump, Thana Road, Najafgarh.

Q01. **(a)** Write an example of a function which is continuous but fails to be differentiable at exactly two points.

(b) The composition of two continuous functions is a continuous function. State if this statement is True / False? Give an example.

(c) At what points, the function $f(x)=\dfrac{1}{x-[x]}$ is discontinuous?

(d) What are the point (s) of non-differentiability for $|2x+3|$?

Q02. **(a)** Given $f(x)=\dfrac{1}{x-1}$. Find the points of the discontinuity of the composite function $y=fof(x)$.

(b) Find the number of points of discontinuity for $f(x)=\dfrac{1}{\log|x|}$.

Q03. Discuss the continuity of $f(x)=\begin{cases}\dfrac{e^{1/x}}{1+e^{1/x}}, & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$ at $x=0$.

OR Show that the function defined by $g(x)=x-[x]$ is discontinuous at all integral points.

Q04. **(a)** Discuss the continuity of greatest integer function, $f(x)=[x]$ at integeral points.

(b) Prove that the greatest integer function defined by $f(x)=[x]$ is not differentiable at $x=2$.

Q05. If $f(x)=\dfrac{\tan(\pi/4-x)}{\cot 2x}$ for $x\neq\pi/4$, find the value which can be assigned to $f(x)$ at $x=\pi/4$ so that the function $f(x)$ becomes continuous at every point in $[0,\pi/2]$.

Q06. Prove that $f(x)=\begin{cases}\dfrac{x}{|x|+2x^2}, & x\neq 0\\ k, & x=0\end{cases}$ remains discontinuous at $x=0$, regardless the choice of k.

Q07. Discuss the differentiability of $f(x)=|x-3|$ at $x=3$. $[4\times 7=28$

Q08. Determine the value of μ; if possible so that the following function $f(x)$ is continuous at $x=0$:

$$f(x)=\begin{cases}\dfrac{1-\cos 6x}{x^2}, & \text{when } x<0\\ \mu, & \text{when } x=0\\ \dfrac{\sqrt{x}}{\sqrt{81+\sqrt{x}}-9}, & \text{when } x>0\end{cases}.$$

Q09. Find the value of k, for which the function f defined below is continuous at $x=\pi/2$:

$$f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, & x<\dfrac{\pi}{2}\\ 3, & x=\pi/2\\ \dfrac{3\tan 2x}{2x-\pi}, & x>\dfrac{\pi}{2}\end{cases}. \qquad [6\times 2=12$$

 # Test held on : 14 May, 2017

Solutions Of CRT-06 Set A

Q01. **(a)** $f(x) = |x| + |x-1|$.

(b) Let $f(x) = x^2$, $g(x) = \sin x$, $\therefore fog(x) = f[g(x)] = f(\sin x) = \sin^2 x$. Here f and g both are continuous functions and the composite function $fog(x)$ is also continuous.

(c) As $x - [x] = 0 \ \forall \ x \in Z$ so, f is discontinuous at all $x \in Z$.

(d) $x = -3/2$.

Q02. **(a)** Note that $f(x) = \dfrac{1}{x-1}$ is discontinuous at $x = 1$. For $x \neq 1$, $fof(x) = \dfrac{x-1}{2-x}$.

Clearly, y is discontinuous at $x = 2$. Hence the points of discontinuity are $x = 1$ and $x = 2$.

(b) The function f (x) is discontinuous at $x = -1, 0, 1$ so, the no. of points of discontinuity is 3.

Q03. Discontinuous.

OR The given function g (x) is defined at all integral points.

Let n be an integer. Then g (n) = n – [n] = n – n = 0.

LHL (at x = n) : $\lim\limits_{x \to n^-} g(x) = \lim\limits_{x \to n^-} (x - [x]) = n - (n-1) = 1$.

RHL (at x = n) : $\lim\limits_{x \to n^+} g(x) = \lim\limits_{x \to n^+} (x - [x]) = n - (n) = 0$.

Since LHL (at x = n) ≠ RHL (at x = n).

Therefore g is not continuous at x = n i.e., g (x) is discontinuous at all integral points.

Q04. **(a)** Discontinuous at all integral points.

Q05. 1/2.

Q06. Show that $\lim\limits_{x \to 0^-} f(x) \neq \lim\limits_{x \to 0^+} f(x)$.

Q07. Non-differentiable at $x = 3$.

Q08. $\mu = 18$.

Q09. $k = 6$.

❖ Dear Student/Teacher,

I would urge you for a little favour. Please notify me about any error (s) which you notice in this (or other Maths) work. It would be beneficial for all the future learners of Maths like us. Any constructive criticism will be well acknowledged.

Please find below my contact info when you decide to offer me your valuable suggestions. I am looking forward for a response.

Also I would wish **if you inform your friend/students** about my efforts for Maths so that they may also be benefitted.

Let's *learn* Maths with smile :-)

☞ For any clarification(s), please contact :

O.P. Gupta, *Math Mentor*

[Maths (Hons.), E & C Engg., Indira Award Winner]

Call or WhatsApp @ +91-9650 350 480 *Mail us at :* ***theopgupta@gmail.com***

Follow us on Twitter ***@theopgupta*** *Official Website :* ***www.theOPGupta.com***