Questions For CRT - 05

By O.P. Gupta

INDIRA AWARD WINNER
M.+91 9650350480

Max. Marks : **40**
Time : **60 Minutes**
Topics : **Matrices and Determinants & Inverse Trigonometric Functions**

Advanced MATH Classes, 1st Floor (Above Master Of Burgers), Opp. HP Petrol Pump, Thana Road, Najafgarh.

SECTION A *(1 mark each.)*

Q01. What is the principle value of $\tan^{-1}\left(\tan(2\pi/3)\right)$?

Q02. If M is a square matrix of order 3 and $|2M| = k|M|$, then find the value of k.

Q03. A and B are square matrices of order 3 each, $|A| = 2$ and $|B| = 3$. Find $|3AB|$.

Q04. Find the value of $\cot(\pi/2 - 2\cot^{-1}\sqrt{3})$.

SECTION B *(2 marks each.)*

Q05. Simplify : $y = \tan^{-1}\dfrac{5x}{1-6x^2}$, $-\dfrac{1}{\sqrt{6}} < x < \dfrac{1}{\sqrt{6}}$.

Q06. Simplify $\cot^{-1}\dfrac{1}{\sqrt{x^2-1}}$ for $x < -1$.

Q07. Prove that the diagonal elements of a skew symmetric matrix are all zero.

Q08. Find the value of $\sin^{-1}\sin 12 + \cos^{-1}\cos 12$.

SECTION C *(4 marks each.)*

Q09. If $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$, then using A^{-1}, solve the system of equations : $x - 2y = -1,\ 2x + y = 2$.

Q10. If $A + B + C = \pi$, then find the value of $\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$.

OR Using properties of determinants, prove that $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$.

Q11. If $p \neq 0,\ q \neq 0$ and $\begin{vmatrix} p & q & p\alpha+q \\ q & r & q\alpha+r \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$, then, using properties of determinants, prove that at least one of the following statements is true :

(a) p, q, r are in G.P.

(b) α is a root of the equation $px^2 + 2qx + r = 0$.

Q12. Show that $\Delta = \Delta_1$, where $\Delta = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix}$, $\Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix}$.

SECTION D *(6 marks each.)*

Q13. Does the equation $\tan^{-1}\dfrac{x+1}{x-1} + \tan^{-1}\dfrac{x-1}{x} = -\tan^{-1}7$ have any solutions? If yes, then solve.

OR Prove that $2\tan^{-1}\left\{\tan\dfrac{\alpha}{2}\tan\left(\dfrac{\pi}{4}-\dfrac{\beta}{2}\right)\right\} = \tan^{-1}\left(\dfrac{\sin\alpha.\cos\beta}{\cos\alpha+\sin\beta}\right)$.

Q14. Find the value of x, y and z, if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A' = A^{-1}$.

This paper is based on CBSE Sample Papers issued by CBSE.

Test held on : 07 May, 2017

Solutions Of CRT-05

Q01. $\tan^{-1}\left(\tan(2\pi/3)\right)=\tan^{-1}\left(-\tan(\pi/3)\right)=-\pi/3$.

Q02. As $|2M|=k|M| \quad \Rightarrow 2^3|M|=k|M| \quad \therefore k=8$.

Q03. $|3AB|=3^3|A||B|=27\times 2\times 3=162$.

Q04. $\cot(\pi/2-2\cot^{-1}\sqrt{3})=\cot(\pi/2-2\pi/6)=\cot(\pi/6)=\sqrt{3}$.

Q05. Here $y=\tan^{-1}\dfrac{5x}{1-6x^2}=\tan^{-1}\left(\dfrac{3x+2x}{1-3x.2x}\right)=\tan^{-1}3x+\tan^{-1}2x$

Q06. Let $y=\cot^{-1}\dfrac{1}{\sqrt{x^2-1}},\ x<-1$.

Put $x=\sec\theta \Rightarrow \theta=\sec^{-1}x$. Then $\sec\theta<-1 \ \Rightarrow \dfrac{\pi}{2}<\theta<\pi$

$\therefore\ y=\cot^{-1}\dfrac{1}{\sqrt{\sec^2\theta-1}}=\cot^{-1}\left(\dfrac{1}{|\tan\theta|}\right) \qquad \Rightarrow y=\cot^{-1}\left(\dfrac{1}{-\tan\theta}\right)=\cot^{-1}\left(-\cot\theta\right)$

$\Rightarrow y=\cot^{-1}\left(\cot(\pi-\theta)\right)=\pi-\theta \qquad \therefore\ y=\pi-\sec^{-1}x$.

Q07. Let A be a skew-symmetric matrix. Then by def., $A^T=-A$

So, the $(i, j)^{th}$ element of A^T = the $(i, j)^{th}$ element of $(-A)$

$\Rightarrow$ the $(j, i)^{th}$ element of A = $-$the $(i, j)^{th}$ element of A

For the diagonal elements, $i=j$ then, the $(i, i)^{th}$ element of A = $-$ the $(i, i)^{th}$ element of A

$\Rightarrow 2\times$ the $(i, i)^{th}$ element of A = 0 $\quad \therefore$ the $(i, i)^{th}$ element of A = 0

Hence the diagonal elements of a skew symmetric matrix are all zero.

See a **simple approach** for this question in **O.P. Gupta's Mathematicia**.

Q08. $\sin^{-1}\sin 12+\cos^{-1}\cos 12=\sin^{-1}(-\sin(4\pi-12))+\cos^{-1}(\cos(4\pi-12))$

$\Rightarrow \qquad =-\sin^{-1}(\sin(4\pi-12))+(4\pi-12)=-(4\pi-12)+(4\pi-12)=0.$

Q09. Here $A=\begin{bmatrix}1 & -2\\ 2 & 1\end{bmatrix}$

Clearly, $|A|=5$, $adj.A=\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix} \qquad \therefore A^{-1}=\dfrac{adj.A}{|A|}=\dfrac{1}{5}\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}$

Given equations are $x-2y=-1,\ 2x+y=2$.

Let $AX=B$ where $A=\begin{bmatrix}1 & -2\\ 2 & 1\end{bmatrix}, B=\begin{bmatrix}-1\\ 2\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}$

So, $A^{-1}AX=A^{-1}B \qquad \Rightarrow IX=A^{-1}B$

$\Rightarrow X=\dfrac{1}{5}\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}\begin{bmatrix}-1\\ 2\end{bmatrix} \qquad \Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{3}{5}\\ \frac{4}{5}\end{bmatrix}$

By equality of matrices, we get : $x=\dfrac{3}{5},\ y=\dfrac{4}{5}$.

Q10. Let $\Delta=\begin{vmatrix}\sin(A+B+C) & \sin B & \cos C\\ -\sin B & 0 & \tan A\\ \cos(A+B) & -\tan A & 0\end{vmatrix} \qquad \Rightarrow \Delta=\begin{vmatrix}\sin\pi & \sin B & \cos C\\ -\sin B & 0 & \tan A\\ \cos(\pi-C) & -\tan A & 0\end{vmatrix}$

$\Rightarrow \Delta=\begin{vmatrix}0 & \sin B & \cos C\\ -\sin B & 0 & \tan A\\ -\cos C & -\tan A & 0\end{vmatrix} \qquad$ Expanding along R_1,

$$\Rightarrow \Delta = 0\begin{vmatrix} 0 & \tan A \\ -\tan A & 0 \end{vmatrix} - \sin B\begin{vmatrix} -\sin B & \tan A \\ -\cos C & 0 \end{vmatrix} + \cos C\begin{vmatrix} -\sin B & 0 \\ -\cos C & -\tan A \end{vmatrix}$$

$$\Rightarrow \Delta = 0 - \sin B\left(0 + \tan A \cos C\right) + \cos C\left(\sin B \tan A - 0\right) = 0.$$

OR Let $\Delta = \begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$ Apply $C_1 \to C_1 + C_3,\ C_2 \to C_2 - C_3$

$\Rightarrow \Delta = \begin{vmatrix} a+b+c & -b & a \\ b+c+a & -c & b \\ c+a+b & -a & c \end{vmatrix}$ Taking $(a+b+c)$ common from C_1,

$\Rightarrow \Delta = (a+b+c)\begin{vmatrix} 1 & -b & a \\ 1 & -c & b \\ 1 & -a & c \end{vmatrix}$ Apply $R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$\Rightarrow \Delta = (a+b+c)\begin{vmatrix} 0 & c-b & a-b \\ 0 & a-c & b-c \\ 1 & -a & c \end{vmatrix}$$

Now expanding along C_1, we get the result.

Q11. We have $\begin{vmatrix} p & q & p\alpha+q \\ q & r & q\alpha+r \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$

By $R_1 \to \alpha R_1$, $\dfrac{1}{\alpha}\begin{vmatrix} p\alpha & q\alpha & p\alpha^2+q\alpha \\ q & r & q\alpha+r \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$

By $R_3 \to R_3 - (R_1 + R_2)$, $\dfrac{1}{\alpha}\begin{vmatrix} p\alpha & q\alpha & p\alpha^2+q\alpha \\ q & r & q\alpha+r \\ 0 & 0 & -p\alpha^2-2q\alpha-r \end{vmatrix} = 0$

Expanding along R_1, $\dfrac{1}{\alpha}\left[0 - 0 + \left(-p\alpha^2 - 2q\alpha - r\right)\left(p\alpha r - q^2\alpha\right)\right] = 0$

$$\Rightarrow -\frac{1}{\alpha}\left(-\alpha(q^2 - pr)\right)\left(2q\alpha + r + p\alpha^2\right) = 0$$

Either $\left(q^2 - pr\right) = 0$ or $\left(2q\alpha + r + p\alpha^2\right) = 0$. That implies, $q^2 = pr$ or $p(\alpha)^2 + 2q(\alpha) + r = 0$

Therefore either p, q, r are in G.P. or, α satisfies the equation $px^2 + 2qx + r = 0$.

Hence either p, q, r are in G.P. **or,** α is a root of the equation $px^2 + 2qx + r = 0$.

Q13. $\tan^{-1}\dfrac{x+1}{x-1} + \tan^{-1}\dfrac{x-1}{x} = -\tan^{-1}7 \quad \Rightarrow \tan^{-1}\left(\dfrac{\dfrac{x+1}{x-1} + \dfrac{x-1}{x}}{1 - \dfrac{x+1}{x-1} \times \dfrac{x-1}{x}}\right) = -\tan^{-1}7$, if $\dfrac{x+1}{x-1} \times \dfrac{x-1}{x} < 1$...(i)

$$\Rightarrow \tan^{-1}\left(\frac{x^2 + x + x^2 - 2x + 1}{x^2 - x - x^2 + 1}\right) = -\tan^{-1}7$$

$$\Rightarrow \tan\tan^{-1}\left(\frac{2x^2 - x + 1}{1 - x}\right) = \tan(-\tan^{-1}7)$$

$$\Rightarrow \frac{2x^2 - x + 1}{1 - x} = -7 \quad \Rightarrow 2x^2 - 8x + 8 = 0 \quad \Rightarrow (x-2)^2 = 0 \quad \therefore x = 2.$$

Let us now check if x = 2 satisfies the condition (i) :

$\frac{x+1}{x-1}\times\frac{x-1}{x}=\frac{2+1}{2-1}\times\frac{2-1}{2}=\frac{3}{2}$ which is not less than 1.

Hence $x=2$ does not satisfy the condition (i). Therefore there is no solution to the given equation.

Q14. Here $A=\begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}$ $\therefore A'=\begin{bmatrix}0 & x & x\\ 2y & y & -y\\ z & -z & z\end{bmatrix}$.

As $A^{-1}=A'$ $\Rightarrow AA^{-1}=AA'$ $\Rightarrow AA'=I\ldots$(i)

Now by (i), $\begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}\begin{bmatrix}0 & x & x\\ 2y & y & -y\\ z & -z & z\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

$\Rightarrow\begin{bmatrix}2x^2 & 0 & 0\\ 0 & 6y^2 & 0\\ 0 & 0 & 3z^2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

By equality of matrices, we get :

$2x^2=1,\ 6y^2=1,\ 3z^2=1$ i.e., $x=\pm\frac{1}{\sqrt{2}},\ y=\pm\frac{1}{\sqrt{6}},\ z=\pm\frac{1}{\sqrt{3}}$.

O.P. Gupta, ***Math Mentor***

[Maths (Hons.), E & C Engg., Indira Award Winner]

Call or WhatsApp @ +91-9650 350 480 *Mail us at :* ***theopgupta@gmail.com***

Follow us on Twitter ***@theopgupta*** *Official Website :* ***www.theOPGupta.com***