# Questions For CRT - 04

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Max. Marks : **40**
Time : **60 Minutes**
Topics : **Inverse Trigonometric Functions**

■ **Advanced MATH Classes**, 1st Floor (Above Master Of Burgers), Opp. HP Petrol Pump, Thana Road, Najafgarh.

**Q01.** **(a)** Evaluate : $\tan^{-1}\left\{2\cos\left(2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right\}$.

**(b)** Simplify : $\cot^{-1}(-1)+2\tan^{-1}(1)-\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)$.

**(c)** Write the value of $\cos\left[\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)+\operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)\right]$.

**(d)** Prove that : $\cos^{-1}(-x)=\pi-\cos^{-1}x,\ x\in[-1,1]$.

**(e)** Write down one of the range of $\sec^{-1}x$ other than its principal branch.

**(f)** Find the principal value of $\operatorname{arcsec} x$ where $x=-\frac{2}{\sqrt{2}}$.

**(g)** Write the principal value of $\sin^{-1}\sin\frac{12\pi}{7}$.

**(h)** Let $\cos^{-1}:[-1,1]\to[-\pi,0]$ then, write the value of $\cos^{-1}\frac{1}{2}$.

**(i)** Write the domain of $\sin^{-1}(2x-1)$.

**(j)** State True/False : "All trigonometric functions have inverse over their respective domains". Justify your answer. [1×10 = 10

**Q02.** **(a)** The minimum value of x for which $\tan^{-1}\frac{x}{\pi}>\frac{\pi}{4}$, $x\in N$, is valid is 5. State True or False? Why?

**(b)** Solve : $\cos\left[\sin^{-1}\left(\frac{2}{5}\right)+\cos^{-1}x\right]=0$.

**(c)** Write the value of $\cot^{-1}x+\cot^{-1}y$ if, $\tan^{-1}x+\tan^{-1}y=\frac{4\pi}{5}$.

**(d)** Find the value of x if $\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$.

**(e)** What is the value of $\sin(2\sin^{-1}0.6)$?

**(f)** Find the value of x if $\tan^{-1}x-\cot^{-1}x=\pi$. [2×6 = 12

**Q03.** Prove that : $\cos^{-1}x+\cos^{-1}\sqrt{x}=\cos^{-1}[x^{3/2}-\sqrt{1-x^2}\sqrt{1-x}]$.

**OR** Simplify : $\sin\cot^{-1}\tan\cos^{-1}x$.

**Q04.** Prove that : $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$.

**Q05.** Evaluate : $\cos\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$. [4×3 = 12

**Q06.** Evaluate : $\cot\left\{\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right\}$ **OR** Simplify : $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)$, $x\in\left(\pi,\frac{3\pi}{2}\right)$.

[6×1 = 6

# Test held on : 30 April, 2017

# HINTS & ANSWERS

**Q01.** **(a)** $-\pi/4$ **(b)** $5\pi/12$ **(c)** 0

**(d)** For complete solution of this, see Theory of Chapter 2 in **MATHEMATICIA by O.P. Gupta**. To buy, click at **http://www.imathematicia.com**

**(e)** $[\pi, 2\pi]-\left\{\frac{3\pi}{2}\right\}$, $[-\pi, 0]-\left\{-\frac{\pi}{2}\right\}$ etc. **(f)** $\frac{3\pi}{4}$ **(g)** $-\frac{2\pi}{7}$

**(h)** $-\pi/3$ **(i)** $[0, 1]$

**(j)** False as trigonometric functions don't have inverse over their respective domains as in their domain trigonometric functions are not one-one. To make them one-one, we have to restrict their respective domains so that their inverse may exist.

**Q02.** **(a)** False because, $\tan^{-1}\frac{x}{\pi}>\frac{\pi}{4}$ $\Rightarrow \frac{x}{\pi}>\tan\frac{\pi}{4}$ $\Rightarrow \frac{x}{\pi}>1$ $\Rightarrow x>\pi$

$\therefore x=4$ is the minimum value of x.

**(b)** $x=2/5$ **(c)** $\pi/5$ **(d)** $\frac{2a}{1-a^2}$ **(e)** $24/25$

**(f)** $\because x=\pm 1$ don't satisfy the given equation so, it has no solution.

**Q03.** $x=\frac{2}{3},\frac{11}{3}$ **OR** x. **Q05.** $\frac{6}{5\sqrt{13}}$.

**Q06.** Let $y=\cot\left\{\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right\}$ $\Rightarrow y=\cot\left\{\sum_{n=1}^{23}\cot^{-1}(1+2+4+6+...+2n)\right\}$

$\Rightarrow y=\cot\left\{\sum_{n=1}^{23}\cot^{-1}(1+2(1+2+3+...+n))\right\}$ $\Rightarrow y=\cot\left\{\sum_{n=1}^{23}\cot^{-1}\left(1+2\times\frac{n(n+1)}{2}\right)\right\}$

$\Rightarrow y=\cot\left\{\sum_{n=1}^{23}\cot^{-1}(1+n(n+1))\right\}$ $\Rightarrow y=\cot\left\{\sum_{n=1}^{23}\tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)\right\}$

$\Rightarrow y=\cot\left\{\sum_{n=1}^{23}\left[\tan^{-1}(n+1)-\tan^{-1}n\right]\right\}$ $\Rightarrow y=\cot\left[\tan^{-1}24-\tan^{-1}1\right]$

$\Rightarrow y=\cot\left[\tan^{-1}\frac{24-1}{1+24\times 1}\right]=\cot\cot^{-1}\frac{25}{23}=\frac{25}{23}$

**OR** See **Mathematicia by O.P. Gupta.** Ans. $\pi/4-x/2$.

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❖ Dear Student/Teacher,

I would urge you for a little favour. Please notify me about any error (s) which you notice in this (or other Maths) work. It would be beneficial for all the future learners of Maths like us. Any constructive criticism will be well acknowledged.

Please find below my contact info when you decide to offer me your valuable suggestions. I am looking forward for a response.

Also I would wish **if you inform your friend/students** about my efforts for Maths so that they may also be benefitted.

**Let's *learn* Maths with smile :-)**

☞ For any clarification(s), please contact :

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