

www.theOPGupta.com
Solutions Of JEE Main (2016) Code F
MATHEMATICS

Prepared By : O. P. GUPTA
Mob. +919650350480
Email : theopgupta@gmail.com

PART B – MATHEMATICS

31. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus?
- (1) $(-3, -8)$ (2) $\left(\frac{1}{3}, -\frac{8}{3}\right)$
 (3) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$ (4) $(-3, -9)$
32. If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is:
- (1) $\frac{4}{3}$ (2) 1
 (3) $\frac{7}{4}$ (4) $\frac{8}{5}$
33. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is:
- (1) $x^2 + y^2 - x + 4y - 12 = 0$ (2) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$
 (3) $x^2 + y^2 - 4x + 9y + 18 = 0$ (4) $x^2 + y^2 - 4x + 8y + 12 = 0$
34. The system of linear equations
 $x + \lambda y - z = 0$
 $\lambda x - y - z = 0$
 $x + y - \lambda z = 0$
 has a non-trivial solution for :
- (1) exactly one value of λ . (2) exactly two values of λ .
 (3) exactly three values of λ . (4) infinitely many values of λ .
35. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x$, $x \neq 0$, and $S = \{x \in \mathbb{R} : f(x) = f(-x)\}$; then S:
- (1) contains exactly one element. (2) contains exactly two elements.
 (3) contains more than two elements. (4) is an empty set.
36. Let $p = \lim_{x \rightarrow 0^+} (1 + \tan^2 \sqrt{x})^{\frac{1}{2x}}$ then $\log p$ is equal to:
- (1) 1 (2) $\frac{1}{2}$
 (3) $\frac{1}{4}$ (4) 2.
37. A value of θ for which $\frac{2 + 3i \sin \theta}{1 - 2i \sin \theta}$ is purely imaginary, is:
- (1) $\frac{\pi}{6}$ (2) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$
 (3) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (4) $\frac{\pi}{3}$

38. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal half of the distance between its foci, is:

(1) $\frac{4}{\sqrt{3}}$ (2) $\frac{2}{\sqrt{3}}$
(3) $\sqrt{3}$ (4) $\frac{4}{3}$

39. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true?

(1) $3a^2 - 32a + 84 = 0$ (2) $3a^2 - 34a + 91 = 0$
(3) $3a^2 - 23a + 44 = 0$ (4) $3a^2 - 26a + 55 = 0$

40. The integral $\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3} dx$ is equal to:

(1) $\frac{x^{10}}{2(x^5 + x^3 + 1)^2} + C$ (2) $\frac{x^5}{2(x^5 + x^3 + 1)^2} + C$
(3) $\frac{-x^{10}}{2(x^5 + x^3 + 1)^2} + C$ (4) $\frac{-x^5}{(x^5 + x^3 + 1)^2} + C$

where C is an arbitrary constant.

41. If the line, $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in the plane, $lx + my - z = 9$, then $l^2 + m^2$ is equal to:

(1) 18 (2) 5
(3) 2 (4) 26

42. If $0 \leq x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ is:

(1) 5 (2) 7
(3) 9 (4) 3

43. The area (in sq. units) of the region $\{(x, y) : y^2 \geq 2x \text{ and } x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ is:

(1) $\pi - \frac{8}{3}$ (2) $\pi - \frac{4\sqrt{2}}{3}$
(3) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$ (4) $\pi - \frac{4}{3}$

44. Let \vec{a} , \vec{b} and \vec{c} be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$. If \vec{b} is not parallel to \vec{c} , then the angle between \vec{a} and \vec{b} is:

(1) $\frac{\pi}{2}$ (2) $\frac{2\pi}{3}$
(3) $\frac{5\pi}{6}$ (4) $\frac{3\pi}{4}$

45. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then:
- (1) $(4 - \pi)x = \pi r$ (2) $x = 2r$
 (3) $2x = r$ (4) $2x = (\pi + 4)r$
46. The distance of the point $(1, -5, 9)$ from the plane $x - y + z = 5$ measured along the line $x = y = z$ is:
- (1) $10\sqrt{3}$ (2) $\frac{10}{\sqrt{3}}$
 (3) $\frac{20}{3}$ (4) $3\sqrt{10}$
47. If a curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation, $y(1 + xy) dx = x dy$, then $f\left(-\frac{1}{2}\right)$ is equal to:
- (1) $-\frac{4}{5}$ (2) $\frac{2}{5}$
 (3) $\frac{4}{5}$ (4) $-\frac{2}{5}$
48. If the number of terms in the expansion of $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is:
- (1) 2187 (2) 243
 (3) 729 (4) 64
49. Consider $f(x) = \tan^{-1}\left(\sqrt{\frac{1 + \sin x}{1 - \sin x}}\right)$, $x \in \left(0, \frac{\pi}{2}\right)$. A normal to $y = f(x)$ at $x = \frac{\pi}{6}$ also passes through the point:
- (1) $\left(0, \frac{2\pi}{3}\right)$ (2) $\left(\frac{\pi}{6}, 0\right)$
 (3) $\left(\frac{\pi}{4}, 0\right)$ (4) $(0, 0)$
50. For $x \in \mathbf{R}$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then:
- (1) $g'(0) = \cos(\log 2)$
 (2) $g'(0) = -\cos(\log 2)$
 (3) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$
 (4) g is not differentiable at $x = 0$
51. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is **NOT true**?
- (1) E_2 and E_3 are independent. (2) E_1 and E_3 are independent.
 (3) E_1, E_2 and E_3 are independent. (4) E_1 and E_2 are independent.

52. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A \text{ adj } A = A A^T$, then $5a + b$ is equal to:
- (1) 5 (2) 4
(3) 13 (4) -1
53. The Boolean Expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to:
- (1) $p \wedge q$ (2) $p \vee q$
(3) $p \vee \sim q$ (4) $\sim p \wedge q$
54. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$ is
- (1) -4 (2) 6
(3) 5 (4) 3
55. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x -axis, lie on:
- (1) an ellipse which is not a circle. (2) a hyperbola.
(3) a parabola. (4) a circle.
56. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is:
- (1) 59th (2) 52nd
(3) 58th (4) 46th
57. $\lim_{n \rightarrow \infty} \left(\frac{(n+1)(n+2)\dots 3n}{n^{2n}} \right)^{1/n}$ is equal to:
- (1) $\frac{27}{e^2}$ (2) $\frac{9}{e^2}$
(3) $3 \log 3 - 2$ (4) $\frac{18}{e^4}$
58. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \dots$, is $\frac{16}{5}m$, then m is equal to:
- (1) 101 (2) 100
(3) 99 (4) 102
59. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S , whose centre is at $(-3, 2)$, then the radius of S is:
- (1) $5\sqrt{3}$ (2) 5
(3) 10 (4) $5\sqrt{2}$
60. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30° . After walking for 10 minutes from A in the same direction, at a point B , he observes that the angle of elevation of the top of the pillar is 60° . Then the time taken (in minutes) by him, from B to reach the pillar, is:
- (1) 10 (2) 20
(3) 5 (4) 6

Solutions

31. Remaining two sides of rhombus are $x - y - 3 = 0$ and $7x - y + 15 = 0$.

So on solving, we get vertices as $\left(\frac{1}{3}, -\frac{8}{3}\right)$, $(1, 2)$, $\left(-\frac{7}{3}, -\frac{4}{3}\right)$ and $(-3, -6)$.

32. Let the G.P. be a, ar, ar^2 and terms of A.P. are $A + d, A + 4d, A + 8d$

$$\text{then } \frac{ar^2 - ar}{ar - a} = \frac{(A + 8d) - (A + 4d)}{(A + 4d) - (A + d)} = \frac{4}{3}$$

$$\Rightarrow r = \frac{4}{3}.$$

Alternate Solution:

Let AP is $a, a + d, a + 2d, \dots$

2^{nd} , 5^{th} and 9^{th} terms $a + d, a + 4d, a + 8d$ are in GP

$$\Rightarrow (a + 4d)^2 = (a + d)(a + 8d)$$

$$\Rightarrow d(8d - a) = 0 \Rightarrow 8d = a \text{ as } d \neq 0$$

$$\text{Hence common ratio of GP } \frac{a + 4d}{a + d} = \frac{8d + 4d}{8d + d} = \frac{12d}{9d} = \frac{4}{3}.$$

33. Equation of normal at P is

$$y = -tx + 4t + 2t^3$$

It passes through C(0, -6)

$$\therefore -6 = 4t + 2t^3$$

$$\Rightarrow t^3 + 2t + 3 = 0$$

$$\Rightarrow t = -1$$

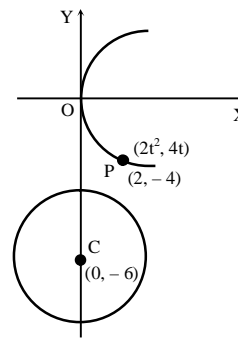
Hence P is (2, -4)

$$r = \sqrt{4+4} = 2\sqrt{2}$$

Equation of required circle

$$(x-2)^2 + (y+4)^2 = 8$$

$$\Rightarrow x^2 + y^2 - 4x + 8y + 12 = 0$$



34. For non-trivial solution

$$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$$1(\lambda + 1) - \lambda(-\lambda^2 + 1) - 1(\lambda + 1) = 0$$

$$\lambda + 1 + \lambda^3 - \lambda - \lambda - 1 = 0$$

$$\lambda(\lambda^2 - 1) = 0 \Rightarrow \lambda = 0, \lambda = \pm 1.$$

35. $f(x) + 2f\left(\frac{1}{x}\right) = 3x$

replace x by $\frac{1}{x}$

$$f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$$

$$\Rightarrow f(x) = \frac{2}{x} - x \text{ as } f(x) = f(-x) \Rightarrow x = \pm \sqrt{2}$$

36. $p = \lim_{x \rightarrow 0^+} (1 + \tan^2 \sqrt{x})^{\frac{1}{2x}}$

$$= e^{\lim_{x \rightarrow 0^+} \frac{\tan^2 \sqrt{x}}{2x}}$$

$$= e^{\lim_{x \rightarrow 0^+} \frac{1}{2} \left(\frac{\tan \sqrt{x}}{\sqrt{x}} \right)^2}$$

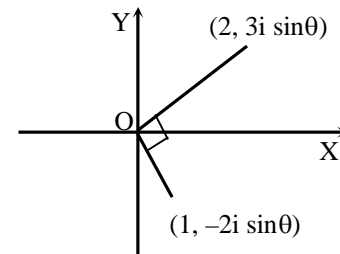
$$p = e^{\frac{1}{2}}$$

37. $\frac{2+3i \sin \theta}{1-2i \sin \theta}$ is purely imaginary $\in \text{Arg} \left(\frac{2+3i \sin \theta}{1-2i \sin \theta} \right) = \frac{\pi}{2}, -\frac{\pi}{2}$

\Rightarrow product of slopes taken as in xy plane is -1

$$\Rightarrow \frac{3 \sin \theta}{2} \cdot \frac{-2 \sin \theta}{1} = -1$$

$$\Rightarrow \sin^2 \theta = \frac{1}{3}, \theta = \sin^{-1} \left(\frac{1}{\sqrt{3}} \right).$$



Alternate Solution:

$$\frac{2+3i \sin \theta}{1-2i \sin \theta} = \frac{2-6 \sin^2 \theta + 7i \sin \theta}{1+4 \sin^2 \theta} \text{ is purely imaginary}$$

$$\Rightarrow \frac{2-6\sin^2\theta}{1+4\sin^2\theta} = 0 \Rightarrow 6\sin^2\theta = 2.$$

38. Given, $\frac{2b^2}{a} = 8$ and $2b = \frac{1}{2}(2ae)$

$$2b = ae$$

$$4b^2 = a^2 \cdot e^2$$

$$3e^2 = 4 \Rightarrow e = \frac{2}{\sqrt{3}}$$

39. $\bar{x} = \frac{2+3+a+11}{4} = \frac{a}{4} + 4$

$$\sigma = \sqrt{\sum \frac{x_i^2}{n} - (\bar{x})^2}$$

$$3.5 = \sqrt{\frac{4+9+a^2+121}{4} - \left(\frac{a}{4} + 4\right)^2}$$

$$\Rightarrow \frac{49}{4} = \frac{4(134+a^2) - (a^2 + 256 + 32a)}{16}$$

$$\Rightarrow 3a^2 - 32a + 84 = 0$$

40. $I = \int \frac{\left(\frac{2}{x^3} + \frac{5}{x^6}\right)}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3} dx, \quad \text{let } 1 + \frac{1}{x^2} + \frac{1}{x^5} = t$

$$\text{Hence } I = \frac{x^{10}}{2(x^5 + x^3 + 1)^2} + c.$$

41. As line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in plane $\ell x + my - z = 9$

So $2\ell - m - 3 = 0$ (as line is perpendicular to normal of the plane) (1)

Also point $(3, -2, -4)$ lies in plane

So $3\ell - 2m - 5 = 0$ (2)

From equation (1) and (2), we get $\ell = 1, m = -1$

So $\ell^2 + m^2 = 2$

42. $2\cos\frac{5x}{2} \cdot \cos\frac{3x}{2} + 2\cos\frac{5x}{2} \cdot \cos\frac{x}{2} = 0$

$$\Rightarrow \cos\frac{5x}{2} \cdot \left(2 \cdot \cos x \cdot \cos\frac{x}{2}\right) = 0$$

$$\Rightarrow \frac{x}{2} = (2n+1)\frac{\pi}{2}, \quad x = (2m+1)\frac{\pi}{2}, \quad \frac{5x}{2} = (2k+1)\frac{\pi}{2}, \quad (\text{where } n, m, k \in \mathbb{Z})$$

$$\Rightarrow x = (2n+1)\pi, \quad x = (2m+1)\frac{\pi}{2}, \quad x = (2k+1)\frac{\pi}{5}$$

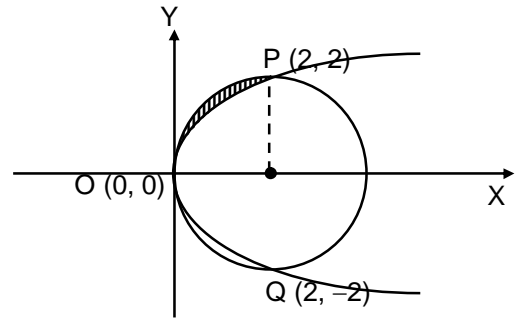
$$\Rightarrow x = \pi, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}.$$

43. The point of intersection of the curve $x^2 + y^2 = 4x$ $y^2 = 2x$ are $(0, 0)$ and $(2, 2)$ for $x \geq 0$ and $y \geq 0$

$$\text{So required area} = \frac{1}{4} \pi \times 4 - \int_0^2 \sqrt{2x} dx$$

$$= \pi - \sqrt{2} \cdot \frac{2}{3} [x^{3/2}]_0^2$$

$$= \pi - \frac{8}{3}$$



44. $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2} (\vec{b} + \vec{c})$

$$\Rightarrow (\vec{a} \cdot \vec{c}) \vec{b} - (\vec{a} \cdot \vec{b}) \vec{c} = \frac{\sqrt{3}}{2} (\vec{b} + \vec{c})$$

$$\Rightarrow \vec{a} \cdot \vec{c} = \frac{\sqrt{3}}{2} \text{ and } \vec{a} \cdot \vec{b} = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos \theta = -\frac{\sqrt{3}}{2} \text{ where } \theta \text{ is angle between } \vec{a} \text{ \& } \vec{b}$$

$$\therefore \theta = \frac{5\pi}{6}$$

45. $f(x) = x^2 + \frac{(1-2x)^2}{\pi}$ (As $r = \frac{1-2x}{\pi}$)

$$f'(x) = 2x - \frac{4(1-2x)}{\pi}$$

$$f''(x) = 2 + \frac{8}{\pi} > 0$$

For minimum value of sum of area $f'(x) = 0$

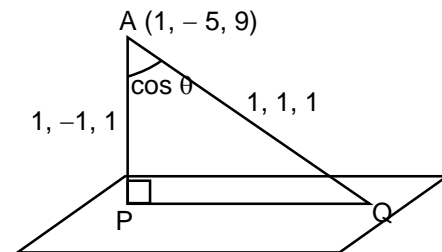
$$x = \frac{2}{\pi+4} \Rightarrow r = \frac{1}{\pi+4}$$

$$\Rightarrow x = 2r.$$

46. $\cos \theta = \frac{1-1+1}{3} = \frac{1}{3}$

$$\cos \theta = \frac{AP}{AQ}$$

$$AQ = \frac{AP}{\cos \theta} = 10\sqrt{3}$$



47. $y(1+xy) dx = x dy$

$$\frac{x dy - y dx}{y^2} = x dx$$

$$\int -d\left(\frac{x}{y}\right) = \int x dx$$

$$-\frac{x}{y} = \frac{x^2}{2} + c \text{ as } y(1) = -1 \Rightarrow c = \frac{1}{2}$$

$$\text{Hence } y = \frac{-2x}{x^2 + 1} \Rightarrow f\left(-\frac{1}{2}\right) = \frac{4}{5}.$$

48. Total number of terms $= {}^{n+2}C_2 = 28$

$$(n+2)(n+1) = 56$$

$$n = 6$$

$$\text{Sum of coefficients} = (1 - 2 + 4)^n$$

$$= 3^6 = 729$$

[*Note: In the solution it is considered that different terms in the expansion having same powers are not merged, as such it should be a bonus question.]

49. $f(x) = \tan^{-1} \sqrt{\frac{1+\sin x}{1-\sin x}}$, where $x \in \left(0, \frac{\pi}{2}\right)$

$$= \tan^{-1} \left(\left| \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right| \right)$$

$$\Rightarrow f(x) = \frac{\pi}{4} + \frac{x}{2}, \quad f\left(\frac{\pi}{6}\right) = \frac{\pi}{3}$$

$$f'(x) = \frac{1}{2}$$

Equation of normal is

$$y - \frac{\pi}{3} = -2 \left(x - \frac{\pi}{6} \right)$$

It passes through $\left(0, \frac{2\pi}{3}\right)$.

50. At $x = 0$, f is differential and $f'(0) = -\cos 0 = -1$

$$g'(0) = f'(f(0)) \cdot f'(0)$$

$$= -\cos(\log 2) \times -1 \quad (\text{at } x = 0, f(0) = \log 2)$$

$$= \cos(\log 2)$$

51. $P(E_1) = \frac{1}{6}$, $P(E_2) = \frac{1}{6}$, $P(E_3) = \frac{1}{2}$

$$\text{Also } P(E_1 \cap E_2) = \frac{1}{36}, \quad P(E_2 \cap E_3) = \frac{1}{12}, \quad P(E_1 \cap E_3) = \frac{1}{12}$$

$$\text{And } P(E_1 \cap E_2 \cap E_3) = 0 \neq P(E_1) \cdot P(E_2) \cdot P(E_3)$$

Hence, E_1, E_2, E_3 are not independent.

52. $A \text{ adj } A = |A| I_n = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$

$$\Rightarrow (10a + 3b) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix}$$

$$\Rightarrow 15a - 2b = 0 \text{ and } 10a + 3b = 13$$

$$\Rightarrow 5a + b = 5 \times \frac{2}{5} + 3 = 5.$$

$$\begin{aligned}
53. \quad & (p \wedge \sim q) \vee q \vee (\sim p \wedge q) \\
& \equiv \{(p \vee q) \wedge (\sim q \vee q)\} \vee (\sim p \wedge q) \\
& \equiv \{(p \vee q) \wedge T\} \vee (\sim p \wedge q) \\
& \equiv (p \vee q) \vee (\sim p \wedge q) \\
& \equiv \{(p \vee q) \vee \sim p\} \wedge (p \vee q \vee q) \\
& \equiv T \wedge (p \vee q) \\
& \equiv p \vee q
\end{aligned}$$

$$\begin{aligned}
54. \quad & \text{Either } x^2 - 5x + 5 = 1 \text{ or } x^2 + 4x - 60 = 0 \\
& x = 1, 4 \text{ or } x = -10, 6 \\
& \text{Also } x^2 - 5x + 5 = -1 \text{ and } x^2 + 4x - 60 \in \text{even number} \\
& x = 2, 3 \\
& \text{For } x = 3 \quad x^2 + 4x - 60 \text{ is odd} \\
& \text{Total solutions are } x = 1, 4, -10, 6, 2 \\
& \Rightarrow \text{Sum} = 3
\end{aligned}$$

$$\begin{aligned}
55. \quad & \text{Let } (h, k) \text{ be the centre of the circle which touch } x\text{-axis and } x^2 + y^2 - 8x - 8y - 4 = 0 \text{ externally.} \\
& \Rightarrow \text{Radius of that circle is } |k| \\
& \Rightarrow (h - 4)^2 + (k - 4)^2 = (|k| + 6)^2 \\
& \Rightarrow x^2 - 8x - 20y - 4 = 0 \text{ if } y \geq 0 \\
& \text{and } x^2 - 8x + 4y - 4 = 0 \text{ if } y < 0 \\
& \Rightarrow \text{The curve is parabola.}
\end{aligned}$$

$$56. \quad \text{Words starting with A, L, M} = \frac{4!}{2!} + 4! + \frac{4!}{2!} = 48$$

$$\text{Words starting with SA, SL} = \frac{3!}{2!} + 3! = 9$$

$$\Rightarrow \text{Rank of the word SMALL} = 58.$$

$$\begin{aligned}
57. \quad \ln y &= \lim_{n \rightarrow \infty} \frac{1}{n} \cdot \log \left(1 + \frac{1}{n} \right) \left(1 + \frac{2}{n} \right) \dots \left(1 + \frac{2n}{n} \right) \\
&= \frac{1}{n} \cdot \sum_{r=1}^{2n} \ln \left(1 + \frac{r}{n} \right)
\end{aligned}$$

$$\begin{aligned}
\ln y &= \int_0^2 \ln(1+x) \, dx, \quad \text{let } t = 1+x \\
&= \int_1^3 \ln t \, dt \\
&= \ln \frac{27}{e^2}
\end{aligned}$$

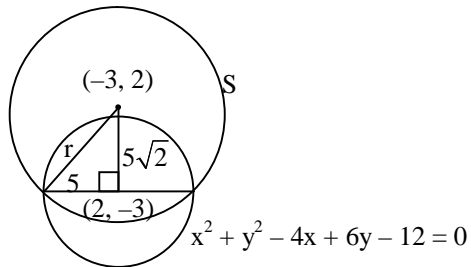
$$58. \quad \text{Let } S = \left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right)^2 + \dots + \left(\frac{44}{5}\right)^2$$

$$\Rightarrow S = \frac{16}{25} [2^2 + 3^2 + 4^2 + 5^2 \dots + 11^2]$$

$$\Rightarrow S = \frac{16}{25} [1^2 + 2^2 \dots + 11^2 - 1] = \frac{16}{5} \times 101$$

$$\Rightarrow m = 101$$

59. Let 'r' be the radius of circle S
 $\Rightarrow r = 5\sqrt{3}$



60. $\tan 60^\circ = \frac{h}{x} \Rightarrow h = \sqrt{3}x$
 $\tan 30^\circ = \frac{h}{x+y} \Rightarrow \sqrt{3}h = x+y$
 $3x = x+y$
 $\Rightarrow 2x = y$
 Time taken from A to B is 10 min
 So time taken from B to pillar is 5 min

