

Questions

For CRT - 03

BY O.P. GUPTA

Max. Marks : 40

INDIRA AWARD WINNER

Time : 60 Minutes

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Topics : Applications Of Matrices & Determinants

Advanced MATH Classes, 1st Floor (Above Master Of Burgers), Opp. HP Petrol Pump, Thana Road, Najafgarh

Q01. (a) Write the value of $|A||\text{adj}A|$ if $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

(b) If the value of an order 3 determinant is given to be 12, then the value of the determinant formed by replacing each element by its co-factor will be 144. True or False? Give reasons. $[1 \times 2 = 2]$

Q02. (a) Without actually expanding, evaluate : $\begin{vmatrix} y^2z^2 & yz & y+z \\ z^2x^2 & zx & z+x \\ x^2y^2 & xy & x+y \end{vmatrix}$.

(b) If $P = \begin{pmatrix} 2 & -3 \\ 4 & -7 \end{pmatrix}$, $Q = \begin{pmatrix} -1 & 5 \\ 2 & -8 \end{pmatrix}$, then find $|P|Q| + |P|Q|$.

Q03. Solve : $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$. $[4 \times 2 = 8]$

Q04. Using properties of determinants, prove that : $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$.

OR Use properties to show that $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$.

Q05. Let $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$, then find the value of $\lim_{t \rightarrow 0} \frac{f(t)}{t^2}$.

Q06. Prove that : $\begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$.

Q07. Using cofactors of elements of 1st column, evaluate $\begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$ & show that it is $(x-y)(y-z)(z-x)$.

Q08. A typist charges ₹ 145 for typing 10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are ₹ 180. Using matrices, find the charges of typing one English and one Hindi page separately. However typist charged only ₹ 2 per page from a poor student Shyam for 5 Hindi pages. How much less was charged from this poor boy?

OR There are 2 families A and B. There are 4 men, 6 women and 2 children in family A, and 2 men, 2 women and 4 children in family B. The recommended daily amount of calories is 2400 for men, 1900 for women, 1800 for children and 45 grams of protein for men, 55 grams for women and 33 grams for children. Represent the above information using matrices. Using matrix multiplication, calculate the total requirement of calories and proteins for each of the 2 families. $[6 \times 5 = 30]$

HINTS & ANSWERS

Q01. (a) $|A||\text{adj}A| = |A||A|^{3-1} = |A|^3 = -1$

(b) True (using $|(\text{adj}.A)^T| = |\text{adj}.A| = |A|^{n-1}$ where n is order of A.

Q02. (a) Ans. 0. See Exemplar Problem Solutions by O.P. Gupta (Q07)

(b) $|P|Q| + |P|Q| = \begin{vmatrix} 2 & -3 \\ 4 & -7 \end{vmatrix}(-2) + (-2)\begin{vmatrix} -1 & 5 \\ 2 & -8 \end{vmatrix} = \begin{vmatrix} -2 & 1 \\ 6 & -15 \end{vmatrix} = (-2)^2 \times (-15 - 12) = -108.$

Q03. $x = \frac{2}{3}, \frac{11}{3}.$

Q05. See Exemplar Problem Solutions by O.P. Gupta (Q30)

Q06. See Exemplar Problem Solutions by O.P. Gupta (Q25)

Q07. (We know that the det. is equal to the sum of the product of the elements of 1st column with their corresponding cofactors).

$$\begin{aligned} \text{Let } \Delta &= \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} = 1(xy^2 - xz^2) - 1(yx^2 - yz^2) + 1(zx^2 - zy^2) \\ &= xy^2 - xz^2 - yx^2 + yz^2 + zx^2 - zy^2 = xy^2 - yx^2 + zx^2 - zy^2 + yz^2 - xz^2 \\ &= xy(y-x) + z(x^2 - y^2) + z^2(y-x) = (x-y)[-xy + z(x+y) - z^2] \\ &= (x-y)[-xy + zx + yz - z^2] = (x-y)[yz - xy + zx - z^2] \\ &= (x-y)[y(z-x) - z(z-x)] = (x-y)(z-x)[y-z] \text{ i.e., } \Delta = (x-y)(y-z)(z-x). \end{aligned}$$

Q08. See Mathematicia by O.P. Gupta

OR The given information can be expressed as :

| | Men | Women | Children | Calories | Proteins | |
|------------------------|---------------------|-------|----------|----------------|----------|-----|
| Family A | 4 | 6 | 2 | 2400 | 45 | |
| Family B | 2 | 2 | 4 | 1900 | 55 | |
| | | | | 1800 | 33 | |
| \Rightarrow Family A | 9600 + 11400 + 3600 | | | 180 + 330 + 66 | 24600 | 576 |
| Family B | 4800 + 3800 + 7200 | | | 90 + 110 + 132 | 15800 | 332 |

So 24600 calories and 579 grams of proteins are needed for Family A and 15800 calories and 332 grams of proteins are needed for Family B.

❖ Dear Student/Teacher,

I would urge you for a little favour. Please notify me about any error (s) which you notice in this (or other Maths) work. It would be beneficial for all the future learners of Maths like us. Any constructive criticism will be well acknowledged.

Please find below my contact info when you decide to offer me your valuable suggestions. I am looking forward for a response.

Also I would wish **if you inform your friend/students** about my efforts for Maths so that they may also be benefitted.

Let's learn Maths with smile :-)

☞ For any clarification(s), please contact :

O.P. Gupta, Math Mentor

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