Questions For CRT - 02

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Max. Marks : **40**
Time : **60 Minutes**
Topics : **Algebra Of Matrices & Determinants**

Advanced MATH Classes, 1st Floor (Above Master Of Burgers), Opp. HP Petrol Pump, Thana Road, Najafgarh

Q01. **(a)** Find the matrix A, if $\begin{bmatrix}4\\1\\3\end{bmatrix}A=\begin{bmatrix}-4&8&4\\-1&2&1\\-3&6&3\end{bmatrix}$.

(b) If $\begin{vmatrix}2x&5\\8&x\end{vmatrix}=\begin{vmatrix}6&-2\\7&3\end{vmatrix}$, write the value of x.

(c) If $A=[a_{ij}]$ is a 3×3 matrix and A_{ij} denotes the co-factors of the corresponding elements a_{ij}'s then, what is the value of $a_{21}A_{11}+a_{22}A_{12}+a_{23}A_{13}$?

(d) Find $A(adj.A)$ without finding adj.A if matrix $A=\begin{bmatrix}3&2&3\\2&1&0\\1&3&1\end{bmatrix}$. Mention the property used.

Q02. If $A=\begin{bmatrix}3&1\\7&5\end{bmatrix}$ then, find x and y so that $A^2+xI-yA=O$. Hence find A^{-1}.

Q03. Find the matrix A, satisfying the matrix equation : $\begin{bmatrix}2&1\\3&2\end{bmatrix}A\begin{bmatrix}-3&2\\5&-3\end{bmatrix}=\begin{bmatrix}-2&4\\3&-1\end{bmatrix}$.

Q04. Find A^{-1}, if $A=\begin{pmatrix}0&1&1\\1&0&1\\1&1&0\end{pmatrix}$. Also show that $A^{-1}=\dfrac{A^2-3I}{2}$.

Q05. If $A=\begin{bmatrix}0&6&7\\-6&0&8\\7&-8&0\end{bmatrix}$, $B=\begin{bmatrix}0&1&1\\1&0&2\\1&2&0\end{bmatrix}$, $C=\begin{bmatrix}2\\-2\\3\end{bmatrix}$, then verify that (A + B) C = AC + BC.

Q06. **(a)** For what value of x, the matrix $\begin{bmatrix}7-2x&x+5\\3&7\end{bmatrix}$ is non-invertible?

(b) If A is 3 × 3 invertible matrix, such that for any scalar k (non-zero), kA is invertible then find the value of $(kA)^{-1}$.

Q07. **(a)** Fill in the blanks : If A is skew symmetric matrix, then A^2 is a …….. matrix.

(b) If $x\in R, 0\le x\le\dfrac{\pi}{2}$, and $\begin{vmatrix}2\sin x&-1\\1&\sin x\end{vmatrix}=\begin{vmatrix}3&0\\-4&\sin x\end{vmatrix}$, then find the values of x. [4×7 = 28

Q08. If $A=\begin{pmatrix}0&-\tan\dfrac{x}{2}\\\tan\dfrac{x}{2}&0\end{pmatrix}$ and I is an identity matrix then, show that $(I+A)=(I-A)\begin{pmatrix}\cos x&-\sin x\\\sin x&\cos x\end{pmatrix}$.

OR By using induction, prove that $A^n=\begin{bmatrix}\cos n\theta&i\sin n\theta\\i\sin n\theta&\cos n\theta\end{bmatrix}$ for all $n\in N$ if $A=\begin{bmatrix}\cos\theta&i\sin\theta\\i\sin\theta&\cos\theta\end{bmatrix}$.

Q09. Find $\begin{pmatrix}3&-4&2\\2&3&5\\1&0&1\end{pmatrix}^{-1}$. Using it, solve : $3x-4y+2z=-1,\ 2x+3y+5z=7,\ x+z=2$. [6×2 = 12

 # Test held on : 16 April, 2017

HINTS & ANSWERS

Q01. (a) $A = [-1 \quad 2 \quad 1]$

(b) Expand the determinants to get : $(2x)(x) - 5\times 8 = 6\times 3 - (-2)\times 7 \Rightarrow 2x^2 = 72 \Rightarrow x = \pm 6$.

(c) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13} = a_{21}(a_{22}a_{33} - a_{32}a_{23}) + a_{22}[-(a_{21}a_{33} - a_{31}a_{23})] + a_{23}(a_{21}a_{32} - a_{31}a_{22}) = 0$.

(d) $A(\text{adj.}A) = 14\,I$. Property used is $A(\text{adj.}A) = (\text{adj.}A)A = |A|\,I$.

Q02. $x = 8,\ y = 8,\ A^{-1} = \begin{bmatrix} 5/8 & -1/8 \\ -7/8 & 3/8 \end{bmatrix}$

Q03. $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}^{-1} \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1}$

$\Rightarrow A = \frac{1}{4-3}\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix} \times \frac{1}{9-10}\begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} -7 & 9 \\ 12 & -14 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} \qquad \therefore A = \begin{bmatrix} 24 & 13 \\ -34 & -18 \end{bmatrix}$

Q04. $A^{-1} = \frac{1}{2}\begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$

Q05. See Mathematicia Q02 (Short & Long Answer Questions)

Q06. (a) See Mathematicia (Very Short Answer Questions)

(b) $\frac{1}{k}A^{-1}$.

Q07. (a) symmetric. (b) See Mathematicia Vol. 1 (Very Short Answer Questions). Ans. $x = \frac{\pi}{2}, \frac{\pi}{6}$.

Q08. See Mathematicia (Short & Long Answer Questions)

OR See Mathematicia (Short & Long Answer Questions)

Q09. $A^{-1} = \frac{1}{9}\begin{pmatrix} -3 & -4 & 26 \\ -3 & -1 & 11 \\ 3 & 4 & -17 \end{pmatrix}$ and $x = 3, y = 2, z = -1$.

❖ Dear Student/Teacher,

I would urge you for a little favour. Please notify me about any error (s) which you notice in this (or other Maths) work. It would be beneficial for all the future learners of Maths like us. Any constructive criticism will be well acknowledged.

Please find below my contact info when you decide to offer me your valuable suggestions. I am looking forward for a response.

Also I would wish **if you inform your friend/students** about my efforts for Maths so that they may also be benefitted.

Let's *learn* Maths with smile :-)

☞ For any clarification(s), please contact :

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