Questions For CRT - 01

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Max. Marks : **40**
Time : **60 Minutes**
Topics : **Algebra Of Matrices**

Advanced MATH Classes, 1st Floor (Above Master Of Burgers), Opp. HP Petrol Pump, Thana Road, Najafgarh

Q01. **(a)** Write the value of $x-2y$ if $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1\\ -1\end{bmatrix}$.

(b) If P and Q are symmetric matrices of same order, then PQ is symmetric if and only if _________.

(c) If $A=\begin{bmatrix}\sin x & \cos x\\ -\cos x & \sin x\end{bmatrix}$ then, write the value of $A'A$. **(d)** If $A=\begin{bmatrix}-5 & 0\\ 0 & -5\end{bmatrix}$, then find A^4.

(e) Matrix $A=\begin{bmatrix}0 & 2b & -2\\ 3 & 1 & 3\\ 3a & 3 & -1\end{bmatrix}$ is given to be symmetric, find the value of $3a+b$.

(f) Write the value of p^2+mk if $A=\begin{bmatrix}p & m\\ k & -p\end{bmatrix}$ satisfies the equation $A^2=I$. $[1\times 6=6$

Q02. **(a)** Determine a matrix B such that $2A+B+X=O$, where $A=\begin{pmatrix}3 & -2\\ 1 & 5\end{pmatrix}$ and $X=\begin{pmatrix}-1 & 2\\ 3 & 4\end{pmatrix}$.

(b) Show that a matrix which is both symmetric and skew-symmetric is a null matrix. $[2\times 2=4$

Q03. **(a)** Show that the elements along the main diagonal of a skew-symmetric matrix are all zero.

(b) If A and B are symmetric matrices, prove that $AB-BA$ is a skew-symmetric matrix. $[2\times 2=4$

Q04. Find the value of x such that $\begin{bmatrix}x & 4 & -1\end{bmatrix}\begin{bmatrix}2 & 1 & -1\\ 1 & 0 & 0\\ 2 & 2 & 4\end{bmatrix}\begin{bmatrix}x & 4 & -1\end{bmatrix}^T=O$, if $x\in Z$.

Q05. If $A=\begin{pmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{pmatrix}$ find $A^2-5A+4I$ and hence find a matrix X such that $A^2-5A+4I+X=O$.

Q06. Construct a 2×2 matrix $A=[a_{ij}]$, where $a_{ij}=\begin{cases}3i-j, & \text{if } i\geq j\\ |i-2j|, & \text{if } i<j\end{cases}$. Also find $a_{12}+a_{21}$.

Q07. Express the matrix $A=\begin{pmatrix}3 & 2 & 3\\ 4 & 5 & 3\\ 2 & 4 & 5\end{pmatrix}$ as the sum of symmetric and a skew-symmetric matrix.

Q08. If $M=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$, $N=\begin{bmatrix}1 & a\\ 4 & b\end{bmatrix}$ such that $M^2+N^2=(M+N)^2$, find the value of a – b. $[4\times 5=20$

Q09. Let $A=\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix}$ and $f(x)=x^2-4x+7$. Show that $f(A)=O$. Use this result to find A^5.

OR Using elementary transformations, find the inverse of $\begin{pmatrix}8 & 4 & 3\\ 2 & 1 & 1\\ 1 & 2 & 2\end{pmatrix}$, if it exists. $[6\times 1=6$

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Q01. **(a)** $x-2y=1-2(-1)=3$.

(b) PQ = QP. **(c)** I, unit matrix of order 2.

(d) Use $A=-5I$. So $A^4=\begin{pmatrix}625 & 0\\ 0 & 625\end{pmatrix}=625\,I$.

(e) $3a+b=-2+3/2=-1/2$. **(f)** $p^2+mk=1$.

Q02. **(a)** $B=\begin{pmatrix}-5 & 2\\ -5 & -14\end{pmatrix}$.

Q04. $x=-4\in Z$.

Q05. See **Mathematicia by O.P. Gupta** (Short & Long Answer Questions).

Q06. $A=\begin{bmatrix}2 & 3\\ 5 & 4\end{bmatrix}$, $a_{12}+a_{21}=8$.

Q07. $\begin{pmatrix}3 & 3 & 5/2\\ 3 & 5 & 7/2\\ 5/2 & 7/2 & 5\end{pmatrix}+\begin{pmatrix}0 & -1 & 1/2\\ 1 & 0 & -1/2\\ -1/2 & 1/2 & 0\end{pmatrix}$.

Q08. Use $(M+N)^2=(M+N)(M+N)=M^2+MN+NM+N^2$

Then, $M^2+N^2=M^2+MN+NM+M^2 \Rightarrow MN+NM=O$

Substitute the matrices and then get $a=1, b=-1 \;\therefore a-b=2$.

Q09. Use $A^2-4A+7I=O$ i.e., $A^2=4A-7I$ to obtain $A^5=-31A-56I=\begin{bmatrix}-118 & -93\\ 31 & -118\end{bmatrix}$.

OR $\begin{pmatrix}0 & 2/3 & -1/3\\ 1 & -13/3 & 2/3\\ -1 & 4 & 0\end{pmatrix}$.

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