

# Questions

For CRT - 01

BY O.P. GUPTA

Max. Marks : 40

INDIRA AWARD WINNER

Time : 60 Minutes

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Topics : Algebra Of Matrices

Advanced MATH Classes, 1<sup>st</sup> Floor (Above Master Of Burgers), Opp. HP Petrol Pump, Thana Road, Najafgarh

- Q01.** (a) Write the value of  $x - 2y$  if  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ .
- (b) If P and Q are symmetric matrices of same order, then PQ is symmetric if and only if \_\_\_\_\_.
- (c) If  $A = \begin{bmatrix} \sin x & \cos x \\ -\cos x & \sin x \end{bmatrix}$  then, write the value of  $A'A$ . (d) If  $A = \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$ , then find  $A^4$ .
- (e) Matrix  $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$  is given to be symmetric, find the value of  $3a + b$ .
- (f) Write the value of  $p^2 + mk$  if  $A = \begin{bmatrix} p & m \\ k & -p \end{bmatrix}$  satisfies the equation  $A^2 = I$ . [1×6 = 6]
- Q02.** (a) Determine a matrix B such that  $2A + B + X = O$ , where  $A = \begin{pmatrix} 3 & -2 \\ 1 & 5 \end{pmatrix}$  and  $X = \begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix}$ .
- (b) Show that a matrix which is both symmetric and skew-symmetric is a null matrix. [2×2 = 4]
- Q03.** (a) Show that the elements along the main diagonal of a skew-symmetric matrix are all zero.
- (b) If A and B are symmetric matrices, prove that  $AB - BA$  is a skew-symmetric matrix. [2×2 = 4]
- Q04.** Find the value of x such that  $\begin{bmatrix} x & 4 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 0 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} x & 4 & -1 \end{bmatrix}^T = O$ , if  $x \in \mathbb{Z}$ .
- Q05.** If  $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$  find  $A^2 - 5A + 4I$  and hence find a matrix X such that  $A^2 - 5A + 4I + X = O$ .
- Q06.** Construct a  $2 \times 2$  matrix  $A = [a_{ij}]$ , where  $a_{ij} = \begin{cases} 3i - j, & \text{if } i \geq j \\ |i - 2j|, & \text{if } i < j \end{cases}$ . Also find  $a_{12} + a_{21}$ .
- Q07.** Express the matrix  $A = \begin{pmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{pmatrix}$  as the sum of symmetric and a skew-symmetric matrix.
- Q08.** If  $M = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ ,  $N = \begin{bmatrix} 1 & a \\ 4 & b \end{bmatrix}$  such that  $M^2 + N^2 = (M + N)^2$ , find the value of  $a - b$ . [4×5 = 20]
- Q09.** Let  $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$  and  $f(x) = x^2 - 4x + 7$ . Show that  $f(A) = O$ . Use this result to find  $A^5$ .

**OR** Using elementary transformations, find the inverse of  $\begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ , if it exists. [6×1 = 6]

# HINTS & ANSWERS

**Q01.** (a)  $x - 2y = 1 - 2(-1) = 3$ .

(b)  $PQ = QP$ . (c)  $I$ , unit matrix of order 2.

(d) Use  $A = -5I$ . So  $A^4 = \begin{pmatrix} 625 & 0 \\ 0 & 625 \end{pmatrix} = 625 I$ .

(e)  $3a + b = -2 + 3/2 = -1/2$ . (f)  $p^2 + mk = 1$ .

**Q02.** (a)  $B = \begin{pmatrix} -5 & 2 \\ -5 & -14 \end{pmatrix}$ .

**Q04.**  $x = -4 \in \mathbb{Z}$ .

**Q05.** See **Mathematicia** by **O.P. Gupta** (Short & Long Answer Questions).

**Q06.**  $A = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$ ,  $a_{12} + a_{21} = 8$ .

**Q07.**  $\begin{pmatrix} 3 & 3 & 5/2 \\ 3 & 5 & 7/2 \\ 5/2 & 7/2 & 5 \end{pmatrix} + \begin{pmatrix} 0 & -1 & 1/2 \\ 1 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{pmatrix}$ .

**Q08.** Use  $(M + N)^2 = (M + N)(M + N) = M^2 + MN + NM + N^2$

Then,  $M^2 + N^2 = M^2 + MN + NM + M^2 \Rightarrow MN + NM = O$

Substitute the matrices and then get  $a = 1, b = -1 \therefore a - b = 2$ .

**Q09.** Use  $A^2 - 4A + 7I = O$  i.e.,  $A^2 = 4A - 7I$  to obtain  $A^5 = -31A - 56I = \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$ .

**OR**  $\begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix}$ .

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❖ Dear Student/Teacher,

I would urge you for a little favour. Please notify me about any error (s) which you notice in this (or other Maths) work. It would be beneficial for all the future learners of Maths like us. Any constructive criticism will be well acknowledged.

Please find below my contact info when you decide to offer your valuable suggestions. I am looking forward for a response.

Moreover, I would wish **if you inform your friends/students** about my efforts for Maths so that they may also be benefited.

**Let's learn Maths with smile :-)**

☞ For any clarification(s), please contact :

**O.P. Gupta, Math Mentor**

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