CHALLENGE 30 ON INTEGRALS

Q01. Put $x=t^2 \Rightarrow dx = 2tdt$ $\quad \Rightarrow \mathrm{I} = \int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\frac{dx}{x} = \int \sqrt{\frac{1-t}{1+t}}\frac{2tdt}{t^2} = \int \frac{1-t}{\sqrt{1-t^2}}\frac{2dt}{t}$

$\Rightarrow \mathrm{I} = 2\int \frac{1}{t\sqrt{1-t^2}}dt - 2\int \frac{dt}{\sqrt{1-t^2}}$ $\quad \Rightarrow \mathrm{I} = 2\int \frac{t}{t^2\sqrt{1-t^2}}dt - 2\sin^{-1}t$

Put $1-t^2 = u^2 \Rightarrow tdt = -udu$ $\quad \Rightarrow \mathrm{I} = -2\int \frac{udu}{(1-u^2)u} - 2\sin^{-1}\sqrt{x}$

$\Rightarrow \mathrm{I} = -2\frac{1}{2}\log\left|\frac{1+u}{1-u}\right| - 2\sin^{-1}\sqrt{x} + \mathrm{C}$ $\quad \Rightarrow \mathrm{I} = \log\left|\frac{1-\sqrt{1-x}}{1+\sqrt{1-x}}\right| - 2\sin^{-1}\sqrt{x} + \mathrm{C}$

Q02. Let $\mathrm{I} = \int \frac{x+\sqrt{1-x^2}\sin^{-1}x}{\sqrt{1-x^2}}dx$ $\quad \Rightarrow \mathrm{I} = \int \frac{x}{\sqrt{1-x^2}}dx + \int \frac{\sqrt{1-x^2}\sin^{-1}x}{\sqrt{1-x^2}}dx$

$\Rightarrow \mathrm{I} = \int \frac{x}{\sqrt{1-x^2}}dx + \int 1.\sin^{-1}xdx$ $\quad \Rightarrow \mathrm{I} = \int \frac{x}{\sqrt{1-x^2}}dx + \sin^{-1}x\int 1.dx - \int\left[\frac{d}{dx}\sin^{-1}x\int 1.dx\right]dx$

$\Rightarrow \mathrm{I} = \int \frac{x}{\sqrt{1-x^2}}dx + x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}dx = x\sin^{-1}x + \mathrm{C}$

Q03. $\int \frac{x}{1+\sec x}dx = \int \frac{x\cos x}{1+\cos x}dx = \int \frac{x(1+\cos x - 1)}{1+\cos x}dx = \int \frac{x(1+\cos x)}{1+\cos x}dx - \int \frac{x}{1+\cos x}dx$

$= \int xdx - \frac{1}{2}\int x\sec^2\frac{x}{2}dx = \frac{x^2}{2} - \frac{1}{2}\left[x\int \sec^2\frac{x}{2}dx - \int\left(\frac{d}{dx}x\int \sec^2\frac{x}{2}dx\right)dx\right]$

$= \frac{x^2}{2} - \frac{1}{2}\left[2x\tan\frac{x}{2} - 2\int \tan\frac{x}{2}dx\right] = \frac{x^2}{2} - \left[x\tan\frac{x}{2} + 2\log|\cos\frac{x}{2}|\right] = \frac{x^2}{2} - x\tan\frac{x}{2} - 2\log\left|\cos\frac{x}{2}\right| + \mathrm{C}$

Q04. Let $\mathrm{I} = \int \cos 2\theta \log\left(\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}\right)d\theta$

$= \log\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)\int \cos 2\theta d\theta - \int\left[\frac{d}{d\theta}\log\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)\int \cos 2\theta d\theta\right]d\theta$

$= \log\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)\frac{\sin 2\theta}{2}$

$\qquad - \int\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)\left[\frac{(\cos\theta-\sin\theta)(\cos\theta-\sin\theta)-(\cos\theta+\sin\theta)(-\sin\theta-\cos\theta)}{(\cos\theta-\sin\theta)^2}\right]\frac{\sin 2\theta}{2}d\theta$

$= \log\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)\frac{\sin 2\theta}{2} - \int \tan 2\theta d\theta = \frac{\sin 2\theta}{2}\log\tan\left(\frac{\pi}{4}+\theta\right) - \frac{1}{2}\log|\sec 2\theta| + \mathrm{C}.$

Q05. Let $\mathrm{I} = \int \sqrt{\frac{x}{a^3-x^3}}dx = \int \frac{\sqrt{x}}{\sqrt{a^3-(x^{3/2})^2}}dx$ $\quad$ Put $\mathrm{x}^{3/2} = \mathrm{t} \Rightarrow \sqrt{x}dx = \frac{2}{3}dt$

$\Rightarrow \mathrm{I} = \frac{2}{3}\int \frac{dt}{\sqrt{(a^{3/2})^2 - t^2}} = \frac{2}{3}\sin^{-1}\left(\frac{t}{a^{3/2}}\right) + \mathrm{C} = \frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{3/2} + \mathrm{C}$

Q06. Let $\mathrm{I} = \int a^x\left[\log x + \log a \log\left(\frac{x^x}{e^x}\right)\right]dx = \int a^x \log xdx + \int a^x \log a\left[x(\log x - 1)\right]dx$

$\Rightarrow \mathrm{I} = a^x\int \log xdx - \int\left(\frac{d}{dx}a^x\int \log xdx\right)dx + \int a^x\log a\left[x(\log x-1)\right]dx$

$$\Rightarrow \mathrm{I} = a^x x(\log x - 1) - \int a^x x \log a(\log x - 1)dx + \int a^x \log a\left[x(\log x - 1)\right]dx = a^x x \log\left(\frac{x}{e}\right) + \mathrm{C}$$

Q07. Let $\mathrm{I} = \int \frac{1}{\sqrt{1-\mathrm{e}^{2x}}}dx = \int \frac{e^{-x}}{\sqrt{\mathrm{e}^{-2x}-1}}dx$ Put $e^{-x} = t \Rightarrow e^{-x}dx = -dt$

$\Rightarrow \mathrm{I} = -\int \frac{dt}{\sqrt{t^2-1}} = -\log|t + \sqrt{t^2-1}| + \mathrm{C}$ $\Rightarrow \mathrm{I} = -\log|e^{-x} + \sqrt{e^{-2x}-1}| + \mathrm{C}$

$\therefore \mathrm{I} = x - \log|1 + \sqrt{1-e^{2x}}| + \mathrm{C}$

Q08. Let $\mathrm{I} = \int \frac{1}{x^{2/3}\sqrt{x^{2/3}-4}}dx = \int \frac{1}{x^{2/3}\sqrt{(x^{1/3})^2-4}}dx$ Put $x^{1/3} = t \Rightarrow \frac{dx}{x^{2/3}} = 3dt$

$\Rightarrow \mathrm{I} = 3\int \frac{dt}{\sqrt{t^2-4}} = 3\log|t + \sqrt{t^2-4}| + \mathrm{C} = 3\log|x^{1/3} + \sqrt{x^{2/3}-4}| + \mathrm{C}$

Q09. Let $\mathrm{I} = \int \frac{1}{(3\sin x + 2\cos x)^2}dx = \int \frac{\sec^2 x}{(3\tan x + 2)^2}dx$ Put $3\tan x + 2 = t \Rightarrow \sec^2 x dx = \frac{1}{3}dt$

$\Rightarrow \mathrm{I} = \frac{1}{3}\int \frac{1}{t^2}dx = -\frac{1}{3t} + C = -\frac{1}{3(3\tan x + 2)} + \mathrm{C}$

Q10. Let $\mathrm{I} = \int x^2\sqrt{ax+b}\,dx$ Put $ax + b = t \Rightarrow dx = \frac{1}{a}dt$ also, $x^2 = \left(\frac{t-b}{a}\right)^2$

$\Rightarrow \mathrm{I} = \int\left(\frac{t-b}{a}\right)^2\sqrt{t}\frac{dt}{a} = \frac{1}{a^3}\int(t^{5/2} + b^2t^{1/2} - 2bt^{3/2})\,dt$

$\Rightarrow \mathrm{I} = \frac{1}{a^3}\left[\frac{2}{7}t^{7/2} + \frac{2}{3}b^2t^{3/2} - \frac{4}{5}bt^{5/2}\right] + \mathrm{C} = \frac{2}{a^3}t^{3/2}\left[\frac{1}{7}t^2 + \frac{1}{3}b^2 - \frac{2}{5}bt\right] + \mathrm{C}$

$\therefore \mathrm{I} = \frac{2}{a^3}(ax+b)^{3/2}\left[\frac{(ax+b)^2}{7} - \frac{2b}{5}(ax+b) + \frac{b^2}{3}\right] + \mathrm{C}$

Q11. Let $\mathrm{I} = \int \frac{2}{1-\sin 2x}dx = \int \frac{2}{1+\cos\left(\frac{\pi}{2}+2x\right)}dx = \int \frac{2}{2\cos^2\left(\frac{\pi}{4}+x\right)}dx$

So, $\mathrm{I} = \int \sec^2\left(\frac{\pi}{4}+x\right)dx = \tan\left(\frac{\pi}{4}+x\right) + \mathrm{C}$

Q12. $\int \frac{\sin^2 x}{(1+\cos x)^2}dx = \int \frac{1-\cos^2 x}{(1+\cos x)^2}dx = \int \frac{(1-\cos x)(1+\cos x)}{(1+\cos x)^2}dx = \int \frac{(1-\cos x)}{(1+\cos x)}dx = \int \tan^2\left(\frac{x}{2}\right)dx$

$= \int[\sec^2\left(\frac{x}{2}\right) - 1]dx = 2\tan\frac{x}{2} - x + \mathrm{C}$

Q13. Let $\mathrm{I} = \int \frac{\sqrt{1+\mathrm{x}^2}}{1-\mathrm{x}^2}\mathrm{dx} = \int \frac{1+\mathrm{x}^2}{(1-\mathrm{x}^2)\sqrt{1+\mathrm{x}^2}}\mathrm{dx} = -\int \frac{-1-\mathrm{x}^2}{(1-\mathrm{x}^2)\sqrt{1+\mathrm{x}^2}}\mathrm{dx} = -\int \frac{1-\mathrm{x}^2-2}{(1-\mathrm{x}^2)\sqrt{1+\mathrm{x}^2}}\mathrm{dx}$

$\Rightarrow \mathrm{I} = -\int \frac{1-\mathrm{x}^2}{(1-\mathrm{x}^2)\sqrt{1+\mathrm{x}^2}}\mathrm{dx} + \int \frac{2}{(1-\mathrm{x}^2)\sqrt{1+\mathrm{x}^2}}\mathrm{dx} = -\int \frac{1}{\sqrt{1+\mathrm{x}^2}}\mathrm{dx} + \mathrm{I}_1.$

Now put $\mathrm{x} = \frac{1}{\mathrm{t}}$ in $\mathrm{I}_1 \Rightarrow \mathrm{dx} = -\frac{1}{\mathrm{t}^2}$. So $\mathrm{I}_1 = -\int \frac{2\mathrm{t}}{(\mathrm{t}^2-1)\sqrt{1+\mathrm{t}^2}}\mathrm{dt}$.

Put $\mathrm{t}^2 + 1 = \mathrm{y}^2 \Rightarrow 2\mathrm{t}\,\mathrm{dt} = 2\mathrm{y}\,\mathrm{dy}$.

We have, $I_1 = -\int \frac{2y}{(y^2-2)y}dy = -2\int \frac{1}{y^2-(\sqrt{2})^2}dy = -2\times\frac{1}{2\sqrt{2}}\log\left|\frac{y-\sqrt{2}}{y+\sqrt{2}}\right| = \frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{t^2+1}+\sqrt{2}}{\sqrt{t^2+1}-\sqrt{2}}\right|$

i.e., $I_1 = \frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{x^2+1}+x\sqrt{2}}{\sqrt{x^2+1}-x\sqrt{2}}\right|$. So, $I = -\log\left|x+\sqrt{1+x^2}\right| + \frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{x^2+1}+x\sqrt{2}}{\sqrt{x^2+1}-x\sqrt{2}}\right| + k$

Q14. $\int \sec^2 x \operatorname{cosec}^2 x\, dx = \int \frac{\sin^2 x+\cos^2 x}{\cos^2 x \sin^2 x}dx = \int(\sec^2 x + \operatorname{cosec}^2 x)\,dx = \tan x - \cot x + C$

Q15. Let $I = \int \frac{x\, e^{m\sin^{-1}x}}{\sqrt{1-x^2}}dx$ Put $\sin^{-1}x = t \Rightarrow x = \sin t$ and $\frac{dx}{\sqrt{1-x^2}} = dt$

$\Rightarrow I = \int \sin t\, e^{mt}\, dt = \sin t\int e^{mt}\, dt - \int\left(\frac{d}{dt}\sin t\int e^{mt}\, dt\right)dt$

$\Rightarrow I = \frac{1}{m}e^{mt}\sin t - \frac{1}{m}\left[\int e^{mt}\cos t\, dt\right]$

$\Rightarrow I = \frac{1}{m}e^{mt}\sin t - \frac{1}{m}\left[\cos t\int e^{mt}\, dt - \int\left(\frac{d}{dt}\cos t\int e^{mt}\, dt\right)dt\right]$

$\Rightarrow I = \frac{1}{m}e^{mt}\sin t - \frac{1}{m}\left[\frac{1}{m}e^{mt}\cos t + \frac{1}{m}\int e^{mt}\sin t\, dt\right]$

$\Rightarrow I = \frac{1}{m}e^{mt}\sin t - \frac{1}{m^2}e^{mt}\cos t - \frac{1}{m^2}I$

$\Rightarrow I\left(1+\frac{1}{m^2}\right) = \frac{1}{m}e^{mt}\sin t - \frac{1}{m^2}e^{mt}\cos t = \frac{e^{mt}}{m^2}\left(m\sin t - \cos t\right)$

$\Rightarrow I = \frac{e^{m\sin^{-1}x}}{1+m^2}[mx - \sqrt{1-x^2}] + C$

Q16. Let $I = \int \frac{e^{2x}}{1+e^x}dx$ Put $1+e^x = t \Rightarrow e^x dx = dt$ $\therefore I = \int\frac{(t-1)}{t}dt$

$\Rightarrow I = \int\left(1-\frac{1}{t}\right)dx = t - \log|t| + C = e^x - \log|1+e^x| + k$, where $k = 1+C$

Q17. $\int \frac{\sin 2x}{\sin^4 x - \cos^4 x}dx = \int\frac{\sin 2x}{(\sin^2 x+\cos^2 x)(\sin^2 x - \cos^2 x)}dx = -\int\frac{\sin 2x}{\cos 2x}dx = \frac{1}{2}\log|\cos 2x| + k$

Q18. Let $I = \int \sin^3\sqrt{x}\, dx$ Put $x = t^2 \Rightarrow dx = 2t dt$ $\therefore I = \int 2t\sin^3 t\, dt$

Use $\sin 3A = 3\sin A - 4\sin^3 A \Rightarrow \sin^3 t = \frac{1}{4}(3\sin t - \sin 3t)$

So, $I = \frac{1}{2}\int(3\sin t - \sin 3t)t\, dt = \frac{1}{2}\left[3\int t\sin t\, dt - \int t\sin 3t\, dt\right]$ Apply integral By Parts

$\therefore I = \frac{2}{9}\sin^3\sqrt{x} + \frac{2}{3}\sqrt{x}\cos^3\sqrt{x} - 2\sqrt{x}\cos\sqrt{x} + \frac{4}{3}\sin\sqrt{x} + k$

Q19. $\int x\, 2^x\, dx = x\int 2^x\, dx - \int\left(\frac{d}{dx}x\int 2^x\, dx\right)dx = x\left(\frac{2^x}{\log 2}\right) - \frac{1}{\log 2}\int 2^x dx = \frac{2^x}{\log_e 2}[x - \log_2 e] + C$

Q20. $\int\sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}}dx = \int\sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}\times\frac{\sin(x-\alpha)}{\sin(x-\alpha)}}dx = \int\frac{\sin(x-\alpha)}{\sqrt{\sin^2 x - \sin^2\alpha}}dx$

Since $\sin(A-B) = \sin A\cos B - \cos A\sin B$ so, $\cos\alpha\int\frac{\sin x\, dx}{\sqrt{\cos^2\alpha - \cos^2 x}} - \sin\alpha\int\frac{\cos x\, dx}{\sqrt{\sin^2 x - \sin^2\alpha}}$

$$\therefore \int \sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}}\,dx = -\cos\alpha\,\sin^{-1}\left(\frac{\cos x}{\sin\alpha}\right) - \sin\alpha \log|\sin x + \sqrt{\sin^2 x - \sin^2\alpha}| + C.$$

Q21. Put $\tan x - x = t \Rightarrow (\sec^2 x - 1)d = dt \Rightarrow \tan^2 x dx = dt \quad \therefore \int (\tan x - x)\tan^2 x\,dx = \int t\,dx = \frac{t^2}{2} + C$

So, $\int (\tan x - x)\tan^2 x\,dx = \frac{1}{2}(\tan x - x)^2 + C$

Q22. Let $I = \int \frac{1+\sin x}{\sin x(1+\cos x)}dx = \int \frac{\sin x}{\sin^2 x(1+\cos x)}dx + \int \frac{1}{1+\cos x}dx$

$\Rightarrow I = \int \frac{\sin x}{(1-\cos x)(1+\cos x)^2}dx + \frac{1}{2}\int \sec^2\left(\frac{x}{2}\right)dx$ Put cos x = t in first integral $\Rightarrow$ sin x dx = –dt

i.e., $I = -\int \frac{dt}{(1-t)(1+t)^2} + \frac{1}{2}\left[2\tan\left(\frac{x}{2}\right)\right]$

Consider $\frac{1}{(1-t)(1+t)^2} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{(1+t)^2} \Rightarrow$ A = 1/4, B = 1/4, C = 1/2.

So, $I = \tan\left(\frac{x}{2}\right) - \frac{1}{4}\int \frac{dt}{(1-t)} - \frac{1}{4}\int \frac{dt}{(1+t)} + \frac{1}{2}\int \frac{dt}{(1+t)^2}$

$\Rightarrow I = \tan\left(\frac{x}{2}\right) + \frac{1}{4}\log|1-t| - \frac{1}{4}\log|1+t| + \frac{1}{2(1+t)} + C$

$\Rightarrow I = \tan\left(\frac{x}{2}\right) + \frac{1}{2}\log\left|\tan\left(\frac{x}{2}\right)\right| + \frac{1}{2(1+\cos x)} + C$

Q23. Let $I = \int \frac{\sqrt{x^2+1}}{x^4}dx = \int \frac{\sqrt{1+\frac{1}{x^2}}}{x^3}dx$ Put $1 + \frac{1}{x^2} = t \Rightarrow \frac{dx}{x^3} = -\frac{1}{2}dt$

$\Rightarrow I = -\frac{1}{2}\int \sqrt{t}\,dx = -\frac{1}{3}t^{3/2} + C = -\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2} + C$

Q24. $\int \frac{\cos x - \cos 2x}{1-\cos x}dx = \int \frac{\cos x - 2\cos^2 x + 1}{1-\cos x}dx = \int \frac{(2\cos x+1)(1-\cos x)}{1-\cos x}dx = 2\sin x + x + C$

Q25. $\int \frac{\cos 5x + \cos 4x}{1-2\cos 3x}dx = \int \frac{2\sin 3x \cos\frac{5x+4x}{2}\cos\frac{5x-4x}{2}}{\sin 3x - 2\sin 3x\cos 3x}dx = \int \frac{2\sin 3x\cos\frac{9x}{2}\cos\frac{x}{2}}{\sin 3x - \sin 6x}dx$

$= \int \frac{2[2\sin\frac{3x}{2}\cos\frac{3x}{2}]\cos\frac{9x}{2}\cos\frac{x}{2}}{2\cos\frac{9x}{2}\sin\frac{-3x}{2}}dx = -\int 2\cos\frac{3x}{2}\cos\frac{x}{2}dx = -\int\left[\cos\left(\frac{3x}{2}+\frac{x}{2}\right) + \cos\left(\frac{3x}{2}-\frac{x}{2}\right)\right]dx$

$= -\int[\cos 2x + \cos x]dx = -\frac{1}{2}[2\sin x + \sin 2x] + C.$

Q26. Let $I = \int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$ Put $x = a\tan^2\theta \Rightarrow dx = 2a\tan\theta\sec^2\theta d\theta$ & $\theta = \tan^{-1}\sqrt{\frac{x}{a}}$

$\Rightarrow I = \int \sin^{-1}\sqrt{\frac{a\tan^2\theta}{a+a\tan^2\theta}}(2a\tan\theta\sec^2\theta)d\theta = 2a\int \theta(\tan\theta\sec^2\theta)d\theta$

$\Rightarrow I = 2a\left[\theta\int(\tan\theta\sec^2\theta)d\theta - \int\left(\frac{d}{d\theta}\theta\int(\tan\theta\sec^2\theta)d\theta\right)d\theta\right]$

$\Rightarrow I = 2a\left[\theta\frac{\tan^2\theta}{2} - \frac{1}{2}\int \tan^2\theta d\theta\right] \quad \Rightarrow I = a\left[\theta\tan^2\theta - (\tan\theta - \theta)\right] + C$

$$\Rightarrow \mathrm{I} = a\left[\theta\tan^2\theta - (\tan\theta - \theta)\right] + \mathrm{C} = (x+a)\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{ax} + \mathrm{C}.$$

Q27. $\int (\tan\log x + \sec^2\log x)\,dx = \int \tan\log x dx + \int \sec^2\log x\,dx$

$$= \tan\log x\int 1dx - \int\left(\frac{d}{dx}\tan\log x\int 1dx\right)dx + \int\sec^2\log x\,dx$$

$$= x\tan\log x - \int\left(\left[\frac{\sec^2\log x}{x}\right]x\right)dx + \int\sec^2\log x\,dx = x\tan\log x + \mathrm{C}$$

Q28. Let $\mathrm{I} = \int \frac{1}{(\sin x - 2\cos x)(2\sin x + \cos x)}dx = \int \frac{\sec^2 x}{(\tan x - 2)(2\tan x + 1)}dx$

Put $\tan x = t \Rightarrow \sec^2 x dx = dt$ $\quad\therefore \mathrm{I} = \int \frac{dt}{(t-2)(2t+1)}$

Use partial fraction now, to get : $\mathrm{I} = \frac{1}{5}\log\left|\frac{\tan x - 2}{2\tan x + 1}\right| + k$

Q29. Change $\int (1 + x - x^{-1}).e^{x+x^{-1}}\,dx$ into $\int\left[e^{\left(x+\frac{1}{x}\right)} + x\left(1 - \frac{1}{x^2}\right)e^{\left(x+\frac{1}{x}\right)}\right]dx$

Then use $\int\left[f(x) + xf'(x)\right]dx = xf(x) + \mathrm{C}$ to get $x.e^{x+x^{-1}} + \mathrm{C}$

Q30. Change $\int \frac{dx}{\cos^3 x\sqrt{\sin 2x}}$ into $\int \frac{dx}{\cos^4 x\sqrt{2\tan x}} = \int \frac{\sec^4 x\,dx}{\sqrt{2\tan x}} \Rightarrow \sqrt{2\tan x}\left(1 + \frac{1}{5}\tan^2 x\right) + k.$

Hii, All!

I hope this texture may have proved beneficial for you.

While going through this material, if you noticed any error(s) or, something which doesn't make sense to you, please bring it in my notice through SMS or Call at +91-9650 350 480 or Email at **theopgupta@gmail.com**.

With lots of Love & Blessings!

- OP Gupta [+91-9650 350 480 | +91-9718 240 480]
Maths (H.), E & C Engg., Indira Award Winner

Visit at : **www.theOPGupta.com/**