CBSE 2014 ALL INDIA COMPARTMENT EXAMINATION (Set I, II, III)

Time Allowed : 3 Hours **Maximum Marks : 100**

SECTION – A

[Question numbers 01 to 10 carry 1 mark each.]

Q01. Let $* : R \times R \to R$, $(a, b) \to a + 4b^2$ is a binary operation. Compute $(-5) * (2*0)$.

Q02. The element a_{ij} of a 3×3 matrix is given by $a_{ij} = \frac{1}{2}\{-3i + j\}$. Write the value of element a_{32}.

Q03. Write the principal value of $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.

Q04. If $\begin{pmatrix} 2x & 4 \end{pmatrix}\begin{pmatrix} x \\ -8 \end{pmatrix} = O$, find the positive value of x.

Q05. Write the value of $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$.

Q06. Find $\int \frac{\sin^6 x}{\cos^8 x}\,dx$.

Q07. Evaluate : $\int_0^1 \frac{\tan^{-1} x}{1+x^2}\,dx$.

Q08. If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, find the angle between $\vec{a}$ and $\vec{b}$.

Q09. Find the angle between x-axis and the vector $\hat{i} + \hat{j} + \hat{k}$.

Q10. Write the equation of the straight line through the point (α, β, γ) and parallel to z-axis.

SECTION – B

[Question numbers 11 to 22 carry 4 marks each.]

Q11. If $f, g : R \to R$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x$, for all $x \in R$. Then find the value of *fog* and *gof*.

Q12. Prove that : $\cos^{-1}(x) + \cos^{-1}\left\{\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right\} = \frac{\pi}{3}$. **(OR)** Solve for x : $\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$.

Q13. Using properties of determinants, prove the following : $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$.

Q14. Find the value of the constant k so that the function *f*, defined below, is continuous at x = 0 where

$$f(x) = \begin{cases} \left(\frac{1-\cos 4x}{8x^2}\right), & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}.$$

Q15. If $y = \tan^{-1}\left(\frac{x}{a}\right) + \log\sqrt{\frac{x-a}{x+a}}$, prove that $\frac{dy}{dx} = \frac{2ax^2}{x^4 - a^4}$.

Q16. Find the intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is (a) strictly increasing and (b) strictly decreasing.

(OR) The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which the area increases when the side is 10 cm.

Q17. Show that : $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}\,dx = \frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$. **(OR)** Find : $\int \frac{x^3}{x^4 + 3x^2 + 2}\,dx$.

Q18. Find the particular solution of the differential equation : $x\frac{dy}{dx} - y + x\,\text{cosec}\left(\frac{y}{x}\right) = 0$; given that y = 0 when x = 1.

Q19. Solve the differential equation $x\frac{dy}{dx} + y = x\cos x + \sin x$, given that $y\left(\frac{\pi}{2}\right) = 1$.

Q20. Find the distance of the point (–1,–5,–10) from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} - \hat{j} + \hat{k}) = 5$.

Q21. Find the vector $\vec{p}$ which is perpendicular to both $\vec{\alpha} = 4\hat{i} + 5\hat{j} - \hat{k}$ and $\vec{\beta} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{p}.\vec{q} = 21$, where $\vec{q} = 3\hat{i} + \hat{j} - \hat{k}$.

(OR) Find the unit vector perpendicular to the plane ABC where the position vectors of A, B and C are $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} + 3\hat{k}$ respectively.

Q22. A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find the mean of X.

SECTION – C

[Question numbers 23 to 29 carry 6 marks each.]

Q23. AB is a diameter of a circle and C is any point on the circle. Show that the area of ΔABC is maximum, when it is isosceles.

Q24. Using integration, find the area of the triangle PQR, coordinates of whose vertices are P(2, 0), Q(4, 5) and R(6, 3).

Q25. Find : $\int \frac{\sqrt{x^2+1}(\log(x^2+1) - 2\log x)}{x^4} dx$. **(OR)** Evaluate : $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx,\ x \in [0,1]$.

Q26. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r}.(2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and parallel to x-axis.

Q27. An urn contains 4 balls. Two balls are drawn at random from the urn (without replacement) and are found to be white. What is the probability that all the four balls in the urn are white?

(OR) In a game, a man wins rupees five for a six and loses rupee one for any other number, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/loses.

Q28. Two schools, P and Q, want to award their selected students for the values of sincerity, truthfulness and hard work at the rate of ₹x, ₹y and ₹z for each respective value per student. School P awards its 2, 3 and 4 students on the above respective values with a total prize money of ₹4600. School Q wants to award its 3, 2 and 3 students on the respective values with a total award money of ₹4100. If the total amount of award money for one prize on each value is ₹1500, using matrices find the award money for each value. Suggest one other value which the school can consider for awarding the students.

Q29. One kind of cake requires 200 g of flour and 25 g of fat, and another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes. Formulate the above as a linear programming problem and solve graphically.

☞ **NOTE:** Only those Questions from Set II are given here which are not in common with Set I.

Q01. Find the value of the following :

$$\cot\left(\frac{\pi}{2} - 2\cot^{-1}\sqrt{3}\right).$$

Q02. Evaluate : $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$.

Q11. Find the particular solution of the differential equation $\left\{x\sin^2\left(\frac{y}{x}\right) - y\right\}dx + x\,dy = 0$, given that $y = \frac{\pi}{4}$, when $x = 1$.

Q12. Show that $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\cos^{-1}\left(\frac{36}{85}\right)$.

(OR) Solve for x : $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}(x),\ [x>0]$.

Q17. If $y=\sin^{-1}\left\{x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right\}$ and $0<x<1$, then find $\frac{dy}{dx}$.

Q18. Find the image of the point having position vector $\hat{i}+3\hat{j}+4\hat{k}$ in the plane $\vec{r}.(2\hat{i}-\hat{j}+\hat{k})+3=0$.

Q28. Find the point P on the curve $y^2=4ax$ which is nearest to the point (11a, 0).

Q29. Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line $y=x$ and the circle $x^2+y^2=18$.

☞ **NOTE:** Only those Questions from Set III are given here which are not in common with Set I & Set II.

Q01. Let *f* : {1, 3, 4} → {1, 2, 5} and g : {1, 2, 5} → {1, 3} given by *f* = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}. Write down *gof*.

Q06. Evaluate : $\int_1^2 \frac{x^3-1}{x^2}dx$.

Q11. Show that $\Delta=\Delta_1$, where $\Delta=\begin{vmatrix}Ax & x^2 & 1\\ By & y^2 & 1\\ Cz & z^2 & 1\end{vmatrix}$, $\Delta_1=\begin{vmatrix}A & B & C\\ x & y & z\\ zy & zx & xy\end{vmatrix}$.

Q14. Evaluate : $\int_{\pi/6}^{\pi/3}\frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$. **(OR)** Find $\int\frac{x\cos^{-1}(x)}{\sqrt{1-x^2}}dx$.

Q17. Solve the differential equation : $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx}=y\cos\left(\frac{y}{x}\right)+x;\ x\neq 0$.

Q22. If $(\tan^{-1}x)^y + y^{\cot x} = 1$, then find $\frac{dy}{dx}$.

Q23. If the length of three sides of a trapezium other than base is 10 cm each, then find the area of the trapezium when it is maximum.

Q24. Using integration, find the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$.

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Q01. 11. Q02. 7/2. Q03. $-\pi/4$. Q04. $x = 4$. Q05. Value of determinant is 0.

Q06. $\frac{\tan^7 x}{7} + C$. Q07. $\frac{\pi^2}{32}$. Q08. $\frac{\pi}{6}$. Q09. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Q10. $\frac{x-\alpha}{0} = \frac{y-\beta}{0} = \frac{z-\gamma}{1}$. Q11. Value of $fog = \begin{cases} 0, \text{ if } x \ge 0 \\ -4x, \text{ if } x < 0 \end{cases}$ and $gof = 0 \ \forall x \in R$.

Q12. Substitute $\cos^{-1}(x) = \theta \Rightarrow \cos\theta = x$ in LHS. (OR) $x = \sqrt{3}$.

Q14. 1. Q16. (a) $(-2, 1) \cup (3, \infty)$ (b) $(-\infty, -2) \cup (1, 3)$. (OR) $10\sqrt{3}\ cm^2/s$.

Q17. (OR) $\log\left|\frac{x^2+2}{\sqrt{x^2+1}}\right| + C$. Q18. $\cos\left(\frac{y}{x}\right) = 1 + \log|x|$.

Q19. $y = \sin x$. Q20. 13 units. Q21. $7\hat{i} - 7\hat{j} - 7\hat{k}$. (OR) $\frac{1}{\sqrt{14}}[3\hat{i} + 2\hat{j} - \hat{k}]$.

Q22. The probability distribution is as shown below :

X	14	15	16	17	18	19	20	21
P(X)	2/15	1/15	2/15	3/15	1/15	2/15	3/15	1/15

Mean of X $= \sum X.\,P(X) = 263/15 = 17.53$. Q24. 7 Sq.units.

Q25. $\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\frac{2}{3} - \log\left(1+\frac{1}{x^2}\right)\right] + C$. (OR) $\frac{\sin^{-1}\sqrt{x}}{\pi}[2(2x-1)] + \frac{2}{\pi}\sqrt{x-x^2} - x + C$.

Q26. $\vec{r}.(3\hat{k} - \hat{j}) = 6$. Q27. 6/10 (OR) Expected value = ₹0.

Q28. Values of x, y and z are respectively 500, 400 and 600.

Q29. To maximize : $Z = x + y$.

Subject to constraints : $x \ge 0, y \ge 0,\ 200x + 100y \le 5000,\ 25x + 50y \le 1000$.

Z is maximum at (20, 10).

CBSE 2014 Compt. Exams. All India – Set 2

Q01. $\sqrt{3}$. Q02. $\frac{\pi}{2}$. Q11. $\cot\frac{y}{x} = 1 + \log|x|$. Q12. (OR) $x = \frac{1}{\sqrt{3}}$.

Q17. $\frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x}\sqrt{1-x}}$. Q18. (–3, 5, 2).

Q28. $(9a, \pm 6a)$. Q29. $9\pi/4$ Sq.units.

CBSE 2014 Compt. Exams. All India – Set 3

Q01. $gof = \{(1,3), (3,1), (4,3)\}$. Q06. 1.

Q11. We have $\Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix} = \begin{vmatrix} A & x & zy \\ B & y & zx \\ C & z & xy \end{vmatrix}$. Apply $R_1 \to xR_1, R_2 \to yR_2, R_3 \to zR_2$ in Δ_1 to get :

$$\Delta_1 = \frac{1}{xyz}\begin{vmatrix} Ax & x^2 & zyx \\ By & y^2 & zxy \\ Cz & z^2 & xyz \end{vmatrix} = \frac{xyz}{xyz}\begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix} = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix} = \Delta$$. Hence proved.

Q14. $2\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$. (OR) $-\sqrt{1-x^2}\cos^{-1}(x) - x + C$. Q17. $\sin\left(\frac{y}{x}\right) = \log|x| + C$.

Q22. $\frac{y^{\cot x}\mathrm{cosec}^2 x \log y - y(\tan^{-1}x)^{y-1}(1+x^2)^{-1}}{(\tan^{-1}x)^y \log\tan^{-1}x + \cot x\ y^{\cot x - 1}}$. Q23. $75\sqrt{3}$ Sq.units. Q24. $\frac{7}{2}$Sq.units.