❖ Evaluate the following limits [Q00 - Q74] :

Q00. $\lim\limits_{x\to\infty} \dfrac{x^4 + 2x^3 + 3}{2x^4 - x + 2}$

Q01. $\lim\limits_{n\to\infty}\left(\dfrac{1}{n^3} + \dfrac{2^2}{n^3} + \dfrac{3^2}{n^3} + \ldots + \dfrac{1}{n}\right)$

Q02. $\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x+c} - \sqrt{x}\right)$

Q03. $\lim\limits_{x\to-\infty}\left[\sqrt{x^2 + 4x} - \sqrt{x^2 - 4x}\right]$

Q04. $\lim\limits_{x\to\infty} 3x\tan\left(\dfrac{1}{x}\right)$

Q05. $\lim\limits_{x\to\infty}\dfrac{\sin x}{x}$

Q06. $\lim\limits_{x\to\infty}\dfrac{x + \cos x}{x + \sin x}$

Q07. $\lim\limits_{x\to\infty}\left(1 + \dfrac{a}{x}\right)^x$

Q08. $\lim\limits_{x\to\infty} x\left[e^{1/x} - e^{-1/x}\right]$

Q09. $\lim\limits_{x\to\infty}\dfrac{2x^3 + 2x^2 + 1}{3x^3 + x + 2}$

Q10. $\lim\limits_{n\to\infty}\left(\dfrac{1}{n^2} + \dfrac{2}{n^2} + \dfrac{3}{n^2} + \ldots + \dfrac{1}{n}\right)$

Q11. $\lim\limits_{n\to\infty}\left(\dfrac{1}{n^4} + \dfrac{2^3}{n^4} + \dfrac{3^3}{n^4} + \ldots + \dfrac{n^3}{n^4}\right)$

Q12. $\lim\limits_{n\to\infty}\left(\dfrac{1^2 + 1}{n^3} + \dfrac{2^2 + 2}{n^3} + \dfrac{3^2 + 3}{n^3} + \ldots + \dfrac{n^2 + n}{n^3}\right)$

Q13. $\lim\limits_{n\to\infty}\left(\dfrac{1.2 + 2.3 + 3.4 + \ldots + n.(n+1)}{n^3}\right)$

Q14. $\lim\limits_{x\to\infty}\dfrac{5}{2}x\sin\left(\dfrac{1}{x}\right)$

Q15. $\lim\limits_{n\to\infty} 2\pi n\sin\dfrac{\pi}{n}$

Q16. $\lim\limits_{x\to\infty}\dfrac{\cos x}{x}$

Q17. $\lim\limits_{x\to\infty}\dfrac{x + \sin x + 1}{x + \cos x}$

Q18. $\lim\limits_{x\to\infty}\cos\left[\log\left(\dfrac{x-1}{x}\right)\right]$

Q19. $\lim\limits_{n\to\infty}\left(1 + \dfrac{1}{2n}\right)^n$

Q20. $\lim\limits_{x\to\infty}\left(1 - \dfrac{1}{x}\right)^{2x}$

Q21. $\lim\limits_{x\to\infty} x\left[e^{3/x} - e^{-3/x}\right]$

Q22. $\lim\limits_{h\to 0}\dfrac{(x+h)^{1/n} - x^{1/n}}{h}$

Q23. $\lim\limits_{x\to 64}\dfrac{x^{1/6} - 2}{x^{1/3} - 4}$

Q24. $\lim\limits_{x\to 4}\dfrac{x^3 - 2x^2 - 9x + 4}{x^2 - 2x - 8}$

Q25. $\lim\limits_{x\to 0}\dfrac{e^x - 1 - x}{x^2}$

Q26. $\lim\limits_{x\to 0}\dfrac{e^{\sin x} - \sin x - 1}{x^2}$

Q27. $\lim\limits_{x\to 1}\dfrac{\log x}{x - 1}$

Q28. $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2 + 8} - \sqrt{10 - x^2}}{\sqrt{x^2 + 3} - \sqrt{5 - x^2}}$

Q29. $\lim\limits_{x\to 0}\dfrac{(1+x)^{1/n} - 1}{x}$

Q30. $\lim\limits_{x\to 0}\dfrac{(1+x)^{1/3} - (1-x)^{1/3}}{x}$

Q31. $\lim\limits_{x\to -3}\dfrac{x^3 + 27}{x^5 + 243}$

Q32. $\lim\limits_{\theta\to 0}\theta\left(3\operatorname{cosec}2\theta - 2\cot 3\theta\right)$

Q33. $\lim\limits_{x\to 0}\dfrac{e^{ax} - e^{-bx}}{x}$

Q34. $\lim\limits_{x\to 0}\dfrac{a^x - b^x}{x}$

Q35. $\lim\limits_{x\to 0}\dfrac{6^x - 2^x}{x}$

Q36. $\lim\limits_{x\to 3}\dfrac{\log(x-2)}{x-3}$

Q37. $\lim\limits_{h\to 0}\dfrac{\log(x+h) - \log x}{h}$

Q38. $\lim\limits_{n\to\infty}\left(\dfrac{1}{3} + \dfrac{1}{3^2} + \dfrac{1}{3^3} + \ldots + \dfrac{1}{3^n}\right)$

Q39. $\lim\limits_{x\to 0}\dfrac{\sqrt[n]{1-x} - 1}{x}$

Q40. $\lim\limits_{x\to 0}\dfrac{\sin(x^2 + 5x)}{x}$

Q41. $\lim\limits_{x\to 0}\dfrac{\sin x^\circ}{x}$

Q42. $\lim\limits_{a\to x}\dfrac{x^n - a^n}{x - a}$

Q43. $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x^2}\right)^x$

Q44. $\lim\limits_{x\to\infty}\left(\dfrac{x+1}{x-2}\right)^{(2x-1)}$

Q45. $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^{\frac{x+1}{x}}$

Q46. $\lim\limits_{n\to\infty}\left(1+\dfrac{1}{2}+\dfrac{1}{4}+\ldots+\dfrac{1}{2^n}\right)$

Q47. $\lim\limits_{n\to\infty}\dfrac{2^n-1}{2^n+1}$

Q48. $\lim\limits_{n\to\infty}\dfrac{2^{1/n}-1}{2^{1/n}+1}$

Q49. $\lim\limits_{x\to\infty}\left(\dfrac{x^2+1}{x^2-1}\right)^{x^2}$

Q50. $\lim\limits_{x\to\infty}x\left[\log(x+a)-\log x\right]$

Q51. $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^{7x}$

Q52. $\lim\limits_{x\to\infty}\left(\dfrac{x}{2+x}\right)^x$

Q53. $\lim\limits_{x\to\infty}\left(\dfrac{x^2+2x-1}{2x^2-3x-2}\right)^{\frac{2x+1}{x-1}}$

Q54. $\lim\limits_{x\to\infty}\left(\dfrac{2x^2+3}{2x^2+5}\right)^{8x^2+3}$

Q55. $\lim\limits_{x\to2}\dfrac{2^x+2^{3-x}-6}{\sqrt{2^{-x}}-2^{1-x}}$

Q56. $\underset{x\to0}{Lt}\left(\dfrac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}}$

Q57. $\underset{x\to\pi/4}{Lt}\dfrac{\log\tan x}{1-\cot x}$

Q58. $\underset{x\to0}{Lt}\dfrac{e^{\sin 3x}-1}{\log(1+\tan 2x)}$

Q59. $\underset{x\to0}{Lt}\dfrac{\sqrt{1+\sin 3x}-1}{\log(1+\tan 2x)}$

Q60. $\underset{x\to0}{Lt}\dfrac{27^x-9^x-3^x+1}{\sqrt{5}-\sqrt{4+\cos x}}$

Q61. $\underset{x\to0}{Lt}\dfrac{e^{\tan x}-e^x}{\tan x-x}$

Q62. $\underset{x\to0}{Lt}\dfrac{\sin\log(1+x)}{\log(1+\sin x)}$

Q63. $\lim\limits_{x\to1}\dfrac{1-\dfrac{1}{x}}{\sin\left[\pi(x-1)\right]}$

Q64. $\lim\limits_{x\to0}\dfrac{e^{\sin 2x}-e^{\sin x}}{x}$

Q65. $\lim\limits_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$

Q66. $\lim\limits_{x\to e}\dfrac{\log x-1}{x-e}$

Q67. $\lim\limits_{n\to\infty}\dfrac{\sin\left(\dfrac{a}{2^n}\right)}{\sin\left(\dfrac{b}{2^n}\right)}$

Q68. $\lim\limits_{x\to\infty}2^{x-1}\tan\left(\dfrac{a}{2^x}\right)$

Q69. $\lim\limits_{x\to0}\dfrac{\sin x^2\left(1-\cos x^2\right)}{x^6}$

Q70. $\lim\limits_{x\to0}\dfrac{\sin^2 4x^2}{x^4}$

Q71. $\lim\limits_{n\to\infty}\dfrac{1+2+3+\ldots+n}{n^2}$

Q72. $\lim\limits_{x\to\infty}\dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$

Q73. $\lim\limits_{x\to1}\dfrac{\left(x+x^2+x^3+\ldots+x^n\right)-n}{x-1}$

Q74. $\lim\limits_{x\to1/2}\dfrac{8x^3-1}{16x^4-1}$

Q75. Find the value of *k*, if $\lim\limits_{x\to1}\dfrac{x^4-1}{x-1}=\lim\limits_{x\to k}\dfrac{x^3-k^3}{x^2-k^2}$.

Q76. If $\lim\limits_{x\to3}\dfrac{x^n-3^n}{x-3}=108$, find the value of *n*.

Q77. If $\lim\limits_{x\to a}\dfrac{x^9-a^9}{x-a}=9$, find all the possible values of *a*.

Q78. Find all the possible values of *a*: $\lim\limits_{x\to a}\dfrac{x^5-a^5}{x-a}=405$.

Q79. Determine the values of $\lambda$ : $\lim\limits_{x\to\lambda}\dfrac{x^9-\lambda^9}{x-\lambda}=\lim\limits_{x\to5}(4+x)$.

Q80. If $f(x)=\begin{cases}1+x^2, & if\ 0\le x\le1\\ 2-x, & if\ x>1\end{cases}$, then evaluate left hand and right hand limits at *x*=1. Check if $\lim\limits_{x\to1}f(x)$ exists.

Q81. If $f(x)=\begin{cases}\dfrac{x-|x|}{x}, & if\ x\ne0\\ 2, & if\ x=0\end{cases}$, then show that $\lim\limits_{x\to0}f(x)$ does not exist.

Q82. Discuss the existence of limits in the followings :

    (a) $\lim\limits_{x \to 0} \dfrac{1}{x}$                 (b) $\lim\limits_{x \to 0} \dfrac{1}{|x|}$                 (c) $\lim\limits_{x \to 0} \dfrac{x}{|x|}$ .

Q83. If $f(x) = \begin{cases} 4x - 5, \ if \ x \le 2 \\ x - \lambda, \ if \ x > 2 \end{cases}$ , find $\lambda$ if $\lim\limits_{x \to 2} f(x)$ exists.

Q84. Find $k$ so that $\lim\limits_{x \to 2} f(x)$ may exist such that $f(x) = \begin{cases} 2x + 3, \ if \ x \le 2 \\ x + k, \ if \ x > 2 \end{cases}$ .

Q85. Evaluate the following one sided limits :

    (a) $\lim\limits_{x \to 2^-} \dfrac{x-3}{x^2 - 4}$             (b) $\lim\limits_{x \to -8^+} \dfrac{2x}{x+8}$             (c) $\lim\limits_{x \to 0^-} (1 + \mathrm{cosec}\, x)$ .

# ❖ ANSWERS ❖

| | | | | | | |
|---|---|---|---|---|---|---|
| Q00. 1/2 | Q01. 1/3 | Q02. $c/2$ | Q03. –4 | Q04. 3 | Q05. 0 | Q06. 1 |
| Q07. $e^a$ | Q08. 2 | Q09. 2/3 | Q10. 1/2 | Q11. 1/4 | Q12. 1/3 | Q13. 1/3 |
| Q14. $5/2$ | Q15. $2\pi^2$ | Q16. 0 | Q17. 1 | Q18. 1 | Q19. $\sqrt{e}$ | Q20. $e^{-2}$ |
| Q21. 6 | Q22. $\dfrac{1}{n} x^{(1-n)/n}$ | Q23. 1/4 | Q24. 23/6 | Q25. 1/2 | Q26. 1/2 | Q27. 1 |
| Q28. 2/3 | Q29. $1/n$ | Q30. 2/3 | Q31. 1/15 | Q32. 5/6 | Q33. $a+b$ | Q34. $\log_e(a/b)$ |
| Q35. $\log_e 3$ | Q36. 1 | Q37. $1/x$ | Q38. 1/2 | Q39. $-1/n$ | Q40. 5 | Q41. $\pi/180$ |
| Q42. $nx^{n-1}$ | Q43. 1 | Q44. $e^6$ | Q45. 1 | Q46. 2 | Q47. 1 | Q48. 0 |
| Q49. $e^2$ | Q50. $a$ | Q51. $e^7$ | Q52. $e^{-2}$ | Q53. $e^{-1}$ | Q54. $e^{-8}$ | Q55. 8 |
| Q56. $1/e$ | Q57. 1 | Q58. 3/2 | Q59. 3/4 | Q60. $8\sqrt{5}(\log 3)^2$ | | Q61. 1 |
| Q62. 1 | Q63. $1/\pi$ | Q64. 1 | Q65. 1/2 | Q66. $1/e$ | Q67. $a/b$ | Q68. $a/2$ |
| Q69. 1/2 | Q70. 16 | Q71. 1/2 | Q72. 1 | Q73. $n(n+1)/2$ | | Q74. 3/4 |
| Q75. 8/3 | Q76. 4 | Q77. 1,–1 | Q78. 3,–3 | Q79. 1,–1 | Q83. –1 | Q84. 5 |

Q85. a) $\infty$      b) $-\infty$      c) $-\infty$

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