Solutions Of Questions On Finding
DOMAIN & RANGE OF FUNCTIONS

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Q01. Express the function $f:\mathrm{N}\to\mathrm{N}$ defined by $f(x)=2x+3$ in a set of ordered pairs. And then find Range (f).

Sol. The Range $f=f(\mathrm{N})=\{f(x):x\in\mathrm{N}\}$.

$=\{f(x):x=1,2,3,...\}=\{2x+3:x=1,2,3,...\}$

$=\{5,7,9,11,...,(2x+3),...\}$.

Hence $f=\{(1,5),(2,7),(3,9),(4,11),...,(x,2x+3),...\}$.

Q02. Determine the range of the function $y=\dfrac{x}{1+x^2}$.

Sol. Clearly Domain of y : $x\in\mathrm{R}$.

Now $y=\dfrac{x}{1+x^2}$ $\Rightarrow yx^2-x+y=0$.

For x to be a real value, $\mathrm{D}\geq 0$ i.e., $(-1)^2-4(y)(y)\geq 0$ $\Rightarrow(1-4y^2)\geq 0$

$\Rightarrow(1-2y)(1+2y)\geq 0$

$\therefore$ Range : $y\in\left[-\dfrac{1}{2},\dfrac{1}{2}\right]$.

–ve +ve –ve
$-\infty$ $-1/2$ $1/2$ ∞

Q03. Let $f=\left\{\left(x,\dfrac{x^2}{1+x^2}\right):x\in\mathrm{R}\right\}$ be a real valued function defined from R into R. Determine the range of f.

Sol. It is evident that Domain of f : $x\in\mathrm{R}$.

Let $y=\dfrac{x^2}{1+x^2}$ $\Rightarrow x^2y+y-x^2=0$ $\Rightarrow x=\pm\sqrt{\dfrac{y}{1-y}}$.

Since x is real so, $\dfrac{y}{1-y}\geq 0$.

$\therefore$ Range : $y\in[0,1)$.

–ve +ve –ve
$-\infty$ 0 1 ∞

Look out for the Method 2 to be discussed in the class!

Q04. Determine the range of $f=\left\{\left(x,\dfrac{1}{1-x^2}\right):x\in\mathrm{R}-\{-1,1\}\right\}$.

Sol. It is clear that the domain of f : $x\in\mathrm{R}-\{-1,1\}$.

Let $y=\dfrac{1}{1-x^2}$ $\Rightarrow x=\pm\sqrt{\dfrac{y-1}{y}}$.

For x to be real, $\dfrac{y-1}{y}\geq 0$.

+ve –ve +ve
$-\infty$ 0 1 ∞

$\therefore$ Range f : $y\in(-\infty,0)\cup[1,\infty)$.

Q05. Determine the range of $f=\left\{\left(x,\dfrac{x^2-1}{x-1}\right):x\in\mathrm{R}-1\right\}$.

Sol. Clearly, domain of f : $x\in\mathrm{R}-1$.

Let $y=\dfrac{x^2-1}{x-1}=x+1$.

Since $x+1$ is defined for all $x \in \mathrm{R}$, but $x \neq 1$, $\therefore y \neq 2$.

Hence, range : $y \in \mathrm{R} - 2$.

Q06. Determine the domain and range of $y = \dfrac{1}{2 - \sin 3x}$.

Sol. Here y is defined for all $x \in \mathrm{R}$. So, domain : $x \in \mathrm{R}$.

Now as for all $x \in \mathrm{R}$, $-1 \le \sin 3x \le 1 \quad \Rightarrow 1 \ge -\sin 3x \ge -1$

$2+1 \ge 2 - \sin 3x \ge 2-1 \quad \Rightarrow 3 \ge 2 - \sin 3x \ge 1 \quad \Rightarrow \dfrac{1}{3} \le \dfrac{1}{2-\sin 3x} \le 1$

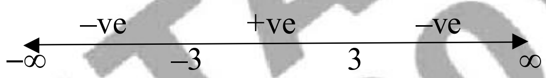
i.e., $\dfrac{1}{3} \le y \le 1$.

Hence, range : $y \in \left[\dfrac{1}{3}, 1\right]$.

Q07. Find the domain and range of $y = \sqrt{9 - x^2}$.

Sol. Clearly y is defined iff $9 - x^2 \ge 0$

$\Rightarrow (3-x)(3+x) \ge 0$



$\therefore$ Domain : $x \in [-3, 3]$.

Now for all $x \in \mathrm{R}$, $0 \le x^2 < \infty \quad \Rightarrow 0 \ge -x^2 > -\infty$

i.e., $9 + 0 \ge 9 - x^2 > 9 - \infty \quad \Rightarrow -\infty < 9 - x^2 \le 9$

$\Rightarrow 0 \le \sqrt{9 - x^2} \le 3 \quad \Rightarrow 0 \le y \le 3 \quad [\because \sqrt{x}$ is defined if $x \ge 0$

Hence, range : $y \in [0, 3]$.

Q08. Determine the range of $y = \dfrac{1}{3 - x^2}$.

Sol. The domain of y : $x \in \mathrm{R} - \{-\sqrt{3}, \sqrt{3}\}$

Now for all $x \in \mathrm{R}$, $0 \le x^2 < \infty \quad \Rightarrow 0 \ge -x^2 > -\infty \quad \Rightarrow 3 + 0 \ge 3 - x^2 > 3 - \infty$

$\Rightarrow 3 \ge 3 - x^2 > -\infty \quad$ i.e., $3 - x^2 > -\infty$ and $3 - x^2 \le 3 \quad \Rightarrow \dfrac{1}{3-x^2} < 0$ and $\dfrac{1}{3-x^2} \ge \dfrac{1}{3}$

$\therefore$ Range : $y \in (-\infty, 0) \cup \left[\dfrac{1}{3}, \infty\right)$.

Hii.

Here is a short message I have to convey. I've devoted myself for the service of Mathematics.. to help the students in need in all possible ways. It will be a thing of pleasure for me if my work/collection serves any purpose in your life.

I would like to put forward a request. If you find anything in this work that is unclear to you or is senseless, please bring it in my notice through SMS/Call on **+91-9650 350 480**. Any suggestion(s) regarding improvement of these works on Maths will be highly acknowledged. Mail your feedback(s) at **theopgupta@gmail.com**

Wish You All The Very Best!

Lots of love and blessings!

– OP Gupta, *MathsGuru* [**INDIRA Award Winner**]

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