Question 1:
Express the given complex number in the form $a + ib$: $(5i) \left( -\frac{3}{5}i \right)$
Answer
\[ (5i) \left( -\frac{3}{5}i \right) = -5 \times \frac{3}{5}i \times i \]
\[ = -3i^2 \]
\[ = -3(-1) \quad \left[ i^2 = -1 \right] \]
\[ = 3 \]

Question 2:
Express the given complex number in the form $a + ib$: $i^9 + i^{19}$
Answer
\[ i^9 + i^{19} = i^{4\times2+1} + i^{4\times4+3} \]
\[ = (i^4)^2 \cdot i + (i^4)^4 \cdot i^3 \]
\[ = 1 \cdot i + 1 \cdot (-i) \quad \left[ i^4 = 1, \ i^3 = -i \right] \]
\[ = i + (-i) \]
\[ = 0 \]

Question 3:
Express the given complex number in the form $a + ib$: $i^{-39}$
Answer
\[ i^{-39} = i^{4\times9-3} = (i^4)^9 \cdot i^{-3} \]
\[ = 1^{-9} \cdot i^{-3} \quad \left[ i^4 = 1 \right] \]
\[ = \frac{1}{i^3} = \frac{1}{-i} \quad \left[ i^3 = -i \right] \]
\[ = -\frac{i}{i} \]
\[ = -1 = i \quad \left[ i^2 = -1 \right] \]
Question 4:
Express the given complex number in the form \(a + ib\): \(3(7 + i7) + i(7 + i7)\)
Answer
\[
3(7 + i7) + i(7 + i7) = 21 + 21i + 7i + 7i^2
= 21 + 28i + 7 \times (-1) \quad \therefore i^2 = -1
= 14 + 28i
\]

Question 5:
Express the given complex number in the form \(a + ib\): \((1 - i) - (-1 + i6)\)
Answer
\[
(1 - i) - (-1 + i6) = 1 - i + 1 - 6i
= 2 - 7i
\]

Question 6:
Express the given complex number in the form \(a + ib\): \(\left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right)\)
Answer
\[
\left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right)
= \frac{1}{5} + \frac{2}{5}i - 4 - \frac{5}{2}i
= \left(\frac{1}{5} - 4\right) + i\left(\frac{2}{5} - \frac{5}{2}\right)
= \frac{-19}{5} + i\left(\frac{-21}{10}\right)
= \frac{-19}{5} - \frac{21}{10}i
\]

Question 7:
Express the given complex number in the form \(a + ib\): \(\left(\frac{1}{3} + i\frac{7}{3}\right) + \left(4 + i\frac{1}{3}\right) - \left(-\frac{4}{3} + i\right)\)
Answer
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\[
\left( \frac{1}{3} + i \frac{7}{3} \right) + \left( 4 + i \frac{1}{3} \right) - \left( -\frac{4}{3} + i \right) \\
= \frac{1}{3} + \frac{7}{3} + 4 + \frac{1}{3} + \frac{4}{3} - i \\
= \left( \frac{1}{3} + 4 + \frac{4}{3} \right) + i \left( \frac{7}{3} + \frac{1}{3} - 1 \right) \\
= \frac{17}{3} + i \frac{5}{3}
\]

**Question 8:**
Express the given complex number in the form \( a + ib \): \( (1 - i)^4 \)

**Answer**

\[
(1 - i)^4 = \left( (1 - i)^3 \right)^2 \\
= \left[ 1^2 + i^2 - 2i \right]^2 \\
= \left[ 1 - 1 - 2i \right]^2 \\
= (-2i)^2 \\
= (-2i)(-2i) \\
= 4i^2 = -4 \quad \left[ i^2 = -1 \right]
\]

**Question 9:**
Express the given complex number in the form \( a + ib \): \( \left( \frac{1}{3} + 3i \right)^5 \)

**Answer**
Question 10:

Express the given complex number in the form \( a + ib \): \( \left(-2 - \frac{1}{3}i\right)^3 \)

Answer

\[
\left(-2 - \frac{1}{3}i\right)^3 = \left(-1\right)^3 \left(2 + \frac{1}{3}i\right)^3 \\
= -\left[ 2^3 + \left(\frac{1}{3}i\right)^3 + 3 \cdot \left(2\right) \left(\frac{i}{3}\right) \left(2 + \frac{1}{3}i\right) \right] \\
= -\left[ 8 + \frac{i}{27} + 2i \left(2 + \frac{i}{3}\right) \right] \\
= -\left[ 8 - \frac{i}{27} + 4i + \frac{2i^2}{3} \right] \\
= -\left[ 8 - \frac{i}{27} + 4i - \frac{2}{3} \right] \\
= -\left[ \frac{22}{3} + \frac{107i}{27} \right] \\
= -\frac{22}{3} - \frac{107i}{27}
\]

Question 11:

Find the multiplicative inverse of the complex number \( 4 - 3i \)
Let $z = 4 - 3i$

Then, $\overline{z} = 4 + 3i$ and $|z|^2 = 4^2 + (-3)^2 = 16 + 9 = 25$

Therefore, the multiplicative inverse of $4 - 3i$ is given by

$$z^{-1} = \frac{\overline{z}}{|z|^2} = \frac{4 + 3i}{25} = \frac{4}{25} + \frac{3}{25}i$$

**Question 12:**

Find the multiplicative inverse of the complex number $\sqrt{5} + 3i$

Answer

Let $z = \sqrt{5} + 3i$

Then, $\overline{z} = \sqrt{5} - 3i$ and $|z|^2 = (\sqrt{5})^2 + 3^2 = 5 + 9 = 14$

Therefore, the multiplicative inverse of $\sqrt{5} + 3i$ is given by

$$z^{-1} = \frac{\overline{z}}{|z|^2} = \frac{\sqrt{5} - 3i}{14} = \frac{\sqrt{5}}{14} - \frac{3i}{14}$$

**Question 13:**

Find the multiplicative inverse of the complex number $-i$

Answer

Let $z = -i$

Then, $\overline{z} = i$ and $|z|^2 = 1^2 = 1$

Therefore, the multiplicative inverse of $-i$ is given by

$$z^{-1} = \frac{\overline{z}}{|z|^2} = \frac{i}{1} = i$$

**Question 14:**
Express the following expression in the form of $a + ib$.

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3} + \sqrt{2}i) - (\sqrt{3} - i\sqrt{2})}$$

Answer

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3} + \sqrt{2}i) - (\sqrt{3} - i\sqrt{2})}$$

$$= \frac{(3)^2 - (i\sqrt{5})^2}{\sqrt{3} + \sqrt{2}i - \sqrt{3} + \sqrt{2}i} \quad \left[(a+b)(a-b) = a^2 - b^2\right]$$

$$= \frac{9 - 5i^2}{2\sqrt{2}i}$$

$$= \frac{9 - 5(-1)}{2\sqrt{2}i}$$

$$= \frac{9 + 5}{2\sqrt{2}i} \times i$$

$$= \frac{14i}{2\sqrt{2}i^2}$$

$$= \frac{14i}{2\sqrt{2}(-1)}$$

$$= \frac{-7i}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{-7\sqrt{2}i}{2}$$
Exercise 5.2

Question 1:

Find the modulus and the argument of the complex number \( z = -1 - i\sqrt{3} \)

Answer

\[ z = -1 - i\sqrt{3} \]

Let \( r \cos \theta = -1 \) and \( r \sin \theta = -\sqrt{3} \)

On squaring and adding, we obtain

\[ (r \cos \theta)^2 + (r \sin \theta)^2 = (-1)^2 + (-\sqrt{3})^2 \]

\[ r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 3 \]

\[ r^2 = 4 \quad \text{[} \cos^2 \theta + \sin^2 \theta = 1 \text{]} \]

\[ r = \sqrt{4} = 2 \quad \text{[} \text{Conventionally, } r > 0 \text{]} \]

\[ \therefore \text{Modulus} = 2 \]

\[ \therefore 2 \cos \theta = -1 \text{ and } 2 \sin \theta = -\sqrt{3} \]

\[ \Rightarrow \cos \theta = -\frac{1}{2} \text{ and } \sin \theta = -\frac{\sqrt{3}}{2} \]

Since both the values of \( \sin \theta \) and \( \cos \theta \) are negative and \( \sin \theta \) and \( \cos \theta \) are negative in III quadrant,

\[ \text{Argument} = -\left( \pi - \frac{\pi}{3} \right) = -\frac{2\pi}{3} \]

Thus, the modulus and argument of the complex number \( -1 - \sqrt{3} \) i are 2 and \( -\frac{2\pi}{3} \) respectively.

Question 2:
Find the modulus and the argument of the complex number \( z = -\sqrt{3} + i \)

Answer

\( z = -\sqrt{3} + i \)

Let \( r \cos \theta = -\sqrt{3} \) and \( r \sin \theta = 1 \)

On squaring and adding, we obtain

\[
r^2 \cos^2 \theta + r^2 \sin^2 \theta = \left( -\sqrt{3} \right)^2 + 1^2
\]

\[
\Rightarrow r^2 = 3 + 1 = 4
\]

\[
\Rightarrow r = \sqrt{4} = 2
\]

Conventionally, \( r > 0 \)

\( \therefore \) Modulus = 2

\( \therefore 2 \cos \theta = -\sqrt{3} \) and \( 2 \sin \theta = 1 \)

\[
\Rightarrow \cos \theta = -\frac{\sqrt{3}}{2} \quad \text{and} \quad \sin \theta = \frac{1}{2}
\]

\( \therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \) \[ \text{As } \theta \text{ lies in the II quadrant} \]

Thus, the modulus and argument of the complex number \( -\sqrt{3} + i \) are 2 and \( \frac{5\pi}{6} \) respectively.

Question 3:
Convert the given complex number in polar form: \( 1 - i \)

Answer

\( 1 - i \)

Let \( r \cos \theta = 1 \) and \( r \sin \theta = -1 \)

On squaring and adding, we obtain
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\[ r^2 \cos^2 \theta + r^2 \sin^2 \theta = 1^2 + (-1)^2 \]
\[ \Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1 \]
\[ \Rightarrow r^2 = 2 \]
\[ \Rightarrow r = \sqrt{2} \quad \text{[Conventionally, } r > 0 \text{]} \]
\[ \therefore \sqrt{2} \cos \theta = 1 \text{ and } \sqrt{2} \sin \theta = -1 \]
\[ \Rightarrow \cos \theta = \frac{1}{\sqrt{2}} \text{ and } \sin \theta = -\frac{1}{\sqrt{2}} \]
\[ \therefore \theta = -\frac{\pi}{4} \quad \text{[As } \theta \text{ lies in the IV quadrant]} \]
\[ \therefore 1 - i = r \cos \theta + ir \sin \theta = \sqrt{2} \cos \left(-\frac{\pi}{4}\right) + i\sqrt{2} \sin \left(-\frac{\pi}{4}\right) = \sqrt{2} \left[ \cos \left(-\frac{\pi}{4}\right) + i\sin \left(-\frac{\pi}{4}\right) \right] \]

This is the required polar form.

Question 4:
Convert the given complex number in polar form: \(-1 + i\)

Answer
\(-1 + i\)

Let \(r \cos \theta = -1\) and \(r \sin \theta = 1\)

On squaring and adding, we obtain
\[ r^2 \cos^2 \theta + r^2 \sin^2 \theta = (-1)^2 + 1^2 \]
\[ \Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1 \]
\[ \Rightarrow r^2 = 2 \]
\[ \Rightarrow r = \sqrt{2} \quad \text{[Conventionally, } r > 0 \text{]} \]
\[ \therefore \sqrt{2} \cos \theta = -1 \text{ and } \sqrt{2} \sin \theta = 1 \]
\[ \Rightarrow \cos \theta = -\frac{1}{\sqrt{2}} \text{ and } \sin \theta = \frac{1}{\sqrt{2}} \]
\[ \therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \quad \text{[As } \theta \text{ lies in the II quadrant]} \]

It can be written,
This is the required polar form.

Question 5:
Convert the given complex number in polar form: \(-1 - i\)
Answer
\(-1 - i\)
Let \(r \cos \theta = -1\) and \(r \sin \theta = -1\)
On squaring and adding, we obtain
\[r^2 \cos^2 \theta + r^2 \sin^2 \theta = (-1)^2 + (-1)^2\]
\[\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1\]
\[\Rightarrow r^2 = 2\]
\[\Rightarrow r = \sqrt{2}\] [Conventionally, \(r > 0\)]
\[\therefore \sqrt{2} \cos \theta = -1\] and \(\sqrt{2} \sin \theta = -1\)
\[\Rightarrow \cos \theta = -\frac{1}{\sqrt{2}}\] and \(\sin \theta = -\frac{1}{\sqrt{2}}\)
\[\therefore \theta = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4}\] [As \(\theta\) lies in the III quadrant]
\[\therefore -1 - i = r \cos \theta + ir \sin \theta = \sqrt{2} \cos \frac{-3\pi}{4} + i\sqrt{2} \sin \frac{-3\pi}{4} = \sqrt{2} \left(\cos \frac{-3\pi}{4} + i\sin \frac{-3\pi}{4}\right)\]
This is the required polar form.

Question 6:
Convert the given complex number in polar form: \(-3\)
Answer
\(-3\)
Let \(r \cos \theta = -3\) and \(r \sin \theta = 0\)
On squaring and adding, we obtain
Question 7:

Convert the given complex number in polar form: \( \sqrt{3} + i \)

Answer

\[ \sqrt{3} + i \]

Let \( r \cos \theta = \sqrt{3} \) and \( r \sin \theta = 1 \)

On squaring and adding, we obtain

\[ r^2 \cos^2 \theta + r^2 \sin^2 \theta = \left( \sqrt{3} \right)^2 + 1^2 \]

\[ \Rightarrow r^2 \left( \cos^2 \theta + \sin^2 \theta \right) = 3 + 1 \]

\[ \Rightarrow r^2 = 4 \]

\[ \Rightarrow r = \sqrt{4} = 2 \quad \text{[Conventionally, } r > 0 \]}

\[ \therefore 2 \cos \theta = \sqrt{3} \text{ and } 2 \sin \theta = 1 \]

\[ \Rightarrow \cos \theta = \frac{\sqrt{3}}{2} \text{ and } \sin \theta = \frac{1}{2} \]

\[ \therefore \theta = \frac{\pi}{6} \quad \text{[As } \theta \text{ lies in the I quadrant]} \]

\[ \therefore \sqrt{3} + i = r \cos \theta + i r \sin \theta = 2 \cos \frac{\pi}{6} + i 2 \sin \frac{\pi}{6} = 2 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \]
Question 8:
Convert the given complex number in polar form: \( i \)

Answer
\( i \)

Let \( r \cos \theta = 0 \) and \( r \sin \theta = 1 \)

On squaring and adding, we obtain
\[
r^2 \cos^2 \theta + r^2 \sin^2 \theta = 0^2 + 1^2
\]
\[
\Rightarrow r^2 \left( \cos^2 \theta + \sin^2 \theta \right) = 1
\]
\[
\Rightarrow r^2 = 1
\]
\[
\Rightarrow r = \sqrt{1} = 1 \quad \text{[Conventionally, } r > 0 \text{]}
\]
\[
\therefore \cos \theta = 0 \text{ and } \sin \theta = 1
\]
\[
\therefore \theta = \frac{\pi}{2}
\]

\[
\therefore i = r \cos \theta + ir \sin \theta = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}
\]

This is the required polar form.
Question 1:
Solve the equation \( x^2 + 3 = 0 \)

Answer

The given quadratic equation is \( x^2 + 3 = 0 \)

On comparing the given equation with \( ax^2 + bx + c = 0 \), we obtain

\( a = 1, \ b = 0, \) and \( c = 3 \)

Therefore, the discriminant of the given equation is

\[ D = b^2 - 4ac = 0^2 - 4 \times 1 \times 3 = -12 \]

Therefore, the required solutions are

\[ \frac{-b \pm \sqrt{D}}{2a} = \frac{\pm \sqrt{-12}}{2 \times 1} = \frac{\pm \sqrt{12} i}{2} \]

\[ = \frac{\pm 2 \sqrt{3} i}{2} = \pm \sqrt{3} i \]

Question 2:
Solve the equation \( 2x^2 + x + 1 = 0 \)

Answer

The given quadratic equation is \( 2x^2 + x + 1 = 0 \)

On comparing the given equation with \( ax^2 + bx + c = 0 \), we obtain

\( a = 2, \ b = 1, \) and \( c = 1 \)

Therefore, the discriminant of the given equation is

\[ D = b^2 - 4ac = 1^2 - 4 \times 2 \times 1 = 1 - 8 = -7 \]

Therefore, the required solutions are
Question 3:
Solve the equation \( x^2 + 3x + 9 = 0 \)
Answer
The given quadratic equation is \( x^2 + 3x + 9 = 0 \)
On comparing the given equation with \( ax^2 + bx + c = 0 \), we obtain
\( a = 1, b = 3, \) and \( c = 9 \)
Therefore, the discriminant of the given equation is
\[ D = b^2 - 4ac = 3^2 - 4 \times 1 \times 9 = 9 - 36 = -27 \]
Therefore, the required solutions are
\[ \frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{-27}}{2} = \frac{-3 \pm 3\sqrt{-1}}{2} = \frac{-3 \pm 3i}{2} \quad \left[ \sqrt{-1} = i \right] \]

Question 4:
Solve the equation \(-x^2 + x - 2 = 0\)
Answer
The given quadratic equation is \(-x^2 + x - 2 = 0\)
On comparing the given equation with \( ax^2 + bx + c = 0 \), we obtain
\( a = -1, b = 1, \) and \( c = -2 \)
Therefore, the discriminant of the given equation is
\[ D = b^2 - 4ac = 1^2 - 4 \times (-1) \times (-2) = 1 - 8 = -7 \]
Therefore, the required solutions are
\[ \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2\times(-1)} = \frac{-1 \pm \sqrt{7}i}{2} \quad \left[ \sqrt{-1} = i \right] \]

Question 5:
Solve the equation \( x^2 + 3x + 5 = 0 \)
Answer
The given quadratic equation is \( x^2 + 3x + 5 = 0 \)
On comparing the given equation with \( ax^2 + bx + c = 0 \), we obtain
a = 1, b = 3, and c = 5
Therefore, the discriminant of the given equation is
\[ D = b^2 - 4ac = 3^2 - 4 \times 1 \times 5 = 9 - 20 = -11 \]
Therefore, the required solutions are
\[ \frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{-11}}{2} = \frac{-3 \pm i\sqrt{11}}{2} \quad \left[ \sqrt{-1} = i \right] \]

Question 6:
Solve the equation \( x^2 - x + 2 = 0 \)
Answer
The given quadratic equation is \( x^2 - x + 2 = 0 \)
On comparing the given equation with \( ax^2 + bx + c = 0 \), we obtain
\( a = 1, \ b = -1, \) and \( c = 2 \)
Therefore, the discriminant of the given equation is
\[ D = b^2 - 4ac = (-1)^2 - 4 \times 1 \times 2 = 1 - 8 = -7 \]
Therefore, the required solutions are
\[ \frac{-b \pm \sqrt{D}}{2a} = \frac{1 \pm \sqrt{7}i}{2} \quad \left[ \sqrt{-1} = i \right] \]

Question 7:
Solve the equation \( \sqrt{2}x^2 + x + \sqrt{2} = 0 \)
Answer
The given quadratic equation is \( \sqrt{2}x^2 + x + \sqrt{2} = 0 \)
On comparing the given equation with \( ax^2 + bx + c = 0 \), we obtain
\( a = \sqrt{2}, \ b = 1, \) and \( c = \sqrt{2} \)
Therefore, the discriminant of the given equation is
\[ D = b^2 - 4ac = 1^2 - 4 \times \sqrt{2} \times \sqrt{2} = 1 - 8 = -7 \]
Therefore, the required solutions are
\[ \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{7}i}{2 \sqrt{2}} = \frac{-1 \pm \sqrt{7}i}{2\sqrt{2}} \quad \left[ \sqrt{-1} = i \right] \]
Question 8:

Solve the equation $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

Answer

The given quadratic equation is $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = \sqrt{3}$, $b = -\sqrt{2}$, and $c = 3\sqrt{3}$

Therefore, the discriminant of the given equation is

$$D = b^2 - 4ac = \left(-\sqrt{2}\right)^2 - 4\left(\sqrt{3}\right)\left(3\sqrt{3}\right) = 2 - 36 = -34$$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-\sqrt{2}) \pm \sqrt{-34}}{2 \times \sqrt{3}} = \frac{\sqrt{2} \pm \sqrt{34}i}{2\sqrt{3}} \quad \left[\sqrt{-1} = i\right]$$

Question 9:

Solve the equation $x^2 + x + \frac{1}{\sqrt{2}} = 0$

Answer

The given quadratic equation is $x^2 + x + \frac{1}{\sqrt{2}} = 0$

This equation can also be written as $\sqrt{2}x^2 + \sqrt{2}x + 1 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = \sqrt{2}$, $b = \sqrt{2}$, and $c = 1$

$\therefore$ Discriminant $(D) = b^2 - 4ac = \left(\sqrt{2}\right)^2 - 4 \times \left(\sqrt{2}\right) \times 1 = 2 - 4\sqrt{2}$

Therefore, the required solutions are
Question 10:

Solve the equation \( x^2 + \frac{x}{\sqrt{2}} + 1 = 0 \)

Answer

The given quadratic equation is \( x^2 + \frac{x}{\sqrt{2}} + 1 = 0 \)

This equation can also be written as \( \sqrt{2}x^2 + x + \sqrt{2} = 0 \)

On comparing this equation with \( ax^2 + bx + c = 0 \), we obtain

\[ a = \sqrt{2}, \quad b = 1, \quad \text{and} \quad c = \sqrt{2} \]

\[ \therefore \text{Discriminant} \ (D) = b^2 - 4ac = 1^2 - 4 \times \sqrt{2} \times \sqrt{2} = 1 - 8 = -7 \]

Therefore, the required solutions are

\[ \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2 \sqrt{2}} = \frac{-1 \pm \sqrt{7}i}{2 \sqrt{2}} \quad [\sqrt{-1} = i] \]
NCERT Miscellaneous Solutions

Question 1:

Evaluate: \[ i^{18} + \left( \frac{1}{i} \right)^{25} \]^3

Answer
Question 2:
For any two complex numbers \( z_1 \) and \( z_2 \), prove that
\[ \text{Re} (z_1 z_2) = \text{Re} z_1 \text{Re} z_2 - \text{Im} z_1 \text{Im} z_2 \]
Answer
Let \( z_1 = x_1 + iy_1 \) and \( z_2 = x_2 + iy_2 \)

\[ z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2) = x_1(x_2 + iy_2) + iy_1(x_2 + iy_2) \]
\[ = x_1 x_2 + ix_1 y_2 + iy_1 x_2 + i^2 y_1 y_2 \]
\[ = x_1 x_2 + iy_1 y_2 + i(y_1 x_2 - x_1 y_2) \]
\[ [i^2 = -1] \]

\[ \Rightarrow \text{Re}(z_1 z_2) = x_1 x_2 - y_1 y_2 \]

\[ \Rightarrow \text{Re}(z_1 z_2) = \text{Re} z_1 \text{Re} z_2 - \text{Im} z_1 \text{Im} z_2 \]

Hence, proved.

**Question 3:**

Reduce \( \left( \frac{1}{1 - 4i} - \frac{2}{1 + i} \right) \left( \frac{3 - 4i}{5 + i} \right) \) to the standard form.

**Answer**

\[ \left( \frac{1}{1 - 4i} - \frac{2}{1 + i} \right) \left( \frac{3 - 4i}{5 + i} \right) = \left( \frac{1 + i - 2(1 - 4i)}{(1 - 4i)(1 + i)} \right) \left( \frac{3 - 4i}{5 + i} \right) \]
\[ = \left( \frac{1 + i - 2 + 8i}{1 + i - 4i - 4i^2} \right) \left( \frac{3 - 4i}{5 + i} \right) \]
\[ = \left( \frac{-3 + 4i + 27i - 36i^2}{25 + 5i - 15i - 3i^2} \right) \left( \frac{33 + 31i}{28 - 10i} \right) \]
\[ = \frac{33 + 31i}{2(14 - 5i)} \times \frac{14 + 5i}{14 + 5i} \]
\[ \text{[On multiplying numerator and denominator by (14 + 5i)]} \]
\[ = \frac{462 + 165i + 434i + 155i^2}{2(14)^2 - (5i)^2} \]
\[ = \frac{307 + 599i}{2(196 - 25i^2)} \]
\[ = \frac{307 + 599i}{442} \]

This is the required standard form.
Question 4:

If \( x - iy = \frac{a - ib}{c - id} \) prove that \( (x^2 + y^2) = \frac{a^2 + b^2}{c^2 + d^2} \)

Answer

\[
\begin{align*}
    x - iy &= \frac{a - ib}{c - id} \\
    &= \frac{a - ib}{c - id} \times \frac{c + id}{c + id} \quad \text{[On multiplying numerator and denominator by \((c + id)\)]} \\
    &= \frac{(ac + bd) + i(ad - bc)}{c^2 + d^2} \\
    \therefore (x - iy)^2 &= \frac{(ac + bd) + i(ad - bc)}{c^2 + d^2} \\
    \Rightarrow x^2 - y^2 - 2ixy &= \frac{(ac + bd) + i(ad - bc)}{c^2 + d^2} \\
    \text{On comparing real and imaginary parts, we obtain} \\
    x^2 - y^2 &= \frac{ac + bd}{c^2 + d^2}, \quad -2xy = \frac{ad - bc}{c^2 + d^2} \quad (1)
\end{align*}
\]
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\[(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2\]
\[= \left(\frac{ac + bd}{c^2 + d^2}\right)^2 + \left(\frac{ad - bc}{c^2 + d^2}\right)^2 \quad [\text{Using (1)}]\]
\[= \frac{a^2c^2 + b^2d^2 + 2acbd + a^2d^2 + b^2c^2 - 2adbc}{(c^2 + d^2)^2}\]
\[= \frac{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}{(c^2 + d^2)^2}\]
\[= \frac{a^2(c^2 + d^2) + b^2(c^2 + d^2)}{(c^2 + d^2)^2}\]
\[= \frac{(c^2 + d^2) (a^2 + b^2)}{(c^2 + d^2)^2}\]
\[= \frac{a^2 + b^2}{c^2 + d^2}\]
Hence, proved.

Question 5:
Convert the following in the polar form:

(i) \(\frac{1+7i}{(2-i)^2}\), (ii) \(\frac{1+3i}{1-2i}\)

Answer

(ii) Here, \(z = \frac{1+7i}{(2-i)^2}\)
\[= \frac{1+7i}{(2-i)^2} = \frac{1+7i}{4 + i^2 - 4i} = \frac{1+7i}{4-1-4i}\]
\[= \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i} = \frac{3+4i + 21i + 28i^2}{3^2 + 4^2}\]
\[= \frac{3+4i + 21i - 28}{25} = \frac{-25 + 25i}{25}\]
\[= -1 + i\]
Let \(r \cos \theta = -1\) and \(r \sin \theta = 1\)
On squaring and adding, we obtain
\[ r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1 \]
\[ \Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 2 \]
\[ \Rightarrow r^2 = 2 \quad \text{[cos}^2 \theta + \sin^2 \theta = 1] \]
\[ \Rightarrow r = \sqrt{2} \quad \text{[Conventionally, } r > 0] \]
\[ \therefore \sqrt{2} \cos \theta = -1 \text{ and } \sqrt{2} \sin \theta = 1 \]
\[ \Rightarrow \cos \theta = \frac{-1}{\sqrt{2}} \text{ and } \sin \theta = \frac{1}{\sqrt{2}} \]
\[ \therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \quad \text{[As } \theta \text{ lies in II quadrant]} \]
\[ \therefore z = r \cos \theta + i r \sin \theta \]
\[ = \sqrt{2} \cos \frac{3\pi}{4} + i \sqrt{2} \sin \frac{3\pi}{4} = \sqrt{2} \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) \]

This is the required polar form.

(ii) Here, \( z = \frac{1 + 3i}{1 - 2i} \)
\[ = \frac{1 + 3i}{1 - 2i} \times \frac{1 + 2i}{1 + 2i} \]
\[ = \frac{1 + 2i + 3i - 6}{1 + 4} \]
\[ = \frac{-5 + 5i}{5} = -1 + i \]

Let \( r \cos \theta = -1 \) and \( r \sin \theta = 1 \)

On squaring and adding, we obtain
\[ r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1 \]
\[ \Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 2 \]
\[ \Rightarrow r^2 = 2 \quad \text{[cos}^2 \theta + \sin^2 \theta = 1] \]
∴ \[ z = r \cos \theta + i \ r \sin \theta \]

\[ = \sqrt{2} \cos \frac{3\pi}{4} + i \sqrt{2} \sin \frac{3\pi}{4} \]

This is the required polar form.

Question 6:

Solve the equation \[ 3x^2 - 4x + \frac{20}{3} = 0 \]

Answer

The given quadratic equation is \[ 3x^2 - 4x + \frac{20}{3} = 0 \]

This equation can also be written as \[ 9x^2 - 12x + 20 = 0 \]

On comparing this equation with \[ ax^2 + bx + c = 0 \], we obtain

\[ a = 9, \ b = -12, \text{ and } c = 20 \]

Therefore, the discriminant of the given equation is

\[ D = b^2 - 4ac = (-12)^2 - 4 \times 9 \times 20 = 144 - 720 = -576 \]

Therefore, the required solutions are

\[ \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-12) \pm \sqrt{-576}}{2 \times 9} = \frac{12 \pm 576 i}{18} \]

\[ = \frac{12 \pm 24i}{18} = \frac{6(2 \pm 4i)}{18} = \frac{2 \pm 4i}{3} \]

Question 7:

Solve the equation \[ x^2 - 2x + \frac{3}{2} = 0 \]

Answer
The given quadratic equation is \( x^2 - 2x + \frac{3}{2} = 0 \)

This equation can also be written as \( 2x^2 - 4x + 3 = 0 \)

On comparing this equation with \( ax^2 + bx + c = 0 \), we obtain
\( a = 2, \ b = -4, \) and \( c = 3 \)

Therefore, the discriminant of the given equation is
\[
D = b^2 - 4ac = (-4)^2 - 4 \times 2 \times 3 = 16 - 24 = -8
\]

Therefore, the required solutions are
\[
\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-4) \pm \sqrt{-8}}{2 \times 2} = \frac{4 \pm 2\sqrt{2}i}{4} \quad [\sqrt{-1} = i]
\]
\[
= \frac{2 \pm \sqrt{2}i}{2} = 1 \pm \frac{\sqrt{2}}{2}i
\]

**Question 8:**

Solve the equation \( 27x^2 - 10x + 1 = 0 \)

**Answer**

The given quadratic equation is \( 27x^2 - 10x + 1 = 0 \)

On comparing the given equation with \( ax^2 + bx + c = 0 \), we obtain
\( a = 27, \ b = -10, \) and \( c = 1 \)

Therefore, the discriminant of the given equation is
\[
D = b^2 - 4ac = (-10)^2 - 4 \times 27 \times 1 = 100 - 108 = -8
\]

Therefore, the required solutions are
\[
\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-10) \pm \sqrt{-8}}{2 \times 27} = \frac{10 \pm 2\sqrt{2}i}{54} \quad [\sqrt{-1} = i]
\]
\[
= \frac{5 \pm \sqrt{2}i}{27} = \frac{5}{27} \pm \frac{\sqrt{2}}{27}i
\]

**Question 9:**

Solve the equation \( 21x^2 - 28x + 10 = 0 \)

**Answer**

The given quadratic equation is \( 21x^2 - 28x + 10 = 0 \)

On comparing the given equation with \( ax^2 + bx + c = 0 \), we obtain
a = 21, b = -28, and c = 10

Therefore, the discriminant of the given equation is

\[ D = b^2 - 4ac = (-28)^2 - 4 \times 21 \times 10 = 784 - 840 = -56 \]

Therefore, the required solutions are

\[ \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-28) \pm \sqrt{-56}}{2 \times 21} = \frac{28 \pm \sqrt{56} i}{42} = \frac{28 \pm 2\sqrt{14} i}{42} = \frac{2 \pm \sqrt{14} i}{3} \]

**Question 10:**

If \( z_1 = 2 - i, z_2 = 1 + i \), find \( \frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} \)

**Answer**

\[
\begin{align*}
\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} &= \frac{(2 - i) + (1 + i) + 1}{(2 - i) - (1 + i) + 1} \\
&= \frac{4}{(2 - 2i)} = \frac{4}{2(1 - i)} \\
&= \frac{2 \times 1 + i}{1 - i \times 1 + i} = \frac{2(1 + i)}{1 - i^2} \\
&= \frac{2(1 + i)}{1 + 1} = \frac{2(1 + i)}{2} \\
&= |1 + i| = \sqrt{2^2 + 1^2} = \sqrt{2}
\end{align*}
\]

Thus, the value of \( \frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} \) is \( \sqrt{2} \).

**Question 11:**
If \(a + ib = \frac{(x+i)^2}{2x^2+1}\), prove that \(a^2 + b^2 = \frac{(x^2+1)^2}{(2x+1)^2}\)

Answer

\[a + ib = \frac{(x+i)^2}{2x^2+1} = \frac{x^2+i^2+2xi}{2x^2+1} = \frac{x^2-1+i2x}{2x^2+1} = \frac{x^2-1}{2x^2+1} + i\left(\frac{2x}{2x^2+1}\right)\]

On comparing real and imaginary parts, we obtain

\[a = \frac{x^2-1}{2x^2+1} \quad \text{and} \quad b = \frac{2x}{2x^2+1}\]

\[\therefore a^2 + b^2 = \left(\frac{x^2-1}{2x^2+1}\right)^2 + \left(\frac{2x}{2x^2+1}\right)^2\]

\[= \frac{x^4+1-2x^2+4x^2}{(2x+1)^2}\]

\[= \frac{x^4+1+2x^2}{(2x^2+1)^2}\]

\[= \frac{(x^2+1)^2}{(2x^2+1)^2}\]

\[\therefore a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}\]

Hence, proved.

**Question 12:**

Let \(z_1 = 2 - i, z_2 = -2 + i\). Find
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(i) \( \text{Re} \left( \frac{z_1 z_2}{z_1} \right) \),  (ii) \( \text{Im} \left( \frac{1}{z_1 \overline{z}_1} \right) \)

Answer

\( z_1 = 2 - i, \ z_2 = -2 + i \)

(i) \( z_1 z_2 = (2 - i)(-2 + i) = -4 + 2i + 2i - i^2 = -4 + 4i - (-1) = -3 + 4i \)

\( \overline{z}_1 = 2 + i \)

\( \therefore \frac{z_1 z_2}{\overline{z}_1} = \frac{-3 + 4i}{2 + i} \)

On multiplying numerator and denominator by \((2 - i)\), we obtain

\[
\frac{z_1 z_2}{\overline{z}_1} = \frac{(2 - i)(-3 + 4i)(2 - i)}{(2 + i)(2 - i)} = \frac{-6 + 3i + 8i - 4i^2}{2^2 + 1^2} = \frac{-6 + 11i - 4(-1)}{2^2 + 1^2} = \frac{-2 + 11i}{5} = \frac{-2}{5} + \frac{11i}{5}
\]

On comparing real parts, we obtain

\[
\text{Re} \left( \frac{z_1 z_2}{z_1} \right) = \frac{-2}{5}
\]

(ii) \( \frac{1}{z_1 \overline{z}_1} = \frac{1}{(2 - i)(2 + i)} = \frac{1}{(2)^2 + (1)^2} = \frac{1}{5} \)

On comparing imaginary parts, we obtain

\[
\text{Im} \left( \frac{1}{z_1 \overline{z}_1} \right) = 0
\]

Question 13:

Find the modulus and argument of the complex number \( \frac{1 + 2i}{1 - 3i} \).

Answer

Let \( z = \frac{1 + 2i}{1 - 3i} \), then
On squaring and adding, we obtain

\[ r^2 \left( \cos^2 \theta + \sin^2 \theta \right) = \left( -\frac{1}{2} \right)^2 + \left( \frac{1}{2} \right)^2 \]

\[ \Rightarrow r^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \]

\[ \Rightarrow r = \frac{1}{\sqrt{2}} \]  

[Conventionally, \( r > 0 \)]

\[ . \quad \frac{1}{\sqrt{2}} \cos \theta = -\frac{1}{2} \quad \text{and} \quad \frac{1}{\sqrt{2}} \sin \theta = \frac{1}{2} \]

\[ \Rightarrow \cos \theta = -\frac{1}{\sqrt{2}} \quad \text{and} \quad \sin \theta = \frac{1}{\sqrt{2}} \]

\[ . \quad \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \]  

[As \( \theta \) lies in the II quadrant]

Therefore, the modulus and argument of the given complex number are \( \frac{1}{\sqrt{2}} \) and \( \frac{3\pi}{4} \) respectively.

**Question 14:**

Find the real numbers \( x \) and \( y \) if \((x - iy)(3 + 5i)\) is the conjugate of \(-6 - 24i\).

**Answer**

Let \( z = (x - iy)(3 + 5i) \)

\[ z = 3x + 5xi - 3yi - 5y i^2 = 3x + 5xi - 3yi + 5y = (3x + 5y) + i(5x - 3y) \]

\[ . \quad \bar{z} = (3x + 5y) - i(5x - 3y) \]

It is given that, \( \bar{z} = -6 - 24i \)
Equating real and imaginary parts, we obtain

3x + 5y = −6 \quad \ldots \text{(i)}

5x − 3y = 24 \quad \ldots \text{(ii)}

Multiplying equation (i) by 3 and equation (ii) by 5 and then adding them, we obtain

\[
9x + 15y = -18 \\
25x - 15y = 120 \\
\frac{34x}{34} = 102
\]

\[
\therefore x = \frac{102}{34} = 3
\]

Putting the value of x in equation (i), we obtain

\[
3(3) + 5y = -6 \\
\Rightarrow 5y = -6 - 9 = -15 \\
\Rightarrow y = -3
\]

Thus, the values of x and y are 3 and -3 respectively.

Question 15:

Find the modulus of \(\frac{1+i}{1-i} \cdot \frac{1-i}{1+i}\).

Answer

\[
\frac{1+i}{1-i} \cdot \frac{1-i}{1+i} = \frac{(1+i)^2 - (1-i)^2}{(1-i)(1+i)} \\
= \frac{1+i^2 + 2i - 1 - i^2 + 2i}{1^2 + 1^2} \\
= \frac{4i}{2} = 2i
\]

\[
\therefore \left|\frac{1+i}{1-i} \cdot \frac{1-i}{1+i}\right| = |2i| = \sqrt{2^2} = 2
\]

Question 16:
If \((x + iy)^3 = u + iv\), then show that \(\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)\)

Answer

\[
(x + iy)^3 = u + iv
\]

\[
\Rightarrow x^3 + (iy)^3 + 3x \cdot iy (x + iy) = u + iv
\]

\[
\Rightarrow x^3 + i^2 y^3 + 3x^2y^2i + 3xy^2i^2 = u + iv
\]

\[
\Rightarrow x^3 - iy^3 + 3x^2yi - 3xy^2 = u + iv
\]

\[
\Rightarrow (x^3 - 3xy^2) + i(3x^2y - y^3) = u + iv
\]

On equating real and imaginary parts, we obtain

\[
u = x^3 - 3xy^2, \ v = 3x^2y - y^3
\]

\[
\therefore \frac{u}{x} + \frac{v}{y} = \frac{x^3 - 3xy^2}{x} + \frac{3x^2y - y^3}{y}
\]

\[
= \frac{x(x^2 - 3y^2)}{x} + \frac{y(3x^2 - y^2)}{y}
\]

\[
= x^2 - 3y^2 + 3x^2 - y^2
\]

\[
= 4x^2 - 4y^2
\]

\[
= 4(x^2 - y^2)
\]

\[
\therefore \frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)
\]

Hence, proved.

Question 17:

If \(\alpha\) and \(\beta\) are different complex numbers with \(|\beta| = 1\), then find \(|\beta - \alpha| \div |1 - \bar{\alpha}\beta|\).

Answer

Let \(\alpha = a + ib\) and \(\beta = x + iy\)

It is given that, \(|\beta| = 1\)

\[
\therefore \sqrt{x^2 + y^2} = 1
\]

\[
\Rightarrow x^2 + y^2 = 1 \quad \text{... (i)}
\]
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\[
\frac{\beta - \alpha}{1 - \bar{\alpha}\beta} = \frac{(x + iy) - (a + ib)}{1 - (a - ib)(x + iy)}
\]

\[
= \frac{(x - a) + i(y - b)}{1 - (ax + aiy - ibx + by)}
\]

\[
= \frac{(x - a) + i(y - b)}{(1 - ax - by) + i(bx - ay)}
\]

\[
= \frac{|(x - a) + i(y - b)|}{(1 - ax - by)^2 + (bx - ay)^2}
\]

\[
= \frac{\sqrt{x^2 + a^2 - 2ax + y^2 + b^2 - 2by}}{\sqrt{1 + a^2 x^2 + b^2 y^2 - 2ax + 2aby - b^2 x^2 + a^2 y^2 - 2abxy}}
\]

\[
= \frac{\sqrt{(x^2 + y^2) + a^2 + b^2 - 2ax - 2by}}{\sqrt{1 + a^2 (x^2 + y^2) + b^2 (y^2 + x^2) - 2ax - 2by}}
\]

\[
= \frac{\sqrt{1 + a^2 + b^2 - 2ax - 2by}}{\sqrt{1 + a^2 + b^2 - 2ax - 2by}}
\]

\[
\Rightarrow \left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right| = 1
\]

Question 18:

Find the number of non-zero integral solutions of the equation \( |1 - i|^x = 2^x \).

Answer
Thus, 0 is the only integral solution of the given equation. Therefore, the number of non-zero integral solutions of the given equation is 0.

Question 19:
If \((a + ib) (c + id) (e + if) (g + ih) = A + iB\), then show that
\((a^2 + b^2) (c^2 + d^2) (e^2 + f^2) (g^2 + h^2) = A^2 + B^2\).

Answer
\[
(a + ib) (c + id) (e + if) (g + ih) = A + iB
\]
\[
\Rightarrow \sqrt{(a^2 + b^2) (c^2 + d^2) (e^2 + f^2) (g^2 + h^2)} = |A + iB|
\]
\[
\Rightarrow \sqrt{a^2 + b^2} \times \sqrt{c^2 + d^2} \times \sqrt{e^2 + f^2} \times \sqrt{g^2 + h^2} = \sqrt{A^2 + B^2}
\]
On squaring both sides, we obtain
\((a^2 + b^2) (c^2 + d^2) (e^2 + f^2) (g^2 + h^2) = A^2 + B^2\)
Hence, proved.

Question 20:

If \(\left(\frac{1+i}{1-i}\right)^n = 1\), then find the least positive integral value of \(m\).

Answer
Therefore, the least positive integer is 1.

Thus, the least positive integral value of \( m \) is 4 (= 4 \times 1).