SAMPLE PAPER CLASS XII

Issued By CBSE for 2014 Examinations & Onwards
Compiled By : OP Gupta [+91-9650 350 480 | +91-9718 240 480]
For more stuffs on Maths, please visit : www.theOPGupta.com

Time Allowed: 180 Minutes **Max. Marks: 100**

SECTION – A

Q01. Write the smallest equivalence relation R on set A = {1, 2, 3}.

Q02. If $|\vec{a}| = a$ then, find the value of $|\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2$.

Q03. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined to x-axis at angles 30^o and 120^o respectively, then write the value of $|\vec{a}+\vec{b}|$.

Q04. Find the sine of the angle between the line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ and the plane 2x – 2y + z = 5.

Q05. Evaluate : $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$.

Q06. If $A=\begin{pmatrix}4 & 6\\ 7 & 5\end{pmatrix}$, then what is the value of A.(adj. A)?

Q07. For what value of k, the matrix $\begin{pmatrix}2k+3 & 4 & 5\\ -4 & 0 & -6\\ -5 & 6 & -2k-3\end{pmatrix}$ is a skew-symmetric matrix?

Q08. If $\begin{vmatrix}\sin\alpha & \cos\beta\\ \cos\alpha & \sin\beta\end{vmatrix}=\frac{1}{2}$, where α and β are acute angles then, write the value of $\alpha+\beta$.

Q09. If $\int_0^1 (3x^2+2x+k)dx = 0$, write the value of k.

Q10. Evaluate : $\int_{ln2}^{ln3} e^x\,dx$.

SECTION – B

Q11. Let S be the set of all rational numbers except 1 and * be defined on S by a*b = a + b – ab, $\forall a, b \in S$. Prove that :
a) * is a binary on S.
b) * is commutative as well as associative. Also find the identity element of *.

Q12. If a + b + c ≠ 0 and $\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}$ = 0, then using properties of determinants, prove that a = b = c.

Q13. Evaluate : $\int (2\sin 2x-\cos x)\sqrt{6-\cos^2 x-4\sin x}\,dx$ OR Evaluate : $\int \frac{5x}{(x+1)(x^2+9)}dx$.

Q14. Find a unit vector perpendicular to the plane of triangle ABC where the vertices are A(3,–1, 2), B(1,–1,–3) and C(4,–3, 1).
OR Find the value of λ, if the points with position vectors $3\hat{i}-2\hat{j}-\hat{k}$, $2\hat{i}+3\hat{j}-4\hat{k}$, $\hat{j}-\hat{i}+2\hat{k}$ and $4\hat{i}+5\hat{j}+\lambda\hat{k}$ are coplanar.

Q15. Show that the lines $\vec{r}=\hat{i}+\hat{j}-\hat{k}+p(3\hat{i}-\hat{j})$ and $\vec{r}=4\hat{i}-\hat{k}+q(2\hat{i}+3\hat{k})$ intersect. Also find their point of intersection.

Q16. Prove that : $\cot^{-1}7+\cot^{-1}8+\cot^{-1}18=\cot^{-1}3$.
OR Find the greatest and least value of $(\sin^{-1}x)^2+(\cos^{-1}x)^2$.

Q17. Show that the differential equation $xdy - ydx = \sqrt{x^2 + y^2}\,dx$ is homogeneous, and hence solve it.

Q18. Find the particular solution of differential equation $\cos x\, dy = \sin x\,(\cos x - 2y)dx$, given that $y = 0$ when $x = \pi/3$.

Q19. Out of a group of 8 highly qualified doctors in a hospital, 6 are very kind and cooperative with their patients and so are very popular, while the other two remain reserved. For a health camp, three doctors are selected at random. Find the probability distribution of the number of very popular doctors. What values are expected from the doctors?

Q20. Show that the function $g(x) = |x - 2|$, $x \in R$, is continuous but not differentiable at $x = 2$.

Q21. Differentiate $\log(x^{\sin x} + \cot^2 x)$ with respect to x.

Q22. Show that the curves $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other.

OR Separate the interval $[0, \pi/2]$ into sub-intervals in which $f(x) = \sin^4 x + \cos^4 x$ is increasing or decreasing.

SECTION – C

Q23. Find the equation of the plane through the points A(1, 1, 0), B(1, 2, 1) and C(–2, 2,–1) and hence find the distance between the plane and the line $\frac{x-6}{3} = \frac{y-3}{-1} = \frac{z+2}{1}$.

OR A plane meets the x, y and z axes at A, B and C respectively, such that the centroid of the triangle ABC is at (1,–2, 3). Find the vector and Cartesian equation of the plane.

Q24. A company manufactures two types of sweaters, type A and type B. It costs ₹360 to make one unit of type A and ₹120 to make a unit of type B. The company can make at most 300 sweaters and can spend at most ₹72000 a day. The number of sweaters of type A cannot exceed the number of type B by more than 100. The company makes a profit of ₹200 on each unit of type A but considering the difficulties of a common man the company charges a nominal profit of ₹20 on a unit of type B. Using LPP, solve the problem for maximum profit.

Q25. Evaluate : $\int_0^1 x(\tan^{-1} x)^2 dx$.

Q26. Using integration, find the area of the region $\{(x, y) : x^2 + y^2 \le 1 \le x + \frac{y}{2};\ x, y \in R\}$.

Q27. A shopkeeper sells three types of flower seeds A_1, A_2 and A_3. They are sold as a mixture where the proportions are 4:4:2 respectively. The germination rates of three types of seeds are 45%, 60% and 35%. Calculate the probability

(a) of a randomly chosen seed to germinate.

(b) that it is of type A_2, given that a randomly chosen seed doesn't germinate.

OR Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then two balls are drawn at random (without replacement) from Bag II. The balls so drawn are found to be both red in colour. Find the probability that the transferred ball is red.

Q28. Two schools A and B want to award their selected teachers on the values of honesty, hard work and regularity. The school A wants to award ₹x each, ₹y each and ₹z each for the three respective values to 3, 2 and 1 teachers with a total award money of ₹1.28 lakhs. School B wants to spend ₹1.54 lakhs to award its 4, 1 and 3 teachers on the respective values (by giving the same award money for the three values as before). If the total amount of award for one prize on each value is ₹57000, using matrices, find the award money for each value.

Q29. A given rectangular area is to be fenced off in a field whose length lies along a straight river. If no fencing is needed along the river, show that the least length of fencing will be required when length of the field is twice its breadth.

Q01. R = {(1, 1), (2, 2), (3, 3)} **Q02.** $2a^2$ **Q03.** $\sqrt{2}$ **Q04.** $\frac{1}{5\sqrt{2}}$

Q05. $-\frac{\pi}{3}$ **Q06.** $\begin{pmatrix}-22 & 0\\ 0 & -22\end{pmatrix}$ **Q07.** $k=-\frac{3}{2}$ **Q08.** $\frac{2\pi}{3}$

Q09. $k=-2$ **Q10.** 1

Q11. a) Let $a_1, a_2 \in S$. So, $a_1 * a_2 = a_1 + a_2 - a_1a_2$. Since a_1 and a_2 both are not equal to 1 that implies $(a_1-1)(a_2-1)\neq 0$ i.e., $a_1a_2 - a_1 - a_2 + 1 \neq 0$ or $a_1 + a_2 - a_1a_2 \neq 1$. So $a_1 * a_2 \in S$, hence * is a binary.

b) $a_1 * a_2 = a_1 + a_2 - a_1a_2 = a_2 + a_1 - a_2a_1 = a_2 * a_1$. So * is commutative.

Also, $(a_1 * a_2) * a_3 = (a_1 + a_2 - a_1a_2) * a_3 = a_1 + a_2 + a_3 - a_1a_2 - a_2a_3 - a_3a_1 + a_1a_2a_3$

And $a_1 * (a_2 * a_3) = a_1 * (a_2 + a_3 - a_2a_3) = a_1 + a_2 + a_3 - a_1a_2 - a_2a_3 - a_3a_1 + a_1a_2a_3 = (a_1 * a_2) * a_3$

So * is associative.

Let e be the identity.

Then a*e = a i.e., a + e – ae = a $\Rightarrow e(1-a)=0$ which implies e = 0 as $1-a\neq 0$.

Q12. $\Delta = \begin{vmatrix} a & b & c\\ b & c & a\\ c & a & b\end{vmatrix} = 0$. On simplifications using properties, we get :

$(a+b+c)(-a^2-b^2-c^2+ab+bc+ca)=0$ but $a+b+c\neq 0$ so, $(-a^2-b^2-c^2+ab+bc+ca)=0$

$\Rightarrow -\frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]=0 \Rightarrow a=b=c$.

Q13. Let $I=\int (2\sin 2x-\cos x)\sqrt{6-\cos^2 x-4\sin x}\,dx = \int (4\sin x-1)[\sqrt{\sin^2 x-4\sin x+5}]\cos x\,dx$

Put $\sin x = t \Rightarrow \cos x\,dx = dt$. So we get : $I=\int (4t-1)\sqrt{t^2-4t+5}\,dx$

$\therefore I=\frac{4}{3}[\sin^2 x-4\sin x+5]^{\frac{3}{2}}+\frac{7}{2}\left[(\sin x-2)\sqrt{\sin^2 x-4\sin x+5}+\log|\sin x-2+\sqrt{\sin^2 x-4\sin x+5}|\right]+C$

OR Let $I=\int \frac{5x}{(x+1)(x^2+9)}dx$. Consider $\frac{5x}{(x+1)(x^2+9)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+9}$

$\Rightarrow A=-\frac{1}{2}, B=\frac{1}{2}, C=\frac{9}{2}$. So, $I=-\frac{1}{2}\int\frac{dx}{x+1}+\frac{1}{2}\int\frac{x+9}{x^2+9}dx$

Hence, $I=-\frac{1}{2}\log|x+1|+\frac{1}{4}\log|x^2+9|+\frac{3}{2}\tan^{-1}\left(\frac{x}{3}\right)+C$.

Q14. A vector perpendicular to the plane of $\Delta ABC = \overrightarrow{AB}\times\overrightarrow{BC} = -10\hat{i}-7\hat{j}+4\hat{k}$ or $10\hat{i}+7\hat{j}-4\hat{k}$

So unit vector perpendicular to the plane of $\Delta ABC = \frac{\overrightarrow{AB}\times\overrightarrow{BC}}{|\overrightarrow{AB}\times\overrightarrow{BC}|}=\frac{1}{\sqrt{165}}(10\hat{i}+7\hat{j}-4\hat{k})$.

OR Let the points be A(3,–2,–1), B(2, 3,–4), C(–1, 1, 2) and D(4, 5, λ)

A, B, C, D will be coplanar iff $[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}]=0 \Rightarrow \lambda=-\frac{146}{17}$.

Q15. Let the coordinates of any random points on the two lines $\vec{r}=\hat{i}+\hat{j}-\hat{k}+p(3\hat{i}-\hat{j})$ and $\vec{r}=4\hat{i}-\hat{k}+q(2\hat{i}+3\hat{k})$ be P(1+3λ, 1–λ, –1) and Q(4+2μ, 0, 3μ–1) respectively. If these lines intersect, P and Q must coincide for some λ and μ.

Now 1+3λ = 4+2μ …(i), 1–λ = 0 …(ii), –1 = 3μ–1 …(iii).

Solving (ii) and (iii), we get : $\lambda = 1$, $\mu = 0$. Putting values of λ and μ in (i), we observe LHS = RHS. Hence the given lines intersect each other. And point of intersection is : P or Q (4, 0, –1).

Q16. LHS: $\cot^{-1}7+\cot^{-1}8+\cot^{-1}18=\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{8}+\tan^{-1}\frac{1}{18}=\tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\times\frac{1}{8}}\right)+\tan^{-1}\frac{1}{18}$

$$=\tan^{-1}\frac{3}{11}+\tan^{-1}\frac{1}{18}=\tan^{-1}\frac{\frac{3}{11}+\frac{1}{18}}{1-\frac{3}{11}\times\frac{1}{18}}=\tan^{-1}\frac{65}{195}=\tan^{-1}\frac{1}{3}=\cot^{-1}3=\text{RHS.}$$

OR We have $(\sin^{-1}x)^2+(\cos^{-1}x)^2=(\sin^{-1}x+\cos^{-1}x)^2-2\sin^{-1}x\cos^{-1}x$

$$\Rightarrow\ =\frac{\pi^2}{4}-2\sin^{-1}x\left(\frac{\pi}{2}-\sin^{-1}x\right)=2\left[(\sin^{-1}x)^2-\frac{\pi}{2}\sin^{-1}x+\frac{\pi^2}{8}\right]=2\left[\left(\sin^{-1}x-\frac{\pi}{4}\right)^2+\frac{\pi^2}{16}\right]$$

So least value = $2\left[\frac{\pi^2}{16}\right]=\frac{\pi^2}{8}$ and greatest value $=2\left[\left(-\frac{\pi}{2}-\frac{\pi}{4}\right)^2+\frac{\pi^2}{16}\right]=\frac{5\pi^2}{4}$.

Q17. We have $x\,dy-y\,dx=\sqrt{x^2+y^2}\,dx\Rightarrow\frac{dy}{dx}=\frac{y}{x}+\frac{\sqrt{x^2+y^2}}{x}=f(x,y)$ *say* …(i).

Put x = kx, y = ky in (i) : $f(kx,ky)=\frac{ky}{kx}+\frac{\sqrt{k^2x^2+k^2y^2}}{kx}=k^0\left[\frac{y}{x}+\frac{\sqrt{x^2+y^2}}{x}\right]=f(x,y)$

$\therefore$ Differential equation is homogeneous.

Now put $y = vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$. By (i), we get : $v+x\frac{dv}{dx}=\frac{vx}{x}+\frac{\sqrt{x^2+v^2x^2}}{x}$

i.e., $\frac{dv}{\sqrt{1+v^2}}=\frac{dx}{x}\Rightarrow\log|v+\sqrt{1+v^2}|=\log|x|+\log|C|\Rightarrow y+\sqrt{x^2+y^2}=Cx^2$.

Q18. We have cos x dy = sin x (cos x – 2y)dx $\Rightarrow\frac{dy}{dx}+(2\tan x)y=\sin x$

It is of the form $\frac{dy}{dx}+P(x)y=Q(x)$. $\therefore P(x)=2\tan x, Q(x)=\sin x$

I.F. $=e^{\int 2\tan x\,dx}=\sec^2x$. So solution is given by : $y(\sec^2x)=\int\sec^2x\sin x\,dx+C$

i.e., $y(\sec^2x)=\sec x+C$. And $\because$ y = 0 when $x=\frac{\pi}{3}$ so, $0\times\left(\sec^2\frac{\pi}{3}\right)=\sec\frac{\pi}{3}+C\Rightarrow C=-2$

Hence required solution is : $y=\cos x-2\cos^2x$.

Q19. Let X be the random variable representing the number of very popular doctors. So, X = 1, 2, 3.

X	1	2	3
P(X)	$\frac{^6C_1\times{}^2C_2}{^8C_3}=\frac{3}{28}$	$\frac{^6C_2\times{}^2C_1}{^8C_3}=\frac{15}{28}$	$\frac{^6C_3}{^8C_3}=\frac{10}{28}$

It is expected that a doctor must be qualified, kind and cooperative with the patients.

Q20. **Continuity at x = 2**

$$g(x)=|x-2|=\begin{cases}x-2, \text{ if } x\ge2\\2-x, \text{ if } x<2\end{cases}$$

LHL (at x = 2) : $\lim_{x\to2^-}(2-x)=2-2=0$

RHL (at x = 2) : $\lim_{x\to 2^+}(x-2) = 2-2 = 0$ and, g(2) = 2 – 2 = 0

Since LHL = RHL = $g(2)$ so, $g(x)$ is continuous at x = 2.

Differentiability at x = 2

LHD (at x = 2) : $\lim_{x\to 2^-}\frac{(2-x)-g(2)}{x-2} = \lim_{x\to 2^-}\frac{(2-x)-0}{x-2} = -1$

RHD (at x = 2) : $\lim_{x\to 2^+}\frac{(x-2)-g(2)}{x-2} = \lim_{x\to 2^+}\frac{(x-2)-0}{x-2} = 1$

Since LHD $\neq$ RHD so, $g(x)$ is not differentiable at x = 2.

Q21. Let y = $\log(x^{\sin x} + \cot^2 x) \Rightarrow \frac{dy}{dx} = \frac{1}{(x^{\sin x} + \cot^2 x)}\frac{d}{dx}(x^{\sin x} + \cot^2 x)\ldots(i)$

Let u = $x^{\sin x} \Rightarrow \frac{du}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \cos x \log x\right)$

Let v = $\cot^2 x \Rightarrow \frac{dv}{dx} = -2\cot x \operatorname{cosec}^2 x$

Substituting the values of du/dx and dv/dx in (i), we get :

$$\Rightarrow \frac{dy}{dx} = \frac{1}{(x^{\sin x} + \cot^2 x)}\left[x^{\sin x}\left(\frac{\sin x}{x} + \cos x \log x\right) - 2\cot x \operatorname{cosec}^2 x\right]$$

Q22. Solving $xy = a^2$ and $x^2 + y^2 = 2a^2$ we get their points of intersections as P(a, a) and Q(–a,–a).

Now $xy = a^2 \Rightarrow \frac{dy}{dx} = -\frac{y}{x} \Rightarrow \left.\frac{dy}{dx}\right]_{at\ P\,\&\,Q} = -1\ldots(i)$

And $x^2 + y^2 = 2a^2 \Rightarrow \frac{dy}{dx} = -\frac{x}{y} \Rightarrow \left.\frac{dy}{dx}\right]_{at\ P\,\&\,Q} = -1\ldots(ii)$

By (i) and (ii), it is clear that the curves touch each other.

OR $\quad f(x) = \sin^4 x + \cos^4 x \Rightarrow f'(x) = 4\sin^3 x\cos x + 4\cos^3 x(-\sin x) = -\sin 4x$

For $f'(x) = 0 \Rightarrow -\sin 4x = 0 \Rightarrow x = 0, \frac{\pi}{4}, \frac{\pi}{2} \in \left[0, \frac{\pi}{2}\right]$

As $f'(x) < 0$ in $\left(0, \frac{\pi}{4}\right)$ so, $f(x)$ is decreasing in $\left(0, \frac{\pi}{4}\right)$

And $f'(x) > 0$ in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ so, $f(x)$ is increasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

Q23. A vector perpendicular to the plane is parallel to $\overrightarrow{AB} \times \overrightarrow{BC} = -2\hat{i} - 3\hat{j} + 3\hat{k}$ or $2\hat{i} + 3\hat{j} - 3\hat{k} = \vec{m}$ *say*.

So, equation of plane is : $\vec{r}.(2\hat{i} + 3\hat{j} - 3\hat{k}) = (\hat{i} + \hat{j}).(2\hat{i} + 3\hat{j} - 3\hat{k})$ i.e., $\vec{r}.(2\hat{i} + 3\hat{j} - 3\hat{k}) = 5$ ...(i)

Given line is $\frac{x-6}{3} = \frac{y-3}{-1} = \frac{z+2}{1}$ so, $\vec{b} = 3\hat{i} - \hat{j} + \hat{k}$

As $\vec{m}.\vec{b} = (2\hat{i} + 3\hat{j} - 3\hat{k}).(3\hat{i} - \hat{j} + \hat{k}) = 0$ so, the given line is parallel to the plane (i).

Now distance between the point on the line (6, 3,–2) and the plane (i) is :

$$d = \frac{|2\times 6 + 3\times 3 - 3(-2) - 5|}{\sqrt{2^2 + 3^2 + (-3)^2}} = \sqrt{22}\text{ units}.$$

OR $\quad$ Let the coordinates of points A, B and C be (a, 0, 0), (0, b, 0) and (0, 0, c) respectively.

$\therefore$ Equation of plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1\ldots(i)$

Now centroid of triangle ABC is (1,–2, 3) = $\left(\frac{a+0+0}{3}, \frac{0+b+0}{3}, \frac{0+0+c}{3}\right) \Rightarrow a = 3, b = -6, c = 9$

Substituting the values of a, b and c in (i), we get : $6x - 3y + 2z - 18 = 0$ and $\vec{r}.(6\hat{i} - 3\hat{j} + 2\hat{k}) = 18$.

Q24. Let the company manufactures x and y sweaters of type A and type B respectively.
To maximize : Z = ₹(200x + 20y)
Subject to constraints : x, y $\geq$ 0; x + y $\leq$ 300, 360x + 120y $\leq$ 72000, x – y $\leq$ 100.

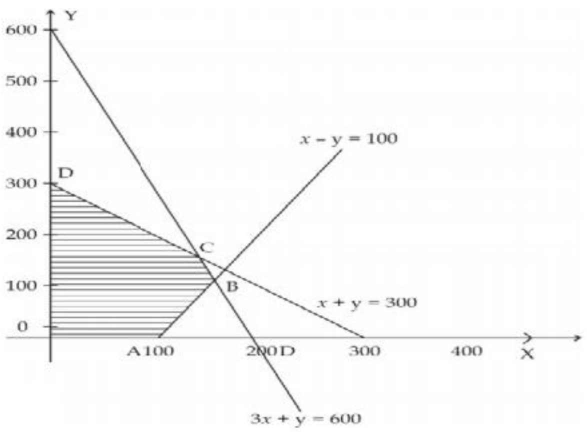

Diagram for Q24

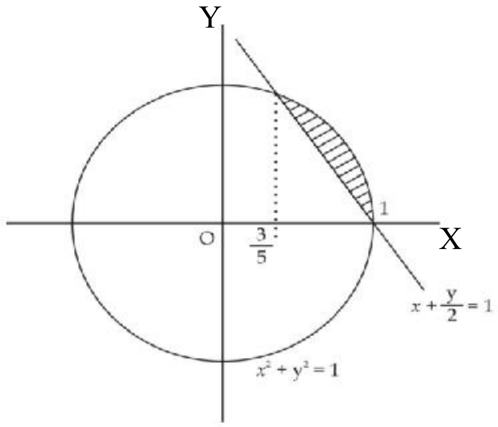

Diagram for Q26

Vertices of the feasible region are A(100, 0), B(175, 75), C(150, 150), D(0, 300).
Maximum profit is obtained at (175, 75).
And maximum value of profit is ₹36500.

Q25. Let $I=\int_0^1 (\tan^{-1}x)^2 x\,dx = \left[(\tan^{-1}x)^2\frac{x^2}{2}\right]_0^1 - \int_0^1 2\tan^{-1}x\frac{1}{1+x^2}\frac{x^2}{2}dx = \frac{\pi^2}{32}-\int_0^1 \tan^{-1}x\frac{x^2}{1+x^2}dx$

[By putting $x=\tan\theta \Rightarrow dx=\sec^2\theta d\theta$. Also when $x=0\Rightarrow\theta=0$ & when $x=1\Rightarrow\theta=\frac{\pi}{4}$]

i.e., $I=\frac{\pi^2}{32}-\int_0^{\pi/4}\theta\tan^2\theta\,d\theta \Rightarrow I=\frac{\pi^2}{32}-\int_0^{\pi/4}\theta\sec^2\theta\,d\theta+\int_0^{\pi/4}\theta\,d\theta$

$=\frac{\pi^2}{32}-\left[\theta\tan\theta\right]_0^{\pi/4}+\int_0^{\pi/4}\tan\theta d\theta+\left[\frac{\theta^2}{2}\right]_0^{\pi/4}$

$\Rightarrow\ =\frac{\pi^2}{32}-\frac{\pi}{4}+\left[\log|\sec\theta|\right]_0^{\pi/4}+\frac{\pi^2}{32}=\frac{\pi^2-4\pi}{16}+\frac{1}{2}\log 2.$

Q26. Solving $x^2+y^2=1$ and $x+y/2=1$ we get : x = 3/5 & x = 1 as the abscissas of point of intersections. See diagram given above.

Required Area = $\int_{3/5}^{1}\sqrt{1-x^2}\,dx-\int_{3/5}^{1}(2-2x)\,dx$

$=\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]_{3/5}^{1}-\left[2x-x^2\right]_{3/5}^{1}$

$=\left[\frac{\pi}{4}-\frac{2}{5}-\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)\right]$sq.units

Q27. Let E_1 : randomly selected seed is of type A_1, E_2 : randomly selected seed is of type A_2,
and E_3 : randomly selected seed is of type A_3.
$\therefore$ $P(E_1)=4/10$, $P(E_2)=4/10$, $P(E_3)=2/10$.
(i) Let A : selected seed germinates.
$\therefore P(A|E_1)=45/100$, $P(A|E_2)=60/100$, $P(A|E_3)=35/100$
So, $P(A)=P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)=49/100$ or 0.49
(ii) Let A : selected seed does not germinates.
$\therefore P(A|E_1)=55/100$, $P(A|E_2)=40/100$, $P(A|E_3)=65/100$

So, $P(E_2|A)=\frac{P(E_2)\,P(A|E_2)}{P(E_1)\,P(A|E_1)+P(E_2)\,P(A|E_2)+P(E_3)\,P(A|E_3)}=\frac{16}{51}$

OR Let E_1 : transferred ball is red, E_2 : transferred ball is black and A : getting both red balls from 2nd bag (after transfer). $\therefore$ $P(E_1)=3/7$, $P(E_2)=4/7$

Also $P(A|E_1) = \frac{{}^5C_2}{{}^{10}C_2} = \frac{10}{45}$ and $P(A|E_2) = \frac{{}^4C_2}{{}^{10}C_2} = \frac{6}{45}$.

So, $P(E_1|A) = \frac{P(E_1)\, P(A\,|\,E_1)}{P(E_1)\, P(A\,|\,E_1) + P(E_2)\, P(A\,|\,E_2)} = \frac{5}{9}$.

Q28. The three equations are : $3x + 2y + z = 1.28$, $4x + y + 3z = 1.54$, $x + y + z = 0.57$.

$$\Rightarrow \begin{pmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1.28 \\ 1.54 \\ 0.57 \end{pmatrix} \text{ i.e., } AX = B \qquad \Rightarrow |A| = -5 \text{ and } X = A^{-1}B$$

$$A^{-1} = \frac{1}{5}\begin{pmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{pmatrix} \qquad \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{pmatrix}\begin{pmatrix} 1.28 \\ 1.54 \\ 0.57 \end{pmatrix} = \begin{pmatrix} 0.25 \\ 0.21 \\ 0.11 \end{pmatrix}$$

$\therefore$ $x = 25000$, $y = 21000$, $z = 11000$.

Q29. Let length be x m and breadth be y m.

$\therefore$ length of fence, $L = x + 2y$

Let given area, $A = xy \Rightarrow y = A/x$

So, $L = x + 2A/x \qquad \Rightarrow \frac{dL}{dx} = 1 - \frac{2A}{x^2}$, $\frac{d^2L}{dx^2} = \frac{4A}{x^3}$

For points of local maxima and minima, $dL/dx = 0 \Rightarrow x = \sqrt{2A}$

$\left.\frac{d^2L}{dx^2}\right]_{at\ x=\sqrt{2A}} = \frac{4A}{(\sqrt{2A})^3} > 0 \qquad \therefore$ L is minimum at $x = \sqrt{2A}$.

Also minimum length = $= \sqrt{2A} + \frac{2A}{\sqrt{2A}} = 2\sqrt{2A}$

So $x = \sqrt{2A}$ and $y = \sqrt{\frac{A}{2}}$ which implies, $x = 2\,y$.

Good Luck & God Bless You!!!