CBSE 2012 FOREIGN ANNUAL EXAMINATION
(Set - 1)

Max. Marks: 100 **Time Allowed: 3 Hours**

SECTION – A
(Question numbers 01 to 10 carry one mark each.)

Q01. Write the value of $\cot\left(\tan^{-1} a + \cot^{-1} a\right)$.

Q02. If $\int\left(\frac{x-1}{x^2}\right)e^x dx = f(x)e^x + c$, then write the value of $f(x)$.

Q03. If A is a square matrix such that $A^2 = A$, then write the value of $(I + A)^2 - 3A$.

Q04. Write the value of $(\hat{i}\times\hat{j}).\hat{k} + (\hat{j}\times\hat{k}).\hat{i}$.

Q05. Write the value of the area of the parallelogram determined by the vectors $2\hat{i}$ and $3\hat{j}$.

Q06. Write the direction cosines of a line parallel to z-axis.

Q07. If $x\begin{bmatrix}2\\3\end{bmatrix} + y\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}10\\5\end{bmatrix}$, write the value of x.

Q08. Write the value of the determinant: $\begin{vmatrix}102 & 18 & 36\\ 1 & 3 & 4\\ 17 & 3 & 6\end{vmatrix}$.

Q09. If the binary operation * on the set Z of integers is defined by $a * b = a + b - 5$, then write the identity element for the operation * in Z.

Q10. If $\int_0^a 3x^2 dx = 8$, then write the value of a.

SECTION – B
(Question numbers 11 to 22 carry four marks each.)

Q11. If $x^m y^n = (x+y)^{m+n}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

Q12. Prove that: $\sin^{-1}\left(\frac{63}{65}\right) = \sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right)$.

OR Solve for x: $2\tan^{-1}(\sin x) = \tan^{-1}(2\sec x)$, $x \neq \frac{\pi}{2}$.

Q13. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of the number of successes and hence find its mean.

Q14. If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that $fof(x) = x$ for all $x \neq \frac{2}{3}$. What is the inverse of f ?

Q15. Using properties of determinants, show that: $\begin{vmatrix}a & a+b & a+b+c\\ 2a & 3a+2b & 4a+3b+2c\\ 3a & 6a+3b & 10a+6b+3c\end{vmatrix} = a^3$.

Q16. Solve the differential equation: $3e^x \tan y\, dx + (2-e^x)\sec^2 y\, dy = 0$, given that when $x = 0$, $y = \frac{\pi}{4}$.

Q17. Find the vector and Cartesian equations of the line passing through the point P(1, 2, 3) and parallel to the planes $\vec{r}.(\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r}.(3\hat{i} + \hat{j} + \hat{k}) = 6$.

Q18. If $\vec{\alpha} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{\beta} = 2\hat{i} + \hat{j} - 4\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$ where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

Q19. Evaluate: $\int x^2 \tan^{-1} x\, dx$. **OR** Evaluate: $\int \frac{3x-1}{(x+2)^2} dx$.

Q20. Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$.

OR Show that the function $y = \log(1+x) - \frac{2x}{2+x}$, $x > -1$, is an increasing function of x throughout its domain.

Q21. Solve the differential equation given as: $\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right]\frac{dx}{dy} = 1,\ x \neq 0$.

Q22. If $y = e^{a\cos^{-1}x}$, $-1 \le x \le 1$, show that: If $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$.

OR If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, $-1 < x < 1$, $x \neq y$, then prove that: $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$.

SECTION – C

(Question numbers 23 to 29 carry six marks each.)

Q23. Find the area of the region in the first quadrant enclosed by x-axis, the line $x = \sqrt{3}\,y$ and the circle $x^2 + y^2 = 4$.

Q24. A company produces soft drinks that has a contract which requires that a minimum of 80 units of the chemical A and 60 units of the chemical B go into each bottle of the drink. The chemicals are available in prepared mix packets from two different suppliers. Supplier S had a packet of mix of 4 units of A and 2 units of B that costs ₹10. The supplier T has a packet of mix of 1 unit of A and 1 unit of B that costs ₹4. How many packets of mixes from S and T should the company purchase to honour the contract requirement and yet maintain the minimum cost? Make a LPP and solve graphically.

Q25. In a certain college, 4% of boys and 1% of girls are taller than 1.75 metres. Furthermore, 60% of the students in the college are girls. A student is selected at random from the college and is found to be taller thane 1.75 metres. Find the probability that the selected student is a girl.

Q26. Using matrices, solve the following system of equations:

$$x - y + z = 4,\ 2x + y - 3z = 0,\ x + y + z = 2.$$

OR If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.

Q27. Evaluate $\int_1^3 \left(x^2 + x\right)dx$ as a limit of a sum.

OR Evaluate: $\int_0^{\pi/4} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x}\,dx$.

Q28. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius R is $\frac{4R}{3}$.

Q29. Find the vector equation of the plane passing through the points (2, 1,–1) and (–1, 3, 4) and perpendicular to the plane $x - 2y + 4z = 10$. Also show that the plane thus obtained contains the line $\vec{r} = -\hat{i} + 3\hat{j} + 4\hat{k} + \lambda(3\hat{i} - 2\hat{j} - 5\hat{k})$.

ANSWERS OF CBSE 2012 FOREIGN – Set 1

Q01. 0 **Q02.** $\frac{1}{x}$ **Q03.** I

Q04. 2 **Q05.** 6 sq.units **Q06.** 0,0,1 **Q07.** 3

Q08. 0 **Q09.** $e = 5$ **Q10.** 2

Q12. **OR** $\frac{\pi}{4}$

Q13.

X	0	1	2	3	4
P(X)	625/1296	500/1296	150/1296	20/1296	1/1296

Mean = $\sum X.P(X) = \frac{2}{3}$

Q14. $f^{-1}(x) = \frac{4x+3}{6x-4}$ **Q16.** $\tan y = \frac{(2-e^x)^3}{8}$

Q17. Cartesian equation: $\frac{1-x}{3} = \frac{y-2}{5} = \frac{z-3}{4}$, Vector equation: $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(5\hat{j} - 3\hat{i} + 4\hat{k})$

Q18. $2\hat{i} + \hat{j} - 4\hat{k} = \left(-\frac{3}{5}\hat{i} - \frac{4}{5}\hat{j} - \hat{k}\right) + \left(\frac{13}{5}\hat{i} + \frac{9}{5}\hat{j} - 3\hat{k}\right)$

Q19. $\frac{1}{3}x^3 \tan^{-1} x - \frac{1}{6}x^2 + \frac{1}{6}\log|1+x^2| + k$ **OR** $3\log|x+2| + \frac{7}{x+2} + k$

Q20. $2x + 3my = am^2(2+3m^2)$ **Q21.** $y\,e^{2\sqrt{x}} = 2\sqrt{x} + k$ **Q23.** $\frac{\pi}{3}$sq.units

Q24. Min.Cost = Rs.260 for 10 packets of mixes from S and 40 packets of mixes from T.

Q25. $\frac{3}{11}$ **Q26.** $x = 2, y = -1, z = 1$ **OR** $\begin{pmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix}$

Q27. $\frac{38}{3}$ **OR** $\frac{\pi}{6}$ **Q29.** $\vec{r}.(18\hat{i} + 17\hat{j} + 4\hat{k}) = 49$