CBSE 2010 DELHI ANNUAL EXAMINATION
(Set - 1, 2, 3)

Max. Marks: 100 **Time Allowed: 3 Hours**

General Instructions:
Same as given in the previous question paper.

SECTION – A
(Question numbers 01 to 10 carry one mark each.)

Q01. Find the value of $\int \frac{\log x}{x}\,dx$.

Q02. **(Set I)** What is the principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$?

(Set II) What is the principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$?

(Set III) Find the principle value of $\sin^{-1}\left(-\frac{1}{2}\right)+\cos^{-1}\left(-\frac{1}{2}\right)$.

Q03. If $A=\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$, then for what value of α is A an identity matrix?

Q04. What is the cosine of the angle which the vector $\sqrt{2}\,\hat{i}+\hat{j}+\hat{k}$ makes with y-axis?

Q05. Write a vector of magnitude 15 units in the direction of vector $\hat{i}-2\hat{j}+2\hat{k}$.

Q06. What is the range of the function $f(x)=\frac{|x-1|}{(x-1)}$?

Q07. **(Set I)** What is the value of the determinant $\begin{vmatrix}0 & 2 & 0\\ 2 & 3 & 4\\ 4 & 5 & 6\end{vmatrix}$?

(Set II) Find the minor of the element of second row and third column (a_{23}) in: $\begin{vmatrix}2 & -3 & 5\\ 6 & 0 & 4\\ 1 & 5 & -7\end{vmatrix}$.

(Set III) If A is a square matrix of order 3 and |3A| = k |A|, then write the value of k.

Q08. Write the vector equation of the line: $\frac{x-5}{3}=\frac{y+4}{7}=\frac{6-z}{2}$.

Q09. What is the degree of the differential equation: $5x\left(\frac{dy}{dx}\right)^2-\frac{d^2y}{dx^2}-6y=\log x$?

Q10. If $\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}\begin{pmatrix}3 & 1\\ 2 & 5\end{pmatrix}=\begin{pmatrix}7 & 11\\ k & 23\end{pmatrix}$, then write the value of k.

SECTION – B
(Question numbers 11 to 22 carry four marks each.)

Q11. **(Set I)** Show that the function f defined as follows, is continuous at $x=2$ but not differentiable:

$$f(x)=\begin{cases}3x-2, & 0<x\le 1\\ 2x^2-x, & 1<x\le 2\\ 5x-4, & x>2\end{cases}.$$

OR Find $\frac{dy}{dx}$, if $y = \sin^{-1}[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}]$.

(Set II) Find all points of discontinuity of f, where f is defined as: $f(x) = \begin{cases} |x|+3, & x \le -3 \\ -2x, & -3 < x < 3 \\ 6x+2, & x \ge 3 \end{cases}$.

OR Find $\frac{dy}{dx}$, if $y = (\cos x)^x + (\sin x)^{\frac{1}{x}}$.

Q12. **(Set I, II)** Prove that: $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$, $x \in (0,1)$.

OR Prove that: $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$.

(Set III) Prove that: $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$

OR If $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$, find the value of x.

Q13. **(Set I, II)** On a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

(Set III) There are two bags, Bag I and Bag II. Bag I contains 4 white and 3 red balls while the other bag contains 3 white and 7 red balls. One ball is drawn at random from one of the bags and it is found to be white. Find the probability that it was drawn from Bag I.

Q14. **(Set I)** Let Z be the set of all integers and R be the relation on Z defined as $R = \{(a,b) : a,b \in Z \text{ and } (a-b) \text{ is divisible by } 5\}$. Prove that R is an equivalence relation.

(Set II) Let $*$ be a binary operation on Q defined by $a * b = \frac{3ab}{5}$. Show that the operation $*$ is commutative as well as associative. Also find its identity element, if it exists.

(Set III) Show that the relation S in the set R of real numbers, defined as $S = \{(a,b) : a,b \in R \text{ and } a \le b^3\}$ is neither reflexive, nor symmetric nor transitive

Q15. Using elementary row operations, find inverse of matrix: $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$.

Q16. **(Set I)** Evaluate: $\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\,dx$. **(Set II)** Evaluate: $\int_0^{\pi} \frac{x}{1+\sin x}\,dx$.

Q17. Find the Cartesian equation of the plane passing through the points $A(0,0,0)$ and $B(3,-1,2)$ and parallel to the line $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$.

Q18. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ respectively, externally in the ratio $1:2$. Also, show that P is the midpoint of the line segment RQ.

Q19. Evaluate: $\int e^x \left(\frac{\sin 4x - 4}{1 - \cos 4x}\right)dx$. **OR** Evaluate: $\int \frac{1-x^2}{x(1-2x)}\,dx$.

Q20. **(Set I)** Find the point on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point.

(Set II) Find equations of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line: $x + 14y + 4 = 0$.

(Set III) Find the equation of tangent to the curve $y = \frac{x-7}{(x-2)(x-3)}$, at a point where it cuts x-axis.

Q21. Find the particular solution of the differential equation satisfying the given conditions: $x^2 dy + (xy + y^2)\,dx = 0$; $y = 1$ when $x = 1$.

Q22. Find the general solution of the differential equation: $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x$.

OR Find the particular solution of the following differential equation satisfying the given conditions: $\frac{dy}{dx}=y\tan x$; given that $y=1$ when $x=0$.

SECTION – C

(Question numbers 23 to 29 carry six marks each.)

Q23. **(Set I)** Using integration, find area of the region bounded by curve $x^2 = 4y$ and the line $x=4y-2$.

OR Evaluate: $\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx$.

(Set II) Using integration, find the area of the region: $\left\{(x,y):\frac{x^2}{9}+\frac{y^2}{4}\le 1\le\frac{x}{3}+\frac{y}{2}\right\}$.

OR Evaluate $\int_1^3\left(3x^2+2x\right)dx$ as the limit of sums.

(Set III) Evaluate $\int_1^4(x^2-x)\,dx$ as limit of sums.

OR Using integration, find the area of the region: $\left\{(x,y):|x-1|\le y\le\sqrt{5-x^2}\right\}$.

Q24. A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and 30 minutes to make a chain. Maximum number of hours available per day is 16. If the profit on a ring is ₹300 and that on a chain is ₹190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an L.P.P. and solve it graphically.

Q25. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to both clubs. Find the probability of the lost card being of clubs.

OR From a lot of 10 bulbs, which includes 3 defectives, a sample of 2 bulbs is drawn at random. Find the probability distribution of the number of defective bulbs.

Q26. Using properties of determinants, show that: $\begin{vmatrix}(b+c)^2 & ab & ca\\ ab & (a+c)^2 & bc\\ ca & bc & (a+b)^2\end{vmatrix}=2abc(a+b+c)^3$.

Q27. **(Set I, II)** Find the value(s) of x for which $f(x)=\left[x(x-2)\right]^2$ is an increasing function. Also find the points on the curve, where the tangent is parallel to x-axis.

(Set III) Find the intervals in which the function f given by $f(x)=\sin x-\cos x,\ 0\le x\le 2\pi$ is strictly increasing or strictly decreasing.

Q28. Show that the right circular cylinder, open at the top, and of given surface area and maximum volume is such that its height is equal to the radius of the base.

Q29. **(Set I)** The points A(4,5,10), B(2,3,4) and C(1,2,–1) are three vertices of a parallelogram ABCD. Find the vector equations of the sides AB and BC and also find the coordinates of point D.

(Set II) Write the vector equations of the lines given below and hence determine the distance between them: $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6};\ \frac{x-3}{4}=\frac{y-3}{6}=\frac{z+5}{12}$.

ANSWERS OF CBSE 2010 DELHI (Set - 1, 2, 3)

Q01. $\frac{(\log x)^2}{2}+k$ **Q02.** **Set I:** $-\frac{\pi}{3}$ **Set II:** $\frac{5\pi}{6}$ **Set III:** $\frac{\pi}{2}$ **Q03.** $\alpha = 0^o$

Q04. $\frac{1}{2}$ **Q05.** $5\hat{i}-10\hat{j}+10\hat{k}$ **Q06.** $\{-1, 1\}$

Q07. **Set I:** 8 **Set II: 13** **Set III: k = 27** **Q08.** $\vec{r}=5\hat{i}-4\hat{j}+6\hat{k}+\lambda(3\hat{i}+7\hat{j}-2\hat{k})$

Q09. Degree: 1 **Q10.** $k = 17$ **Q11.** **Set I: OR** $\frac{1}{\sqrt{1-x^2}}-\frac{1}{2\sqrt{x(1-x)}}$

Set II: Discontinuity at $x=3$ **OR** $\frac{dy}{dx}=(\cos x)^x\left[\log\cos x - x\tan x\right]+(\sin x)^{\frac{1}{x}}\left[\frac{\cot x}{x}-\frac{\log\sin x}{x^2}\right]$

Q12. **Set III: OR** $\pm\frac{1}{\sqrt{2}}$ **Q13.** **Set I, II:** $P(X\geq 4)=\frac{11}{243}$ **Set III:** $\frac{40}{61}$

Q14. **Set II:** Identity element: $\frac{5}{3}$ **Q15.** $\begin{pmatrix}3 & -5\\ -1 & 2\end{pmatrix}$ **Q16.** **Set I:** $2\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$

Set II: π **Q17.** $x-19y-11z=0$ **Q18.** $\overrightarrow{OR}=(3\vec{a}+5\vec{b})$

Q19. $e^x\cot 2x+k$ **OR** $\frac{x}{2}-\frac{3}{4}\log|1-2x|+\log|x|+k$ **Q20.** **Set I:** (0,0), (3,27)

Set II: $x+14y-254=0,\ x+14y+86=0$ **Set III:** $x-20y=7$

Q21. $3x^2y=y+2x$ **Q22.** $y\log x+\frac{2}{x}(1+\log x)=k$ **OR** $y=\sec x$

Q23. **Set I:** $\frac{9}{8}$sq.units **OR** $\frac{\pi}{2}(\pi-2)$ **Set II:** $\left(\frac{3\pi}{2}-3\right)$sq.units **OR** 34

Set III: $\frac{27}{2}$ **OR** $\left(\frac{5\pi-2}{4}\right)$sq.units **Q24.** 8 Gold rings, 16 Chains

Q25. $\frac{11}{50}$ **OR**

X	0	1	2
P(X)	7/15	7/15	1/15

Q27. **Set I, II:** Function is increasing for $0<x<1$ and $x>2$. Required points: (0,0), (1,1), (2,0)

Set III: Function is strictly increasing in $\left(0,\frac{3\pi}{4}\right)\cup\left(\frac{7\pi}{4},2\pi\right)$ and strictly decreasing in $\left(\frac{3\pi}{4},\frac{7\pi}{4}\right)$.

Q29. **Set I:** Vector eq. of AB: $\vec{r}=4\hat{i}+5\hat{j}+10\hat{k}+\lambda(2\hat{i}+2\hat{j}+6\hat{k})$,

Vector eq. of BC: $\vec{r}=2\hat{i}+3\hat{j}+4\hat{k}+\mu(\hat{i}+\hat{j}+5\hat{k})$. Coordinates of D are (3,4,5).

Set II: Vector eq. of line I: $\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$,

Vector eq. of line II: $\vec{r}=3\hat{i}+3\hat{j}-5\hat{k}+\mu(4\hat{i}+6\hat{j}+12\hat{k})$. Required distance: $\frac{\sqrt{293}}{7}$units.

Hii. Here is a short message I have to convey.

I've devoted myself for the service of Mathematics… to help the students in need in all possible ways. It will be a thing of pleasure for me if my work/collection serves any purpose in your life.

The **Solved CBSE Papers** of previous years will offer you a fair insight about the examination patterns over the years and the kind of questions which are asked in exams. This section has been prepared with utmost care. Still if you find **any mistake(s)/misprint(s)** or want to seek **any clarification(s)** regarding any solution(s) or make **any suggestion(s)**, please feel free to interact with me on my mobile **+91-9650 350 480** or get in touch with me at **theopgupta@gmail.com** any time!

For many other solved CBSE papers, please visit: **www.theOPGupta.com**

Wish You All The Very Best!

With lots of Love and Blessings!

- OP Gupta [Electronics & Communications Engineering, Indira Award Winner] ❑❑❑